V.I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

MATEMATIKA INSTITUTI

MUHIDDINOVA OQILA TULQIN QIZI

YUQORI TARTIBLI TENGLAMALAR UCHUN BOSHLANG‘ICH-CHEGARAVIY VA TESKARI MASALALAR

01.01.02 – Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

TOSHKENT – 2022

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Vazirlar Mahkamasi huzuridagi Oliy attestatsiya komissiyasida B2021.3.PhD/FM raqam bilan ro‘yxatga olingan.

Dissertatsiya V.I. Romanovskiy nomidagi Matematika institutida bajarilgan.

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Dissertatsiya bilan V.I.Romanovskiy nomidagi Matematika Institutining Axborot-resurs markazida tanishish mumkin (153-raqami bilan ro‘yxatga olingan). (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko‘chasi, 9-uy. Tel.: (+99871)-207-91-40.

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida olib borilayotgan ko‘plab ilmiy-amaliy tadqiqotlar ba’zi hollarda matematik fizikaning tenglamalari, masalan, sterjenlar, plastinalar va h.k. jismlarning tebranishlariga doir ko‘plab masalalar yuqori tartibli differensial tenglamalariga keltiriladi. Tabiiyki, bu kabi masalalarning fizik ma’nosidan kelib chiqib, xos funksiyalarning ko‘rinishi noma’lum bo‘lgan holida, ixtiyoriy sohalar uchun shunga o‘xshash masalalarni ko‘rib chiqish juda katta qiziqish uyg’otadi. Ushbu dissertatsiya ishi elliptik qismi ixtiyoriy tartibdagi differensial ifodaga ega bo‘lgan va ixtiyoriy ko‘p o‘lchamli yetarlicha silliq chegarali sohada aniqlangan klassik diffuziya va subdiffuziya differensial tenglamalari uchun to‘g’ri va teskari masalalarini klassik Furye metodi yordamida o‘rganishga bag’ishlangan.

Hozirgi kunda kasr tartibli va xususiy hosilali differensial tenglamalar nazariyasi bo‘yicha izlanishlarni rivojlantirish, mamlakatimiz taraqqiyotida muhim rol o‘ynaydi. So‘nggi yillarda mamlakatimiz iqtisodiy rivojlanishida muhim ahamiyatga ega bo‘lgan matematika, fizika, geologiya va biologiya fanlariga e’tibor borgan sari yanada kuchaymoqda. Xususan, matematikaning asosiy yo‘nalishlaridan biri bo‘lgan, amaliy xususiyatga ega bo‘lgan differensial tenglamalar va matematik fizika fanini rivojlantirishga alohida e’tibor qaratilmoqda. Shuningdek, ko‘pgina tadqiqotlar subdiffuziya tenglamalarining o‘ng tomonini aniqlash bo‘yicha teskari masalalarga keltiriladi. Lekin masalalarning barchasida subdiffuziya tenglamasining elliptik qismi oddiy differensial ifoda bo‘lgan holi qaralgan. Shuning uchun elliptik qismi ixtiyoriy ko‘p o‘lchamli sohalarda aniqlangan ixtiyoriy tartibdagi elliptik operatori qatnashgan parabolik, giperbolik hamda subdiffuziya tenglamalari uchun to‘g’ri va teskari masalalarning yechimlarini qurish maqsadli ilmiy tadqiqotlardan hisoblanadi.

Mamlakatimizda biologiya, geologiya, matematika va fizika kabi tabiiy va hayotiy fanlarni rivojlantirishga katta e’tibor qaratilmoqda. Jumladan, kasr tartibli va xususiy hosilali differensial tenglamalar nazariyasini ishlab chiqishga ham alohida e’tibor qaratilmoqda. Chunki bu nazariya mexanika, elektronika, boshqaruv nazariyasi, fiziologiya va biologik sistemalardagi ko‘plab jarayon va hodisalarni tushunishga yordam beradi. Yuqorida ta’kidlanganidek, muhim yo‘nalishlar bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish, matematika fanining asosiy vazifasi va faoliyat yo‘nalishi etib belgilandi[1]. Mamlakatimiz mutaxassislari tomonidan mazkur sohalarda salmoqli natijalar qo‘lga kiritilib, ular yuqoridagi farmon ijrosini ta’minlashda muhim ahamiyat kasb etmoqda. Mamlakatimizda differensial tenglamalar va matematik fizika kabi muhim yo‘nalishlari bo‘yicha xalqaro miqiyosda olib borilayotgan tadqiqotlar, fundamental tadqiqotlarning asosiy yo‘nalishi sifatida qaralayotgani quvonarlidir.

Mazkur dissertatsiya ishining predmeti va tadqiqot ob’yekti O‘zbekiston

[1] O’zbekiston Respublikasi Vazirlar Mahkamasining 2017 yil 18 maydagi «O’zbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy-tadqiqot muassasalari faoliyatini tashkil etish to’g’risida » gi № 292-sonli qarori.

Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-sonli "O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi" haqidagi farmonlarida belgilangan vazifalariga mos keladi, 2017 yil 17-fevraldagi PQ-2789 "Fanlar Akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g’risida"gi Prezident qarori, 2018 yil 27 apreldagi "Innovatsion g’oyalar, texnologiyalar va loyihalarni amaliy joriy qilish tizimini yanada takomillashtirish chora-tadbirlari to‘g’risida"gi PQ-3682-sonli qarori va 2019 yil 9 iyuldagi "Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akademiyasining V.I.Romanovskiy nomidagi matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g’risida"gi PQ-4387 sonli Prezident qarori hamda 2020 yil 7 maydagi "Matematika sohasidagi ta’lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora-tadbirlari to‘g’risida"gi PQ-4708 sonli Prezident qarori, bundan tashqari boshqa qonun va fundamental fanlarga doir boshqa qarorlar ijrosiga mos keladi.

Tadqiqotning Respublikada fan va texnologiyalari rivojlanishi ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot O‘zbekiston Respublikasi fan va texnalogiyalar rivojlanishining IV "Matematika, mexanika va informatika" ustuvor yo‘nalishi doirasida bajarildi.

Muammoning o‘rganilganlik darajasi. Boshlang’ich-chegaraviy masalalarning klassik yechimining mavjudligi va yagonaligi V.A.Il’in , O.A. Oleinik, O.A. Ladyzhinskiy, P.E. Sobolevskiy, M.A.Krasnoselskiy va boshqalar kabi ko‘plab mashhur matematiklar tomonidan o‘rganilgan. V.A. Il’inning 1960-yilda nashr etilgan fundamental ishida ushbu yo‘nalishdagi ishlarga sharh berilgan va muallifning so‘nggi natijalari taqdim etilgan. Bu yerda elliptik qismi ikkinchi tartibli simmetrik differensial ifoda bo‘lgan tenglamalarga klassik Furye usuli qo‘llanilgan. Elliptik qismi ikkinchidan yuqoriroq tartibga ega bo‘lgan holat Krasnoselskiy va boshqalarning mashhur monografiyasida ko‘rib chiqilgan. Bu ishda Furye usuli yordamida boshlang’ich-chegaraviy masalaning boshlang’ich berilganlari elliptik operator darajasining aniqlanish sohasiga tegishli bo‘lgan holda yechimning mavjudligi isbotlanadi. Shuni ta’kidlash kerakki, bir tomondan, ushbu monografiyada yechimning yagonaligi masalalari ko‘rib chiqilmagan, ikkinchi tomondan, ushbu monografiyada keltirilgan tasdiqlarning shartlarini tekshirish oson emas. Yana shuni ham ta’kidlash kerakki, ushbu monografiyada umumlashgan yechim tushunchasi kiritilmagan va ularning mavjudligi masalalari o‘rganilmagan.

Kasr hosilali differensial tenglamalar nazariyasi so‘nggi bir necha o‘n yilliklarda, asosan, fan va texnologiyaning ko‘plab bir-biridan yiroq ko‘rinadigan sohalarida qo‘llanilishi tufayli katta mashhurlik va ahamiyatga ega bo‘ldi. O‘z navbatida, nazariyaning yetarlicha ommalashganligi mutaxassislar e’tiborini tortdi, bu esa kasr tartibli differensial tenglamalarning matematik jihatlari va ularni yechish usullari bo‘yicha ko‘plab tadqiqotlarga sabab bo‘ldi. Subdiffuziya tenglamalari uchun boshlang’ich-chegaraviy masalalar ko‘plab mutaxassislar tomonidan o‘rganilgan. Ulardan xususan, A.A. Kilbasning maqolalaridagi tenglamalarning elliptik qismi asosan bir o‘lchovli differensial ifoda u_{xx} ko‘rinishda bo‘lgan holi o‘rganilgan. Laplas

operatori ishtirok etgan bunday tenglamalar A.V. Pskhu maqolalarida ko‘rib chiqilgan va R^N butun fazoda aniqlangan o‘zgarmas koeffitsientli psevdodifferensial operatorli tenglamalar S. Umarovning ishlarida o‘rganilgan. M. Ruzhansky, N. Tokmagambetov va B.T.Torebeklarning birgalikda yozgan maqolasida elliptik qismi sifatida o‘z-o‘ziga qo‘shma operator qaralgan, ammo bu maqolada faqat umumlashgan yechim uchun yechimning mavjudligi o‘rganilgan.

Shuningdek, ko‘pgina tadqiqotlar subdiffuziya tenglamalarining o‘ng tomonini aniqlash bo‘yicha teskari masalalarga bag’ishlangan. Bu maqolalarning deyarli barchasida subdiffuziya tenglamasining elliptik qismi oddiy differensial ifoda bo‘lgan holi o‘rganilgan. Bulardan istisno bo‘lgan maqolalar N.A. Asl va D.Rostamy, Z. Li, Y. Liu va M. Yamamoto, S.A. Malik va S. Aziz, W. Rundell va Z. Zhang, L. Sun, Y. Zhang va T. Weilarning maqolalari bo‘lib, ularda elliptik qismi Laplas operatori yoki ikkinchi tartibli operator bo‘lgan holi ko‘rilgan va ushbu maqolalarda umumlashgan yechim mavjudligi o‘rganilgan. M. Ruzhansky, N. Tokmagambetov va B.T. Torebeklarning maqolasida elliptik qismi diskret spektrga ega bo‘lgan ixtiyoriy elliptik operator bo‘lgan holi uchun Kaputo kasr hosilali subdiffuziya tenglamasining o‘ng tomonini aniqlash bo‘yicha teskari masalasi o‘rganib chiqilgan va mualliflar umumlashgan yechimning mavjudligi va yagonaligini Furye usuli yordamida isbotlagan. M. Ruzhansky, N.Tokmagambetov va B.T. Torebeklarning maqolasida va boshqa ko‘pgina maqolalarda, jumladan N.A. Asl va D. Rostamy, Z. Li, Y. Liu va M. Yamamoto, W. Rundell va Z. Zhang, L. Sun, Y. Zhang va T. Weilarning maqolalarida Kaputo hosilasi kasr hosila sifatida qaralgan. K.M. Furati, O.S. Iyiola, M. Kirane hamda S.A. Malik, S. Azizlarning ishlarida subdiffuziya tenglamasidagi kasr hosilasi ikki parametrli umumlashtirilgan Xilfer kasr hosilasidir. Shuningdek, K.M. Furati, O.S. Iyiola va M. Kirane, Z. Li, Y. Liu va M. Yamamoto, L. Sun, Y. Zhang va T. Weilarning maqolalarida subdiffuziya tenglamasining o‘ng tomonini aniqlashning teskari masalalari bilan bog’liq bo‘lgan maqolalar tahlili borligiga e’tibor berishimiz mumkin. Shuni ta’kidlash kerakki, elliptik qismi ixtiyoriy chegaralangan sohada aniqlangan ixtiyoriy tartibdagi elliptik operator bo‘lgan subdiffuziya tenglamasining o‘ng tomonini klassik ma’noda aniqlash ilgari hech kim tomonidan ko‘rib chiqilmagan, o‘rganilmagan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti V.I. Romanovskiy nomidagi Matematika institutida O‘zbekiston Respublikasi Innovatsion rivojlanish vazirligining F-FA-2021-424-sonli ilmiy tadqiqot grantining rejalashtirilgan mavzusiga muvofiq amalga oshirildi.

Tadqiqotning maqsadi ko‘p o‘lchamli sohada aniqlangan ixtiyoriy tartibdagi elliptik operatori qatnashgan parabolik, giperbolik hamda subdiffuziya tenglamalari uchun to‘g’ri va teskari masalalarning klassik yechimlarini topishdan iborat.

Tadqiqotning vazifalari quyidagilardan iborat:

Elliptik qismi ixtiyoriy N o‘lchovli sohada aniqlangan ixtiyoriy elliptik operator bo‘lgan giperbolik va parabolik tenglamalar uchun boshlang’ich-chegaraviy masalalarning klassik ma’nosida yagona yechimga ega ekanligini isbotlash.

Umumlashgan yechim tushunchasini kiritish va ularning mavjudligi masalalarini o‘rganish.

Elliptik qismi ixtiyoriy *N* o‘lchovli sohada aniqlangan ixtiyoriy elliptik operator bo‘lgan Kaputo va Riman-Liuvill hosilali subdiffuziya tenglamalari uchun boshlang’ich-chegaraviy masalalarining klassik ma’nosida yagona yechimga ega ekanligini isbotlash.

Elliptik qismi ixtiyoriy *N* o‘lchovli sohada aniqlangan ixtiyoriy elliptik operator bo‘lgan Kaputo va Riman-Liuvill hosilali subdiffuziya tenglamalarining o‘ng tomonini aniqlash bo‘yicha teskari masalalarning klassik ma’nosida yagona yechimga ega ekanligini isbotlash. Teskari masalaning umumlashgan yechimi tushunchasini kiritish va ularning mavjudligi masalalarini tekshirish.

Tadqiqot ob’ekti. Klassik giperbolik va parabolik tenglamalar, elliptik qismi ixtiyoriy N o‘lchovli sohada aniqlangan ixtiyoriy elliptik operator bo‘lgan Kaputo va Riman-Liuvill hosilali subdiffuziya tenglamalari.

Tadqiqot predmeti. Boshlang’ich-chegaraviy masalalari va tenglamalarning o‘ng tomonini aniqlashning teskari masalalari.

Tadqiqot usullari. Tadqiqotda funksional analiz usullari, spektral nazariya va Furye usullari qo‘llaniladi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

elliptik qismi ixtiyoriy o‘lchovli sohada aniqlangan ixtiyoriy elliptik operator bo‘lgan giperbolik va parabolik tenglamalar uchun boshlang’ich-chegaraviy masalalarning klassik ma’nosida yagona yechimga ega ekanligi isbotlangan;

elliptik qismi ixtiyoriy o‘lchovli sohada aniqlangan ixtiyoriy elliptik operator bo‘lgan Kaputo va Riman-Liuvill hosilali subdiffuziya tenglamalari uchun boshlang’ich-chegaraviy masalalar yagona yechimga ega ekanligi isbotlangan;

ixtiyoriy o‘lchovli sohada aniqlangan ixtiyoriy elliptik operator bo‘lgan Kaputo va Riman-Liuvill hosilali subdiffuziya tenglamalarining o‘ng tomonini aniqlash bo‘yicha teskari masalalarning klassik va umumlashgan yechimlarning mavjud va yagonaligi isbotlangan.

Tadqiqotning amaliy natijasi. Olingan natijalar va dissertatsiyada qo‘llanilgan usullar oliy o‘quv yurtlari magistrantlari va doktorantlari uchun bitiruv kursi sifatida o‘qitilishi mumkin.

Tadqiqot natijalarining ishonchliligi. Natijalar funksional analiz usullari, spektral nazariya va Furye usuli yordamida olingan. Olingan barcha natijalar matematik jihatdan to‘g’ri.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati shundan iboratki, tadqiqotda olingan ilmiy natijalar turli xususiy hosilali differensial tenglamalar uchun to‘g’ri va teskari masalalarni yanada chuqurroq tadqiq qilishda qo‘llanilishi mumkin. Xususan, ushbu dissertatsiya ishida ishlab chiqilgan texnika va usullardan to‘g’ri va teskari masalalarning yechimini topishda foydalanish mumkin.

Dissertatsiyaning amaliy ahamiyati shundan iboratki, olingan natijalar differensial operatorlar, jumladan, kasr tartibli chiziqli differensial tenglamalarning sistemalari, involyutsiyali differensial modellar, kasr tartibli Shturm-Liuvil operatorlari va boshqalari uchun to‘g’ri va teskari masalalar nazariyasida qo‘llanilishi mumkin.

Tadqiqot natijalarining joriy qilinishi. Yuqori tartibli tenglamalar uchun

boshlang’ich-chegaraviy va teskari masalalari bo‘yicha olingan natijalar asosida:

subdiffuziya tenglamalarining boshlang’ich-chegaraviy va teskari masalalari uchun topilgan yechimlaridan AAAA-A19-119072290002-9 raqamli «Kamchatkaning tabiiy ofatlar-zilzilalar va vulqon otilishi» mavzusidagi xorijiy grant loyihasida zilzilaning operativ xabarchisi bo‘lgan radioaktiv radon gazining tuproqdan atmosferaning sirt qatlamiga o‘tishini tasvirlashda foydalanilgan (Vitus Bering nomidagi Kamchatka davlat universitetining 2022 yil 24-oktyabrdagi №44-12-sonli ma’lumotnomasi, Rossiya Federatsiyasi). Ilmiy natijaning qo‘llanilishi geofizikaning teskari masalalarini yechish algoritmlarini ishlab chiqish imkonini bergan;

kasr tartibli hosila qatnashgan subdiffuziya tenglamalarining boshlang’ich-chegaraviy va teskari masalalari uchun topilgan yechimlaridan NIOKTR AAAA-A19-119013190077-1 raqamli «Kasr hisobi va kasr taqsimlangan tartibli differensial tenglamalar» mavzusidagi xorijiy grant loyihasida turli fizikaviy va biologik jarayonlarni matematik modellashtirishda foydalanilgan (Kabardin-Balkar ilmiy markazi Amaliy matematika va avtomatlashtirish institutining 2022 yil 5 maydagi №01-14/30-sonli ma’lumotnomasi, Rossiya Federatsiyasi). Ilmiy natijaning qo‘llanilishi so‘nggi yillarda turli fizikaviy va biologik jarayonlarni matematik modellashtirishda samarali qo‘llanilayotgan kasr tartibli hosila qatnashgan evolyutsion tenglamalar uchun lokal va nolokal chegaraviy masalalarni yechish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Tadqiqotning asosiy natijalari 6 ta xalqaro va 5 ta respublika ilmiy anjumanlarida muhokama qilingan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha jami 18 ta ilmiy ishlar chop etilgan bo‘lib, shundan 6 ta maqola O‘zbekiston Respublikasi Oliy attestatsiya komissiyasining falsafa doktorlik dissertatsiyalarining asosiy ilmiy natijalarini chop etish uchun tavsiya etilgan ilmiy nashrlarda chop etilgan, shulardan 4 tasi xalqaro jurnallarda chop etilgan bo’lib, ulardan 3 tasi SCOPUS ma’lumotlar bazalarida indekslangan jurnallarda chop etilgan, 3 tasi respublika jurnallarida chop etilgan va 11 tasi tezisdir.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish, to‘rtta bob, xulosa va adabiyotlar ro‘yxatidan iborat. Dissertatsiyaning umumiy hajmi 126 bet.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertasiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, mavzu bo‘yicha xorijiy ilmiy-tadqiqotlar sharhi, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, ob’ekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertasiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning **“Dastlabki ma’lumotlar”** deb nomlangan birinchi bobi yordamchi xususiyatga ega bo‘lib, dissertatsiyani o‘qish qulay bo‘lishi uchun yaratilgan. Bu yerda yangi natijalar yo‘q va faqat kerakli ta’riflar va tasdiqlar to‘plangan.

Asosan, teoremalar isbotsiz holda berilgan. Biroq, ko‘rilayotgan soha $\mathbb{T}^N$ tor bo‘lgan holda, ba’zi teoremalarning juda sodda isbotlari keltirilgan. Masalan, Sobolevning joylashtish teoremasi, Krasnoselskiy va boshqalarning monografiyasidan olingan lemma, A o‘zgarmas koefitsientlik elliptik operatorini kasr tartibli darajasining aniqlanish sohasining Liuvill fazosi bilan ustma-ust tushishi, hamda yetarlicha silliq funksiya Furye qatorining berilgan funksiyaga tekis yaqinlashishi juda sodda tarzda isbotlangan. Ma’lumki, umumiy holatda bu teoremalar isboti ancha murakkabdir.

Keling, birinchi bobdagi ba’zi ta’riflar va tasdiqlarni keltiraylik.

Aytaylik Ω ixtiyoriy N o‘lovli soha bo‘lsin, u N o‘lchovli fazo $\mathbb{R}^N$ bilan ustma-ust tushishi ham mumkin. Juft $m = 2k$ tartibli

$$A(x,D)u = \sum_{0\le|\alpha|\le m} a_\alpha(x)D^\alpha u,\ (x\in\Omega) \tag{1}$$

differensial ifoda $x_0 \in \Omega$ nuqtada elliptik deyiladi, agar unga mos xarakteristik polinom

$$a(x_0,\xi) = \sum_{|\alpha|=m} a_\alpha(x_0)\xi^\alpha \tag{2}$$

ixitiyoriy noldan farqli ξ lar uchun musbat bo‘lsa. (1) differensial ifoda barcha $x_0 \in \Omega$ nuqtalarida elliptik bo‘lsa, u holda bu differensial ifoda Ω sohada elliptik deyiladi. Biz $A(D)$ orqali $A(D) = \sum_{|\alpha|=m} a_\alpha D^\alpha$ ko‘rinishda bo‘lgan bir jinsli elliptik differensial ifodani belgilaymiz

Agar $f \in L_2\left(\mathbb{T}^N\right)$ sinfdan olingan funksiyalar uchun quyidagi norma

$$\|f\|_{L_2^a(\mathbb{T}^N)} = \sum_{n\in Z^N} \left(1+|n|^2\right)^a |\ f_n\ |^2,\ a>0, \tag{3}$$

biror $a>0$ haqiqiy son uchun chekli bo‘lsa, u holda bu sinf *Liuvill sinfi* $L_2^a\left(\mathbb{T}^N\right)$ deb ataladi. $L_2^a\left(\mathbb{T}^N\right)$ sinflari a natural son bo‘lganda $W_2^a\left(\mathbb{T}^N\right)$ Sobolev sinflari bilan ustma-ust tushadi. Demak, $L_2^a(\Omega)$ Liuvill sinfi Sobolev $W_2^a(\Omega)$ sinfining a butun bo‘lmagan hollari uchun umumlashmasidir.

Dissertatsiyaning asosiy natijalari ikkinchi bobdan boshlanadi. **"Giperbolik va parabolik tenglamalar uchun boshlang’ich-chegaraviy masalalar"** deb nomlangan ikkinchi bobning birinchi bo‘limida N o‘lchovli $\mathbb{T}^N$ torda ixtiyoriy tartibli elliptik differensial operator qatnashgan parabolik tenglama uchun boshlang’ich-chegaraviy masala o‘rganildi.

Masala: Quyidagi

$$u_t(x,t) + A(D)u(x,t) = f(x,t),\ \ x\in\mathbb{T}^N,\ 0<t\le T, \tag{4}$$

tenglamaning

$$u(x,0) = \varphi(x),\ x\in\mathbb{T}^N, \tag{5}$$

boshlang'ich shartni qanoatlantiruvchi va $u_t(x,t),\ A(D)u(x,t)\in C\left(\mathbb{T}^N\times(0;T]\right)$ bo'lgan hamda har bir o'zgaruvchisi bo'yicha 2π-davriyikka ega bo'lgan $u(x,t)\in C\left(\mathbb{T}^N\times[0,T]\right)$ funksiyani toping. Biz bu qidirilayotgan $u(x,t)$ funksiyani masalaning klassik yechimi (yoki yechim) deb ataymiz.

Bu yerda berilgan $\varphi(x)$ va $f(x,t)$ funksiyalar har bir o'zgaruvchisi bo'yicha 2π davriydir.

1-teorema. *Aytaylik* $\tau>\dfrac{N}{2}$ *va*

(a) $\varphi(x)\in L_2^{\tau}\left(\mathbb{T}^N\right)$;

(b) $f(x,t)\in L_2^{\tau}\left(\mathbb{T}^N\right)$ *ixtiyoriy* $t\in[0,T]$*uchun;*

(c) $F(t)=\| f(\cdot,t)\|^2_{L_2^{\tau}(\mathbb{T}^N)}$ *funksiya* $[0,T]$ *segmentda uzluksiz bo'lsin.*

U holda (4)-(5), masalaning yagona davriy yechimi mavjud bo'ladi va uni $x\in\mathbb{T}^N$ *da tekis va absolyut yaqinlashuvchi quyidagi qator ko'rinishida yozish mumkin:*

$$u(x,t)=\sum_{n\in Z^n}\left[e^{-A(n)t}\varphi_n+\int_0^t e^{-A(n)(t-s)}f_n(s)ds\right]e^{inx},t\in[0,T]. \qquad (6)$$

Shuni ta'kidlash kerakki, ushbu bo'limda o'zgarmas koeffitsientli elliptik operator uchun olingan natijalarni, ixtiyoriy N o'lchovli sohada aniqlangan ixtiyoriy tartibli elliptik operator bilan almashtirilgan holda ham osonlik bilan olish mumkin. Buni ko'rsatish maqsadida ikkinchi bobning ikkinchi bo'limida elliptik qismi eng umumiy shaklga ega bo'lgan va ixtiyoriy ko'p o'lchovli sohada (yetarli darajada silliq chegarali) aniqlangan giperbolik tenglamalar uchun boshlang'ich-chegaraviy masalalar o'rganilgan.

Bu bo'limda

$$u_{tt}+A(x,D)u(x,t)=f(x,t),\quad x\in\Omega,\quad t>0; \qquad (7)$$

tenglama

$$u(x,0)=\varphi(x),\quad u_t(x,0)=\psi(x),\quad x\in\Omega; \qquad (8)$$

boshlang'ich va

$$B_ju(x,t)=\sum_{|\alpha|\le m_j}b_{\alpha,j}(x)D^{\alpha}u(x,t)=0, \qquad (9)$$

$$0\le m_j\le m-1,\quad j=1,2,...,l;\quad x\in\partial\Omega;$$

chegaraviy shart bilan birga o'rganilgan. Bu yerda $f(x,t),\varphi(x),\psi(x)$ va $b_{\alpha,j}(x)$ koeffitsientlar berilgan funksiyalardir.

Qo'yilgan masalani biz Furye metodini qo'llab klassik yechimni topamiz. Bunda quyidagi spektral masalaga kelamiz:

$$A(x,D)v(x)=\lambda v(x)\quad x\in\Omega; \qquad (10)$$

$$B_j(x,D)v(x)=0,\quad j=1,2,...,l;\quad x\in\partial\Omega. \qquad (11)$$

Ushbu spektral masala Agmonning 1962-yildagi mashhur maqolasida o‘rganilgan. Muallif Ω sohanining chegarasi va $A(x,D)$, $B_j(x,D)$ operatorlarining koeffitsientlari ma’lum shartlarni qanoatlantirganda, bu spektral masala $L_2(\Omega)$ da to‘la ortonormal $\{v_k\}$ xos funksiyalar sistemasiga va ularga mos manfiy bo‘lmagan xos sonlari $\{\lambda_k\}$ to‘plamiga ega bo‘lishi isbotlagan. Bu shartlarni (A) deb belgilaymiz va har doim bu shartlar bajariladi deb faraz qilamiz.

Aytaylik τ ixtiyoriy haqiqiy son bo‘lsin. Biz

$$D(\hat{A}^{\tau}) = \{ f(x) \in L_2(\Omega) : \sum |\lambda_k|^{2\tau} |f_k|^2 < \infty \}. \qquad (12)$$

sohada aniqlangan

$$\hat{A}^{\tau} f(x) = \sum \lambda_k^{\tau} f_k v_k(x), \qquad (13)$$

operatorni kiritamiz.

2-teorema. *Aytaylik $f(x,t), \varphi(x), \psi(x)$ funksiyalari (7) va (8) larni qanoatlantirib, berilgan sohalarda uzluksiz bo‘lsin. U holda (7)-(9) aralash masalaning klassik yechimi mavjud bo‘lsa, u yagonadir.*

Endi biz quyidagi formal qatorni tuzib olamiz (bu yerda A musbat operator bo‘lgani uchun hamma k larda $\lambda_k > 0$ dir)

$$u(x,t) = \sum_{k=1}^{\infty} \left[\frac{1}{\sqrt{\lambda_k}} \psi_k \sin\sqrt{\lambda_k}\, t + \varphi_k \cos\sqrt{\lambda_k}\, t + \frac{1}{\sqrt{\lambda_k}} \int_0^t f_k(\xi) \sin\sqrt{\lambda_k}\,(t-\xi)\, d\xi \right] v_k(x). \qquad (14)$$

Bu yerda ψ_k, φ_k va $f_k(t)$ lar mos ravishda, $\psi(x)$, $\varphi(x)$ va $f(x,t)$ funksiyalarining Furye koeffitsientlaridir, masalan $\psi_k = (\psi, v_k)$.

3-teorema. *Aytaylik $\tau > 1 + \frac{N}{2m}$ bo‘lib, $\varphi \in D(\hat{A}^{\tau})$, $\psi \in D(\hat{A}^{\tau-\frac{1}{2}})$ va ixtiyoriy $t \in [0,T]$ lar uchun $f(x,t) \in D(\hat{A}^{\tau-\frac{1}{2}})$ bo‘lsin. Bundan tashqari,*

$$F(t) = \| \hat{A}^{\tau-\frac{1}{2}} f(x,t) \|_{L_2(\Omega)}$$

funksiya $[0,T]$ da uzluksiz bo‘lsin. U holda (7)-(9) aralash masalaning yechimi mavjud va u (14) qator ko‘rinishida bo‘ladi.

4-teorema. *Aytaylik 3-teorema shartlari bajarilsin. U holda (7)-(9) masalaning (14) yechimi uchun quyidagi baho o‘rinlidir:*

$$\sup_{t\in[0,T]} \| u(x,t) \|^2_{L_2(\Omega)} \le C\left[\| \varphi \|^2_{L_2(\Omega)} + \| \psi \|^2_{L_2(\Omega)} + T \int_0^T \| f(x,\xi) \|^2_{L_2(\Omega)}\, d\xi \right], \qquad (15)$$

bu yerda C konstanta T ga bog’liq emas.

Quyida K.B. Sabitovning ta’rifiga asoslanib, (7)-(9) masala uchun umumlashgan yechim tushunchasini kiritamiz:

1-ta’rif. *Aytaylik $\varphi_k(x)$, $\psi_k(x)$ va $f_k(x,t)$ 3-teoremaning shartlarini qanoatlantiruvchi funksiyalar ketma-ketligi bo‘lib, bu funksiyalar $L_2(\Omega)$ fazo normasi*

bo'yicha (oxirgi funksiyalar ketma-ketligi $t \in [0,T]$ *bo'yicha tekis) mos ravishda* $L_2(\Omega)$ *dagi* $\varphi(x)$, $\psi(x)$ *va* $f(x,t)$ *funksiyalarga yaqinlashsin. Agar (7)-(9) masalaning* $\varphi_k(x)$, $\psi_k(x)$ *va* $f_k(x,t)$ *funksiyalarga mos kelgan shunday* $u_k(x,t)$ *yechimlari ketma-ketligi mavjud bo'lsaki, bu ketma-ketlik* $u(x,t)$ *funksiyaga* $L_2(\Omega)$ *da* t *bo'yicha* $[0,T]$ *da tekis yaqinlashsa, u holda bu* $u(x,t)$ *funksiya (7)-(9) masalaning umumlashgan yechimi deyiladi*

5-teorema. *Aytaylik,* $\varphi(x)$, $\psi(x)$ *va* $f(x,t)$ *(barcha* $t \in [0,T]$ *larda)* $L_2(\Omega)$ *fazoga tegishli bo'lsin. U holda (7)-(9) masalaning (14) qator bilan aniqlangan shunday yagona va turg'un umumlashgan yechimi mavjudki, uning* $\| u(x,t) \|_{L_2(\Omega)}$ *normasi* $t \in [0,T]$ *bo'yicha uzluksiz funksiya bo'ladi.*

Endi parabolik tenglama uchun quyidagi boshlang'ich-chegaraviy masalani qaraylik: shunday $u(x,t) \in C(\overline{\Omega} \times [0,T])$ funksiya topilsinki, $u_t(x,t), A(x,D)u(x,t) \in C(\overline{\Omega} \times (0,T])$ bo'lib, u

$$u_t + A(x,D)u(x,t) = f(x,t), \quad x \in \Omega, \quad t > 0; \tag{16}$$

tenglamani,

$$u(x,0) = \varphi(x), \quad x \in \Omega; \tag{17}$$

boshlang'ich shartni va (9) chegaraviy shartlarni qanoatlantirsin.

6-teorema. *Aytaylik* $\tau > \dfrac{N}{2m}$ *va* $\varphi(x) \in D(\hat{A}^{\tau})$ *bo'lsin. Bundan tashqari, barcha* $t \in [0,T]$ *larda* $f(x,t) \in D(\hat{A}^{\tau})$ *bo'lib,*

$$F(t) = \| \hat{A}^{\tau} f(x,t) \|_{L_2(\Omega)}$$

funksiya t *bo'yicha* $[0,T]$ *da uzluksiz bo'lsin. U holda (16), (17), (9) masalaning yechimi mavjud bo'lib, u* $\overline{\Omega}$ *sohada tekis va absolyut yaqinlashuvchi*

$$u(x,t) = \sum_{k=1}^{\infty} [e^{-\lambda_k t} \varphi_k + \int_0^t e^{-\lambda_k (t-\xi)} f_k(\xi) d\xi] v_k(x), \quad t \in (0,T],$$

qator ko'rinishiga ega bo'ladi, bunda φ_k *va* $f_k(t)$ *lar mos Furye koeffisientlaridir.*

Uchinchi bob **"Subdiffuziya tenglamalari uchun boshlang'ich-chegaraviy masalalar"** deb nomlangan. Uchinchi bobning birinchi bo'limida vaqt bo'yicha kasr tartibli hosilali va elliptik qismi ixtiyoriy elliptik differensial ifoda bo'lgan sudiffuziya tenglamasi uchun boshlang'ich-chegaraviy masala qaralgan.

Aytaylik, $\rho \in (0,1)$ o'zgarmas son bo'lib, $f(x,t), \varphi(x)$ funksiyalar va $b_{\alpha,j}(x)$ koeffitsientlar berilgan bo'lsin. Quyidagi

$$\partial_t^{\rho} u(x,t) + A(x,D)u(x,t) = f(x,t), \quad x \in \Omega, \quad t > 0; \tag{18}$$

tenglama,

$$\lim_{t \to 0} \partial_t^{\rho-1} u(x,t) = \varphi(x), \quad x \in \Omega; \tag{19}$$

boshlang'ich va

$$B_j u(x,t) = \sum_{|\alpha| \le m_j} b_{\alpha,j}(x) D^{\alpha} u(x,t) = 0, \qquad (20)$$

$$0 \le m_j \le m-1, \quad j = 1,2,...,l; \quad x \in \partial\Omega;$$

chegaraviy shartlar bilan berilgan masalani qaraymiz. Uchinchi bobning birinchi bo‘limida klassik Furye metodi bilan bu masalan yechimining mavjud va yagonaligi isbotlangan. Bunda tenglamaning o‘ng tarafi va boshlang’ich funksiyalarga mos Furye qatorini tekis va absolyut yaqinlashishini ta’minlovchi shartlar topilgan.

7-teorema. *Agar* $f(x,t) \in C(\Omega \times (0,\infty))$ *va* $\varphi \in C(\Omega)$ *bo‘lsa, (18)-(20) masala oshib borsa bitta yechimga ega.*

8-teorema. *Aytaylik* $\tau > \dfrac{N}{2m}$ *va* $\varphi \in D(\hat{A}^{\tau})$ *bo‘lsin. Bundan tashqari, barcha* $t \in [0,T]$ *uchun* $f(x,t) \in D(\hat{A}^{\tau})$ *bo‘lsin, va*

$$F(t) = \| \hat{A}^{\tau} f(x,t) \|_{L_2(\Omega)}$$

funksiya barcha $t \in [0,T]$ *lar uchun uzluksiz bo‘lsin. U holda (18)-(20) boshlang’ich-chegaraviy masalaning yechimi mavjud va u quyidagi qator ko‘rinishiga ega*

$$u(x,t) = \sum_{1}^{\infty} [\varphi_j t^{\rho-1} E_{\rho,\rho}(-\lambda_j t^{\rho}) + \int_0^t f_j(t-\xi)\xi^{\rho-1} E_{\rho,\rho}(-\lambda_j \xi^{\rho}) d\xi] v_j(x), \quad (21)$$

va qator $x \in \bar{\Omega}$ *da va barcha* $t \in (0,T]$ *lar uchun absolyut va tekis yaqinlashadi.*

Boshlang’ich-chegaraviy masala N o‘lchovli torda qaralgan holda, bu shartlarni nafaqat yetarli, balki zarur ekanligini osongina tekshirishimiz mumkin.

Haqiqatan, aytaylik $A(D) = \sum_{|\alpha|=m} a_{\alpha} D^{\alpha}$ o‘zgarmas koeffisientlarga ega bo‘lgan bir jinsli simmetrik nomanfiy elliptik differensial ifoda bo‘lsin. Differensial tenglama va uning boshlang’ich sharti quydagicha bo‘lsin

$$\partial_t^{\rho} u(x,t) + A(D)u(x,t) = f(x,t), \quad x \in \mathbb{T}^N, \quad t > 0; \qquad (22)$$

$$\lim_{t \to 0} \partial_t^{\rho-1} u(x,t) = \varphi(x), \quad x \in \mathbb{T}^N; \qquad (23)$$

(20) chegaraviy shartlar o‘rniga biz har bir x_j argumenti bo‘yicha 2π -davriylikka ega funksiyalarni qaraymiz va $\varphi(x)$ va $f(x,t)$ funksiyalar ham x_j o‘zgaruvchilari bo‘yicha 2π -davriylikka ega bo‘lsin deb faraz qilamiz.

9-teorema. *Aytaylik* $\tau > \dfrac{N}{2}$ *va* $\varphi \in L_2^{\tau}(\mathbb{T}^N)$ *bo‘lsin. Bundan tashqari, ixtiyoriy* $t \in [0,T]$ *uchun* $f(x,t) \in L_2^{\tau}(\mathbb{T}^N)$ *bo‘lsin va ushbu*

$$F(t) = \| f(\cdot,t) \|^2_{L_2^{\tau}(\mathbb{T}^N)}$$

funksiya $[0,T]$ *segmentda uzluksiz bo‘lsin. U holda (22)-(23) boshlang’ich-chegaraviy masalaning yechimi mavjud va u quyidagi qator ko‘rinishiga ega boladi*

$$u(x,t)=\sum_{n\in\mathbb{Z}^N}[\varphi_n t^{\rho-1}E_{\rho,\rho}(-A(n)t^{\rho})+\int_0^t f_n(t-\xi)\xi^{\rho-1}E_{\rho,\rho}(-A(n)\xi^{\rho})d\xi]e^{inx},$$

va bu qator $x\in\mathbb{T}^N$ *da va har bir* $t\in(0,T]$ *uchun absolyut va tekis yaqinlashadi.*

1-eslatma. *E'tibor beringki,* $\tau>\frac{N}{2}$ *bo'lganda, Sobolev joylashtirish teoremasiga asosan,* $L_2^{\tau}(\mathbb{T}^N)$ *dagi barcha funksiyalar* 2π*-davrli uzluksiz funksiyalar bo'ladi. Teskari* $\tau\le\frac{N}{2}$ *tengsizlikning bajarilishi esa* $L_2^{\tau}(\mathbb{T}^N)$ *da chegaralanmagan funksiyalar mavjudligini keltirib chiqaradi. Shuning uchun, bu teoremaning* $\tau>\frac{N}{2}$ *sharti yetarli shart bo'libgina qolmay, balki u zaruriy shart hamdir.*

Uchinchi bobning ikkinchi bo'limida N o'lchovli torda Riman-Liuvill kasr tartibli hosilasi ishtirok etgan subdiffuziya tenglamasi uchun boshlang'ich-chegaraviy masala o'rganilgan.

Aytaylik $\rho\in(0,1)$ o'zgarmas son bo'lsin. Quyidagi boshlang'ich-chegaraviy masalani qaraylik

$$\partial_t^{\rho}u(x,t)-\Delta u(x,t)=f(x,t),\quad x\in\mathbb{T}^N,\quad 0<t\le T, \tag{24}$$

$$\lim_{t\to 0}\partial_t^{\rho-1}u(x,t)=\varphi(x),\quad x\in\mathbb{T}^N. \tag{25}$$

Bu yerda chegaraviy shartlar o'rniga har bir x_j argumenti bo'yicha 2π-davriylikka ega bo'lish sharti qo'yilgan hamda $\varphi(x)$ va $f(x,t)$ funksiyalar ham x_j o'zgaruvchilari bo'yicha 2π-davriylikka ega funksiyalar bo'lsin deb faraz qilingan.

Uchinchi bobning birinchi bo'limidagi boshlang'ich funksiyaga qo'yilgan shart $\varphi\in L_2^{\tau}(\mathbb{T}^N)$, $\tau>\frac{N}{2}$, ko'rinishiga ega edi. Ushbu $\tau>\frac{N}{2}$ tengsizlik dissertatsiyaning yana ikkinchi va to'rtinchi boblarida ham uchraydi. Albatta, bu shartni $\varphi\in L_2^{\frac{N}{2}}(\mathbb{T}^N)$ bilan almashtirib bo'lmaydi, chunki bu holda $L_2^{\frac{N}{2}}(\mathbb{T}^N)$ sinfidagi funksiyalar, yuqoridagi 1-eslatmada aytilganidek, chegaralanmagan bo'lishi ham mumkin. Tabiiyki, quyidagicha savol tug'iladi: $\varphi\in L_2^{\tau}(\mathbb{T}^N)$, $\tau>\frac{N}{2}$ shartni, $\varphi\in C(\mathbb{T}^N)\cap L_2^{\frac{N}{2}}(\mathbb{T}^N)$ shart bilan almashtirish mumkinmi? Ushbu bo'limda buning mumkinligi isbotlangan. Shuningdek, uchinchi bobning birinchi bo'limida tenglamaning o'ng tomoni uchun qo'yilgan $\|f(\cdot,t)\|^2_{L_2^a(\mathbb{T}^N)}\in C[0,T]$ shartni $\|t^{1-\rho}f(\cdot,t)\|^2_{L_2^a(\mathbb{T}^N)}\in C[0,T]$ shart bilan almashtirish mumkinligi ko'rsatilgan.

10-teorema. *Aytaylik* $\varphi \in C(\mathbb{T}^N) \cap L_2^{\frac{N}{2}}(\mathbb{T}^N)$ *bo'lsin. Bundan tashqari, barcha* $t \in [0,T]$ *lar uchun* $f(x,t) \in L_2^a(\mathbb{T}^N)$, $a > \frac{N}{2}$, *bo'lib,* $\| t^{1-\rho} f(\cdot,t) \|^2_{L_2^a(\mathbb{T}^N)} \in C[0,T]$ *bo'lsin. U holda (24)-(25) boshlang'ich chegaraviy masalaning yechimi mavjud va quyidagi ko'rinishga ega*

$$u(x,t) = \sum_{n \in \mathbb{Z}^N} [\varphi_n t^{\rho-1} E_{\rho,\rho}(-|n|^2 t^\rho) + \int_0^t f_n(t-\xi)\xi^{\rho-1} E_{\rho,\rho}(-|n|^2 \xi^\rho) d\xi] e^{inx}, \tag{26}$$

bo'lib, bu qator barcha $t \in (0,T]$ *lar uchun* $x \in \mathbb{T}^N$ *bo'yicha absolyut va tekis yaqinlashadi. Bu yerda* φ_n *va* $f_n(t)$ *Furye koeffitsientlaridir. Bundan tashqari, bu qatorga* ∂_t^ρ *kasr tartibli hosilani va* A *operatorni ta'sir ettirganda ham xosil bo'lgan qator barcha* $t \in (0,T]$ *lar uchun* $x \in \mathbb{T}^N$ *bo'yicha absolyut va tekis yaqinlashadi.*

Uchinchi bobning uchinchi bo'limida N o'lchamli torda Caputo hosilasi qatnashgan subdiffuziya tenglamasi uchun boshlang'ich-chegaraviy masala Furye usuli yordamida o'rganilgan. O'rganilgan masala quyidagi ko'rinishga ega:

$$D_t^\rho u(x,t) + Au(x,t) = f(x,t), \quad x \in \mathbb{T}^N, \quad 0 < t \le T, \tag{27}$$

$$u(x,0) = \varphi(x), \quad x \in \mathbb{T}^N, \tag{28}$$

bu yerda $A = -\Delta$-Laplas operatori, f va φ lar esa berilgan 2π-davriy uzluksiz funksiyalardir. Chegaraviy shart o'rnida 2π-davriylik sharti olingan.
Masala Furye usuli bilan yechilgani uchun A operator sifatida to'la ortonormal xos funksiyalar to'plamiga ega bo'lgan istalgan elliptik operatorini olishimiz mumkin.

11-teorema. *Aytaylik* $a > \frac{N}{2}$ *va* $\varphi \in L_2^a(\mathbb{T}^N)$ *bo'lsin. Bundan tashqari,* $t \in [0,T]$ *lar uchun* $f(x,t) \in L_2^a(\mathbb{T}^N)$ *bo'lib,* $\| f(\cdot,t) \|^2_{L_2^a(\mathbb{T}^N)} \in C[0,T]$ *bo'lsin. U holda (27)- (28) boshlang'ich-chegaraviy masalaning yechimi mavjud va u*

$$u(x,t) = \sum_{n \in Z^N} [\varphi_n E_{\rho,1}(-|n|^2 t^\rho) + \int_0^t f_n(t-\xi)\xi^{\rho-1} E_{\rho,\rho}(-|n|^2 \xi^\rho) d\xi] e^{inx}, \tag{29}$$

ko'rinishdagi har bir $t \in (0,T]$ *da* $x \in \mathbb{T}^N$ *bo'yicha absolyut va tekis yaqinlashuvchi qator kabi aniqlanadi. Bundan tashqari,* D_t^ρ *va* A *operatorlar hadma-had ta'sir qildirilganidan keyin olingan qatorlar ham har bir* $t \in (0,T]$ *da* $x \in \mathbb{T}^N$ *bo'yicha absolyut va tekis yaqinlashadi.*

Ushbu teoremaga o'xshash natija Riman-Liuvill hosilasiga ega bo'lgan subdiffuziya tenglamasi uchun 10-teoremada keltirilgan edi. Ammo uchinchi bobning ikkinchi bo'limida $\varphi(x)$ boshlang'ich funksiya uchun topilgan shart ancha kengroq funksiyalar sinfini qarashga imkon beradi. Masalan, bir o'lchamli holda, uchinchi

bobning ikkinchi bo‘limida $\varphi(x) \in C(\mathbb{T}^1) \cap L_2^{\frac{1}{2}}(\mathbb{T}^N)$ ni talab qilish yetarli ekanligi isbotlangan. Bu bo‘limning oxirida kasr tartibli hosila Kaputo ma’nosida bo‘lsa, bu shart yetarli emasligi ko‘rsatilgan.

Dissertatsiyaning to‘rtinchi bobi **"Subdiffuziya tenglamasi uchun teskari masala"** deb nomlangan to‘rtinchi bobning birinchi bo‘limida quyidagi teskari masala o‘rganilgan.

Masala: Shunday $\{u(x,t), f(x)\}$ funksiyalar juftligi topilsinki, ular har bir x_j o‘zgaruvchisi bo‘yicha 2π-davriy bo‘lsin hamda ushbu xossalarga $f(x) \in C(\mathbb{T}^N)$ va $u(x,t) \in C(\mathbb{T}^N \times [0,T])$, $D_t^\rho u(x,t), A(D)u(x,t) \in C(\mathbb{T}^N \times (0,T])$ ega bo‘lsin, shunindek quyidagi differensial tenglamani

$$D_t^\rho u(x,t) + A(D)u(x,t) = f(x), \quad x \in \mathbb{T}^N, \quad t > 0, 0 < \rho \le 1; \tag{30}$$

va quyidagi boshlang’ich shartni

$$u(x,0) = \varphi(x), \quad x \in \mathbb{T}^N; \tag{31}$$

va quyidagi qo‘shimcha shartni

$$u(x,T) = \Psi(x), \quad x \in \mathbb{T}^N. \tag{32}$$

qanoatlantirsin. Berilgan $\varphi(x)$ va $\Psi(x)$ funksiyalarni har bir x_j o‘zgaruvchi bo‘yicha 2π-davriy bo‘lsin deb faraz qilinadi.

Ushby bo‘limda yuqoridagi teskari masala Furye usuli yechilganligi sababli, ushbu bo‘lim natijalarini umumiyroq ko‘rinishga ega bo‘lgan tenglamalar uchun ham umumlashtirish mumkin.

12-teorema. *Aytaylik* $\tau > \frac{N}{2}$ *bo‘lib,* $\varphi \in L_2^\tau(\mathbb{T}^N)$ *va* $\Psi \in L_2^{\tau+m}(\mathbb{T}^N)$ *bo‘lsin. U holda (30)-(32) teskari masalaning davriy* $\{u(x,t), f(x)\}$ *yagona yechimi mavjud*

$$u(x,t) = \sum_{n \in \mathbb{Z}^N} \varphi_n E_{\rho,1}(-A(n)t^\rho) e^{inx} + \sum_{n \in \mathbb{Z}^N} f_n t^\rho E_{\rho,\rho+1}(-A(n)t^\rho) e^{inx}, \tag{33}$$

va

$$f(x) = \sum_{n \in \mathbb{Z}^N} \frac{\Psi_n}{T^\rho E_{\rho,\rho+1}(-A(n)T^\rho)} e^{inx} - \sum_{n \in \mathbb{Z}^N} \frac{\varphi_n E_{\rho,1}(-A(n)T^\rho)}{T^\rho E_{\rho,\rho+1}(-A(n)T^\rho)} e^{inx}, \tag{34}$$

tekis va absolyut yaqinlashuvchi qatorlar ko‘rinishiga ega bo‘ladi.

13-teorema. *Aytaylik,* $\mathbb{T}^N$ *da* $\varphi(x)$ *va* $\Psi(x)$ *funksiyalar uzluksiz bo‘lsin. U holda, agar (30)-(32) teskari masalaning* $\{u(x,t), f(x)\}$ *klassik yechimi mavjud bo‘lsa, u yagona bo‘ladi.*

To‘rtinchi bobning ikkinchi bo‘limida elliptik qismi umumiy ko‘rinishda va kasr tartibli hosila Riman-Liuvill ma’nosida bo‘lgan subdiffuziya tenglamasining o‘ng tomonini aniqlash bo‘yicha teskari masala o‘rganilgan.

Aytaylik $0 < \rho \le 1$ bo‘lsin. Quyidagi subdiffuziya tenglamasini

$$\partial_t^\rho u(x,t) + A(x,D)u(x,t) = f(x), \quad x \in \Omega, \quad 0 < t \le T, \tag{35}$$

qaraymiz. Bu yerda issiqlik manbalarining harakatini tavsiflovchi $f(x)$ funksiyasi faqat x ga bog'liq bo'lsin deb faraz qilamiz. Agar bu funksiya ma'lum bo'lsa, u holda $u(x,t)$ harorat taqsimotini yagona ravishda topish uchun qo'shimcha shartlar qo'yish zarur (uchinchi bobga qarang). Masalan, boshlang'ich shartni quydagicha

$$\lim_{t\to 0}\partial_t^{\rho-1}u(x,t)=\varphi(x),\quad x\in\overline{\Omega}, \tag{36}$$

va chegaraviy shartni esa,

$$B_j u(x,t)=\sum_{|\alpha|\le m_j} b_{\alpha,j}(x)D^{\alpha}u(x,t)=0, \tag{37}$$

$$0\le m_j\le m-1, j=1,2,...,l; x\in\partial\Omega, 0\le t\le T,$$

ko'rinishda olish mumkin, bu yerda $\varphi(x)$ va $b_{\alpha,j}(x)$ koeffitsientlar berilgan funksiyalaridir. Natijada biz uchinchi bobda ko'rib chiqilgan to'g'ri boshlang'ich-chegaraviy masalani olamiz. Ushbu paragrafda quyidagi teskari masalani qaraymiz, boshlang'ich-chegaraviy masalaning $u(x,t)$ yechimi bilan bir qatorda,

$$u(x,T)=\Psi(x),\quad x\in\overline{\Omega}, \tag{38}$$

qo'shimcha shart asosida tenglamaning o'ng tarafi ham topilsin.

Qaralayotgan teskari masala klassik yechimining mavjudligi va yagonaligini isbotlash uchun Furye metodi qo'llanildi. Bunda boshlang'ich va qo'shimcha shartlardagi funksiyalar uchun Furye metodini qo'llashni taminlaydigan shartlar aniqlandi. Bundan tashqari, teskari masalaning umumlashgan yechimining ta'rifi kiritildi va umumlashgan yechimning mavjud va yagona ekani haqida teoremalar isbotlandi. Klassik va umumlashgan yechimlarning turg'uligi ham ushbu paragrafda ko'rsatildi.

14-teorema. *Aytaylik, $\varphi(x),\Psi(x)$ funksiyalar Ω sohada uzluksiz bo'lsin. U holda, agar (35)-(37), (38) teskari masalaning klassik $\{u(x,t), f(x)\}$ yechimi mavjud bo'lsa, u yagona bo'ladi.*

15-teorema. *Aytaylik, $\tau>\dfrac{N}{2m}$ bo'lib, $\varphi\in D(\hat{A}^{\tau})$ va $\Psi\in D(\hat{A}^{\tau+1})$ bo'lsin. U holda (35)-(37), (38) teskari masalaning klassik $\{u(x,t), f(x)\}$ yechimi mavjud va bu yechim barcha $x\in\overline{\Omega}$ va $t\in(0,T]$ larda tekis va absolyut yaqinlashuvchi quyidagi qatorlar orqali aniqlanadi:*

$$u(x,t)=\sum_{k=1}^{\infty}\varphi_k t^{\rho-1}E_{\rho,\rho}(-\lambda_k t^{\rho})v_k(x)+\sum_{k=1}^{\infty}f_k t^{\rho}E_{\rho,\rho+1}(-\lambda_k t^{\rho})v_k(x), \tag{39}$$

va

$$f(x)=\sum_{k=1}^{\infty}\frac{\Psi_k}{T^{\rho}E_{\rho,\rho+1}(-\lambda_k T^{\rho})}v_k(x)-\sum_{k=1}^{\infty}\frac{\varphi_k E_{\rho,\rho}(-\lambda_k T^{\rho})}{TE_{\rho,\rho+1}(-\lambda_k T^{\rho})}v_k(x). \tag{40}$$

Ushbu teoremaning tadbiqi sifatida teskari masalani N o'lchovli $\mathbb{T}^N$ torda qaraymiz. Aytaylik, $A(D)=\sum_{|\alpha|=m}a_{\alpha}D^{\alpha}$ - o'zgarmas koeffitsientli, simmetrik

nomanfiy elliptik differensial ifoda bo‘lsin. Quyidagi teskari masalani qaraymiz $(0 < \rho \le 1)$:

$$\partial_t^\rho u(x,t) + A(D)u(x,t) = f(x), \quad x \in \mathbb{T}^N, \quad t > 0; \tag{41}$$

$$\lim_{t\to 0} \partial_t^{\rho-1} u(x,t) = \varphi(x), \quad x \in \mathbb{T}^N; \tag{42}$$

$$u(x,T) = \Psi(x), \quad x \in \mathbb{T}^N. \tag{43}$$

(37) ko‘rinishdagi chegaraviy shart o‘rniga $u(x,t)$ va $f(x)$ funksiyalar har bir x_j argument bo‘yicha 2π - davriy bo‘lsin deb shart qo‘yamiz. Bundan tashqari, berilgan $\varphi(x)$ va $\Psi(x)$ funksiyalar ham har bir x_j bo‘yicha 2π -davriy bo‘lsin deb faraz qilamiz.

16-teorema. *Aytaylik,* $\tau > \dfrac{N}{2}$ *bo‘lib,* $\varphi \in L_2^\tau(\mathbb{T}^N)$ *va* $\Psi \in L_2^{\tau+m}(\mathbb{T}^N)$ *bo‘lsin. U holda (41)-(43) teskari masalaning yagona* $\{u(x,t), f(x)\}$ *yechimi mavjud va u*

$$u(x,t) = \sum_{n\in\mathbb{Z}^N} [\varphi_n t^{\rho-1} E_{\rho,\rho}(-A(n)t^\rho) + f_n t^\rho E_{\rho,\rho+1}(-A(n)t^\rho)]e^{inx},$$

$$f(x) = \sum_{n\in\mathbb{Z}^N} f_n e^{inx},$$

ko‘rinishga ega bo‘ladi. Bu yerda

$$f_n = \frac{\Psi_n}{T^\rho E_{\rho,\rho+1}(-A(n)T^\rho)} - \frac{\varphi_n E_{\rho,\rho}(-A(n)T^\rho)}{TE_{\rho,\rho+1}(-A(n)T^\rho)},$$

va φ_n, Ψ_n *lar mos ravishda* $\varphi(x)$ *va* $\Psi(x)$ *funksiyalarning Furye koeffisientlaridir. Bundan tashqari, yuqoridagi qatorlar* $x \in \mathbb{T}^N$ *bo‘yicha barcha* $t \in (0,T]$ *larda tekis va absolyut yaqinlashadi.*

Endi (35)-(37), (38) masalaning umumlashgan yechimi mavjudligini o‘rganamiz. Buning uchun biz $C((0,T],L_2(\Omega))$ simvol orqali shunday $u(x,t)$ funksiyalar sinfini belgilaymizki, ular kvadrati bilan Ω da integrallanuvchi bo‘lib, $U(t) = \| u(\cdot,t) \|_{L_2(\Omega)}$ funsiya $(0,T]$ da uzluksiz bo‘lsin. Bunday $u(x,t)$ funksiyalar uchun (36) shart quyidagi umumlashgan ma’noda tushiniladi:

$$\lim_{t\to 0} \| \partial_t^{\rho-1} u(x,t) - \varphi(x) \|_{L_2(\Omega)} = 0. \tag{44}$$

2-ta’rif. Biz *(35), (44), (37), (38) masalaning umumlashgan yechimi deb,* $A(x,D)u(x,t), \partial_t^\rho u(x,t) \in C((0,T],L_2(\Omega))$ *bo‘lib, (35), (37), (38) shartlar* Ω *ning deyarli barcha nuqtalarida bajaruvchi va (44) munosabat umumlashgan ma’noda bajaruvchi* $\{u(x,t), f(x)\}$ *funksiyalar juftligiga aytamiz, bu yerda* $f \in L_2(\Omega)$, .

17-teorema. *Aytaylik,* $\varphi \in L_2(\Omega)$ *va* $\Psi \in W_2^m(\Omega)$ *bo‘lsin. Bundan tashqari,* Ψ *funksiya (37) chegaraviy shartni qanoatlantirsin. U holda, (35), (44), (37), (38) chegaraviy masalaning yagona* $\{u(x,t), f(x)\}$ *umumlashgan yechimi mavjud va bu yechim uchun quyidagi baholar o‘rinli*

$$\| f \|_{L_2(\Omega)} \le C[\| \varphi \|_{L_2(\Omega)} + \| \Psi \|_{W_2^m(\Omega)}]. \quad (45)$$

$$c \| u \|_{W_2^m(\Omega)} \le \| Au \|_{L_2(\Omega)} \le C[\frac{1}{t} \| \varphi \|_{L_2(\Omega)} + \| \Psi \|_{W_2^m(\Omega)}], \quad t > 0, \quad (46)$$

$$\| \partial_t^\rho u \|_{L_2(\Omega)} \le C[\frac{1}{t} \| \varphi \|_{L_2(\Omega)} + \| \Psi \|_{W_2^m(\Omega)}], \quad t > 0. \quad (47)$$

XULOSA

Ushbu dissertatsiya giperbolik va parabolik tipdagi klassik tenglamalar hamda subdiffuziya tenglamalari uchun boshlang‘ich-chegaraviy va teskari masalalarni o‘rganishga bag‘ishlangan. Ushbu masalalarni yechishning asosiy usuli klassik Furye usulidir. Shuning uchun butun ishda tenglamaning elliptik qismini L_2 da to'la ortonormal xos funksiyalar sistemasiga ega bo'lishi talab qilinadi. Avvalgi o'rganilgan maqolalarda mualliflar, asosan, elliptik qismlarining xos funksiyalari aniq ko‘rinishga ega bo'lgan tenglamalar uchun klassik yechimni topishni o‘rganganishgan. Masalan intervalda u_{xx} yoki to'g'ri to'rtburchakda Laplas tenglamasi bolgan hollarda xos funksiyalarning ko‘rinishi trigonometrik funksiyalar yoki ularning ko‘paytmasidan iborat bo‘ladi. Ushbu dissertatsiyaning asosiy maqsadi Furye usulini xos funksiyalarning aniq ko‘rinishi ma'lum bo'lmagan hollarda qo'llashdir. Biz bu maqsadga erishish uchun V.A. Il’in, M.A. Krasnoselskiy va Sh.A. Alimovning fundamental natijalariga tayandik.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1. Elliptik qismi umumiy shaklga ega bo‘lgan va ixtiyoriy ko‘p o‘lchovli sohada (yetarli darajada silliq chegarali) va N o‘lchovli torda aniqlangan giperbolik va parabolik tenglamalar uchun boshlang‘ich-chegaraviy masalalarning klassik va umumlashgan yechimlari topilgan.

2. Ixtiyoriy tartibli elliptik operator qatnashgan Kaputo va Riman-Liuvill vaqt bo‘yicha kasr hosilali subdiffuziya tenglamasi uchun boshlang‘ich-chegaraviy masalalaring klassik yechimining yagonaligi va mavjudligi Furye usuli bilan isbotlangan.

3. Subdiffuziya tenglamasida Riman-Liuvill kasr tartibli hosilasi qatnashgan vaqtda Mittag-Leffler funksiyalarini aniqroq bahosidan foydalanib, boshlang‘ich shartdagi funksiyaning kengroq sinfi uchun aralash masalaning yechimi mavjudligi isbotlangan. Shuningdek, subdiffuziya tenglamasida Riman-Liuvill kasr hosilasining o‘rniga Kaputo kasr hosilasi olinganda ushbu natijani olib bo‘lmasligiga doir misol keltirilgan.

4. Riman-Liuvill va Kaputo ma’nosidagi kasr hosilali, elliptik qismi umumiy shaklga ega va ixtiyoriy ko‘p o‘lchovli sohada (yetarli darajada silliq chegara bilan) va N o‘lchovli torda aniqlangan subdiffuziya tenglamasining o‘ng tomonini aniqlash bo‘yicha teskari masalasining klassik va umumlashgan yechimlari topilgan.

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INSTITUTE OF MATHEMATICS

OQILA MUKHIDDINOVA TULQIN QIZI

INITIAL - BOUNDARY VALUE AND INVERSE PROBLEMS FOR HIGHER ORDER EQUATIONS

01.01.02 – Differential equations and mathematical physics

ABSTRACT OF THESIS OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

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INTRODUCTION

Actuality and demand of the theme of the dissertation. Many scientific and practical studies at the world level, in many cases of the equations of mathematical physics for example, that a large number of problems on vibrations of rods, plates, etc. leads to high-order differential equations. Naturally, proceeding from the physical meaning of the problem, it is of interest to consider similar problems for arbitrary bounded spatial domains, where the explicit form of the eigenfunctions is not known. This dissertation is devoted to the classical Fourier method study of forward and inverse problems for the classical diffusion and subdiffusion differential equations, the elliptic part of which is a differential expression of arbitrary order and defined in an arbitrary multidimensional domain with a smooth boundary.

In recent years, in our country, more attention is paid to mathematics, physics, geology and biological sciences, which are of fundamental importance in the economic development of our country. In particular, special attention is paid to the development of differential equations and mathematical physics, one of the main areas of mathematics, which has an applied character. A lot of research is also leads to inverse problems of determining the right-hand side of subdiffusion equations. In almost all of these papers, the elliptic part of the subdiffusion equation is an ordinary differential expression. Therefore, for parabolic, hyperbolic and subdiffusion equations, the elliptic part of which is an arbitrary differential operator defined in an arbitrary domain, the solution of direct and inverse problems is a targeted scientific research.

In our country, much attention has been paid to develop disciplines natural and life sciences such as biology, geology, mathematics and physics. Particularly, the developing of the theory of fractional and partial differential equations. Because this theory helps to understand the behavior of many phenomena in the mechanics, electronics, control theory, physiology and biological systems. Investigations on the international level in such important areas as noted above has been considered the main task of fundamental research[1]. Currently, the development of investigations on the theory fractional and partial differential equations plays an important role in the implementation of this decree. Significant results have been obtained by specialists of our country in these areas and they play an important role in the implementation of the above decree. It is gratifying to note that research at the international level in such important areas as differential equations and mathematical physics is considered in our country as the main direction in fundamental research.

The subject and object of research of this thesis are in line with tasks identified in the Decrees of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 “On the strategy of action for the further development Of the Republic of Uzbekistan", UP-2789 dated February 17, 2017 “On measures to further improve of the activities of the Academy of Sciences, organization, management and financing of research activities”, PP-3682 from April 27, 2018 “On measures to further improve the system of practical implementation of innovative ideas, technologies

[1] Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May « On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan» № 292 dated May 17, 2017.

and projects" and PP-4387 from July 9, 2019 “On measures to further development of mathematical education and science, total improvement of the activity of the Uzbekistan Academy of Sciences V.I.Romanovsky Institute of Mathematics" and also PD-4708 from May 7, 2020 "On measures to improve the quality of education and research in mathematics" as well as in other regulations related to basic science.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV, «Mathematics, Mechanics and Computer Science» .

The degree of scrutiny of the problem. The study of the existence and uniqueness of classical solutions of initial-boundary value problems was carried out by many famous mathematicians such as V.A. Il’in, O.A. Oleinik, O.A. Ladyzhinskaya, P.E. Sobolevskii, M.A. Krasnosel’skii and others. In the fundamental work of V.A. Il’in published back in 1960, a review of works in this direction is given and the latest results of the author are presented. Here the classical Fourier method has been applied to equations whose elliptic part is a second-order symmetric differential expression. The case when the elliptic part has an order higher than the second was considered in the well-known monograph by Krasnosel’skii et al. Here, the applicability of the Fourier method is proved in the case when the input data of the initial boundary value problem belong to the domain of a power of the elliptic operator. It should be noted that, on the one hand, uniqueness issues are not considered in this monograph, and on the other hand, the conditions on the input data in this monograph are difficult to check. It should also be noted that the authors of this work did not introduce the concept of a generalized solution and did not investigate the issues of their existence.

The theory of differential equations with fractional derivatives has gained considerable popularity and importance in the past few decades, mainly due to its applications in numerous seemingly distant fields of science and technology. In tern, the well-deserved popularity of the theory attracted the attention of specialists, causing a large number of investigations on mathematical aspects of fractional differential equations and methods to solve them.

Initial-boundary value problems for subdiffusion equations have been studied by many specialists. However, the elliptic part in these papers of Kilbas A.A. was mainly the one-dimensional differential expression u_{xx}. With the participation of the Laplace operator, such equations were considered in the papers of A.V. Pskhu and with pseudodifferential operators with constant coefficients defined in the entire space R^N in the monographs of S. Umarov. The article of M. Ruzhansky takes an abstract self-adjoint operator as the elliptic part, but this article investigates the existence of only a generalized solution.

A lot of research is also devoted to inverse problems of determining the right-hand side of subdiffusion equations. In almost all of these papers, the elliptic part of the subdiffusion equation is an ordinary differential expression. Exceptions are the papers of N.A. Asl and D. Rostamy, Z. Li, Y. Liu and M. Yamamoto, S.A. Malik and S. Aziz, W. Rundell and Z. Zhang, L. Sun, Y. Zhang and T. Wei in which the

elliptic part is either the Laplace operator or a second-order operator; these papers examine the existence of a generalized solution. The paper of M. Ruzhansky, N. Tokmagambetov and B.T. Torebek studied the inverse problem of determining the right-hand side of the subdiffusion equation with Caputo fractional derivatives whose elliptic part is an arbitrary elliptic operator with discrete spectrum and the author proved the existence and uniqueness of a generalized solution by the Fourier method. The paper of M. Ruzhansky, N. Tokmagambetov, B.T. Torebek and most of the other papers, including N.A. Asl and D. Rostamy, Z. Li, Y. Liu and M. Yamamoto, W. Rundell and Z. Zhang, L. Sun, Y. Zhang and T. Wei take the Caputo derivative as the fractional derivative. We point out that the elementary property of the Caputo fractional derivative that the derivative of a constant is zero greatly facilitates the study of inverse problems for the subdiffusion equation with these derivatives. In the work of K.M. Furati, O.S. Iyiola, M. Kirane and S.A. Malik, S. Aziz the fractional derivative in the subdiffusion equation is a two-parameter generalized Hilfer fractional derivative. Note also that the papers of K.M. Furati, O.S. Iyiola and M. Kirane, Z. Li, Y. Liu and M. Yamamoto, L. Sun, Y. Zhang and T. Wei contain a survey of papers dealing with inverse problems of determining the right-hand side of the subdiffusion equation. It should be noted that the inverse problem of determining (in the classical sense) the right-hand side of the subdiffusion equation, the elliptic part of which is an elliptic operator of any order, defined in an arbitrary bounded domain, has not been considered by anyone before.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation research is done in accordance with the planned theme of scientific research grant no. F-FA-2021-424 of the Ministry of Innovative Development of the Republic of Uzbekistan, at the Institute of Mathematics after named V.I. Romanovskiy.

The aim of research work is to prove unique solvability in the classical sense of forward and inverse problems for parabolic, hyperbolic and subdiffusion equations whose elliptic part is an arbitrary elliptic operator defined in an arbitrary domain.

Research problems:

To prove unique solvability in the classical sense of initial-boundary value problems for hyperbolic and parabolic equations whose elliptic part is an arbitrary elliptic operator defined in an arbitrary N dimensional domain. To introduce the concept of a generalized solution and to investigate the questions of their existence.

To prove unique solvability in the classical sense of initial-boundary value problems for subdiffusion equations with the Caputo and Riemann-Liouville derivatives whose elliptic part is an arbitrary elliptic operator defined in an arbitrary N dimensional domain.

To prove unique solvability in the classical sense of the inverse problems on determining the right-hand side of the subdiffusion equations with the Caputo and Riemann-Liouville derivatives whose elliptic part is an arbitrary elliptic operator defined in an arbitrary N dimensional domain. To introduce the concept of a generalized solution of the inverse problem and to investigate the questions of their existence.

The research object. The classical hyperbolic and parabolic equations,

subdiffusion equations with the Caputo and Riemann-Liouville derivatives whose elliptic part is an arbitrary elliptic operator defined in an arbitrary *N* dimensional domain.

The research subject. The initial-boundary value problems and the inverse problems on determining the right-hand side of the equations.

Research methods. In the research the methods of functional analysis, spectral theory and the Fourier methods are used.

Scientific novelty of the research work consists of the following:

It is proved unique solvability in the classical sense of initial-boundary value problems for hyperbolic and parabolic equations whose elliptic part is an arbitrary elliptic operator defined in an arbitrary dimensional domain;

It is proved unique solvability in the classical sense of initial-boundary value problems for subdiffusion equations with the Caputo and Riemann-Liouville derivatives whose elliptic part is an arbitrary elliptic operator defined in an arbitrary dimensional domain;

It is proved unique solvability in the classical sense and proved the existence of a generalized sense of the inverse problems on determining the right-hand side of the subdiffusion equations with the Caputo and Riemann-Liouville derivatives whose elliptic part is an arbitrary elliptic operator defined in an arbitrary dimensional domain.

Practical results of the research. The obtained results and used methods in the dissertation can be taught as a graduate course for masters and doctoral students of higher education institutions.

The reliability of the results of the study. The results were obtained using the methods of functional analysis, spectral theory and the Fourier method. All obtained results are mathematically correct.

Scientific and practical significance of the research results. The scientific significance of the research results is that they can be used for further study of the forward and inverse problems for different partial differential equations. In particular, the techniques and methods developed in this dissertation can be used to find the solutions of forward and inverse problems. The practical significance of the thesis is that the results can be used in the theory of forward and inverse problems for differential operators, including linear systems of fractional differential equations, differential models with involution, fractional Sturm-Liouville operators, and others.

Implementation of the research results. Based on the results obtained on the problem of the initial-boundary value and inverse problems for higher order equations:

the solutions to the initial-boundary and inverse problems of subdiffusion equations, were used in the framework No. AAAA-A19-119072290002-9 of the international project «Natural disasters of Kamchatka-earthquakes and volcanic eruptions», to describe the mass transfer of radioactive radon gas, an operational precursor of earthquakes, from the ground to the surface layer of the atmosphere. (Reference from Kamchatka State University named after Vitus Bering dated october 24, 2022, № 44-12 Russian Federation). It should also be noted that

implementations of scinetific results were used in the development of algorithms for solving inverse problems of geophysics;

the solutions to the initial-boundary and inverse problems of subdiffusion equations involving fractional derivatives were used in the performance of research work within the framework No. NIOKTR AAAA-A19-119013190077-1, 2019-2021 of the topic «Fractional calculus and differential equations of fractional and distributed order» (Reference from institute of Applied Mathematics and Automation of the Kabardino-Balkarian Scientific Center of the Russian Academy of Sciences dated May 20, 2022, № 01-14/30). The implementations of scinetific results gave opportunity to solve local and nonlocal boundary value problems for evolutionary equations with fractional time derivatives, which in recent decades have been effectively used in mathematical modeling of various physical and biological processes.

Approbation of the research results. The main results of the research have been discussed at 6 international and 5 national scientific conferences.

Publications of the research results. On the topic of the dissertation 18 research papers have been published in the scientific journals, 6 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis and 4 of them were published in international journals, 3 of them are included the SCOPUS information database, 3 papers published in national mathematical journals and 11 theses.

The structure and volume of the dissertation. The dissertation consists of an introduction, four chapters, conclusion and bibliography. The total volume of the thesis is 126 pages.

MAIN CONTENT OF THE DISSERTATION

The introduction substantiates the actuality and demand of the theme of the dissertation, determines the correspondence of the study to priority areas of development of science and technology, provides an overview of foreign scientific research on the dissertation topic and the degree of study of the problem, formulates goals and objectives, identifies the object and subject of the study, outlines the scientific novelty and practical results of the study, the theoretical and practical significance of the results obtained is disclosed, information is given on the implementation of the research results, on published works and information on the structure of the dissertation.

The first chapter of the dissertation, titled **“Preliminary information”**, is auxiliary in nature and it was created for the convenience of reading the dissertation. There are no new results here, and only the necessary definitions and assertions are collected. Basically, the theorems are given without proofs. However, in the case when the domain under consideration coincides with the torus $\mathbb{T}^N$ give very simple proofs of some theorems. For example, the Sobolev embedding theorem, the lemma from the monograph by Krasnoselsky et al., the coincidence of the domain of definition of the fractional power of the elliptic operator with the Liouville space and uniformly

convergence of the Fourier series of a smooth function are proved quite simply. It is well known that, in the general case, these assertions are difficult to prove.

Let us present some definitions and assertions from Chapter one. Let Ω be any N dimensional domain, possibly coinciding with the whole N dimensional space $\mathbb{R}^N$. A differential expression

$$A(x,D)u = \sum_{0\le|\alpha|\le m} a_\alpha(x)D^\alpha u, (x\in\Omega) \tag{1}$$

of even order $m=2k$ is said to be elliptic at a point $x_0\in\Omega$ if the corresponding characteristic polynomial

$$a(x_0,\xi) = \sum_{|\alpha|=m} a_\alpha(x_0)\xi^\alpha \tag{2}$$

is positive for every nonzero real vector ξ. Differential expression (1) is called elliptic in the domain Ω if it is elliptic at every point $x_0\in\Omega$. We denote by $A(D)$ a homogeneous elliptic differential expression: $A(D)=\sum_{|\alpha|=m}a_\alpha D^\alpha$.

The class of functions $f\in L_2\left(\mathbb{T}^N\right)$ for which the norm

$$\|f\|_{L_2^a(\mathbb{T}^N)} = \sum_{n\in Z^N}\left(1+|n|^2\right)^a |f_n|^2,\ a>0, \tag{3}$$

is finite for a given fixed real $a>0$ is called the *Liouville class* $L_2^a\left(\mathbb{T}^N\right)$. The classes $L_2^a\left(\mathbb{T}^N\right)$ coincide for natural a with the Sobolev classes $W_2^a\left(\mathbb{T}^N\right)$ and are their natural extensions to fractional values of a. Similarly, the Liouville class $L_2^a(\Omega)$ is a generalization of the Sobolev class $W_2^a(\Omega)$ to a non-integer a.

The main results of the dissertation begin with the second chapter, titled **"Initial-boundary value problems for hyperbolic and parabolic equations"** In first section we investigated an initial-boundary value problem for a parabolic equation defined on the N dimensional torus $\mathbb{T}^N$ with an elliptic differential operator of arbitrary order.

Problem: Find a 2π-periodic on each variable function $u(x,t)\in C\left(\mathbb{T}^N\times[0;T]\right)$ with properties $u_t(x,t),\ A(D)u(x,t)\in C\left(\mathbb{T}^N\times(0;T]\right)$, that satisfies the following equation:

$$u_t(x,t)+A(D)u(x,t)=f(x,t),\quad x\in\mathbb{T}^N,\ 0<t\le T, \tag{4}$$

and the initial condition

$$u(x,0)=\varphi(x),\quad x\in\mathbb{T}^N. \tag{5}$$

Here we assume that the given functions $\varphi(x)$ and $f(x,t)$ have a periodicity of 2π for each variable.

Theorem 1. *Let* $\tau > \frac{N}{2}$ *and*

(a) $\varphi(x) \in L_2^{\tau}\left(\mathbb{T}^N\right)$;

(b) $f(x,t) \in L_2^{\tau}\left(\mathbb{T}^N\right)$ *for any* $t \in [0,T]$;

(c) the $L_2\left(\mathbb{T}^N\right)$*-norm* $F(t) = \| f(\cdot,t) \|^2_{L_2^{\tau}(\mathbb{T}^N)}$ *be continuous on the segment* $[0,T]$.

Then there exists a unique periodic solution of problem (4)-(5), moreover for $x \in \mathbb{T}^N$ *, it can be represented in the form of the following absolute convergent series:*

$$u(x,t) = \sum_{n \in Z^n} \left[e^{-A(n)t} \varphi_n + \int_0^t e^{-A(n)(t-s)} f_n(s) ds \right] e^{inx}, t \in [0,T]. \qquad (6)$$

It should be noted that the results of this section can be easily translated to the case when an elliptic operator with constant coefficients is replaced by an arbitrary elliptic operator defined in an arbitrary N dimensional domain. To illustrate this, in second section we studied initial-boundary value problems for hyperbolic equations, the elliptic part of which has the most general form and is defined in an arbitrary multidimensional domain (with a sufficiently smooth boundary).

Consider the equation

$$u_{tt} + A(x,D)u(x,t) = f(x,t), \quad x \in \Omega, \quad t > 0; \qquad (7)$$

with the initial

$$u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x), \quad x \in \Omega; \qquad (8)$$

and with the boundary

$$B_j u(x,t) = \sum_{|\alpha| \le m_j} b_{\alpha,j}(x) D^{\alpha} u(x,t) = 0, \qquad (9)$$

$$0 \le m_j \le m-1, \quad j = 1,2,...,l; \quad x \in \partial\Omega;$$

conditions. Here $f(x,t), \varphi(x), \psi(x)$ and coefficients $b_{\alpha,j}(x)$ - are given functions.

Application of the Fourier method leads us to the following spectral problem

$$A(x,D)v(x) = \lambda v(x) \quad x \in \Omega; \qquad (10)$$

$$B_j(x,D)v(x) = 0, \quad j = 1,2,...,l; \quad x \in \partial\Omega; \qquad (11)$$

This spectral problem was studied in the well-known paper by Agmon (1962). He proved that under certain conditions on the coefficients of the operators $A(x,D)$, $B_j(x,D)$ and on the boundary of the domain Ω, this spectral problem has a complete system of eigenfunctions $\{v_k\}$ orthonormal in $L_2(\Omega)$ and a countable set of non-negative eigenvalues $\{\lambda_k\}$. We will always assume that these conditions are met.

Let τ be any real number. We introduce the following operator

$$\hat{A}^{\tau} f(x) = \sum \lambda_k^{\tau} f_k v_k(x), \qquad (12)$$

with the domain of definition

$$D(\hat{A}^{\tau}) = \{ f(x) \in L_2(\Omega) : \sum | \lambda_k |^{2\tau} | f_k |^2 < \infty \}. \qquad (13)$$

Theorem 2. *Let the functions* $f(x,t),\varphi(x),\psi(x)$ *be continuous in the domains indicated in (7) and (8). Then there only one classical solution to the mixed problem (7)-(9).*

We compose the following formal series (note, $\lambda_k > 0$ for all k, since operator A is positive)

$$u(x,t) = \sum_{k=1}^{\infty}[\frac{1}{\sqrt{\lambda_k}}\psi_k \sin\sqrt{\lambda_k}t + \varphi_k \cos\sqrt{\lambda_k}t + \frac{1}{\sqrt{\lambda_k}}\int_0^t f_k(\xi)\sin\sqrt{\lambda_k}(t-\xi)d\xi]v_k(x). \quad (14)$$

Here ψ_k, φ_k and $f_k(t)$ are the Fourier coefficients, respectively, of the functions, $\psi(x)$, $\varphi(x)$, and $f(x,t)$, by the system of eigenfunctions $\{v_k(x)\}$ defined as the scalar product in $L_2(\Omega)$; for example $\psi_k = (\psi, v_k)$.

Theorem 3. *Let* $\tau > 1 + \frac{N}{2m}$. *Let further* $\varphi \in D(\hat{A}^{\tau})$, $\psi \in D(\hat{A}^{\tau-\frac{1}{2}})$ *and* $f(x,t) \in D(\hat{A}^{\tau-\frac{1}{2}})$ *for all* $t \in [0,T]$, *moreover, let the function*

$$F(t) = \| \hat{A}^{\tau-\frac{1}{2}} f(x,t) \|_{L_2(\Omega)}$$

be continuous on $[0,T]$. *Then the solution to the mixed problem (7)-(9) exists and it can be represented in the form of series (14).*

Theorem 4. *Let conditions of theorem 3. be satisfied. Then for solution (14) to problem (7)-(9) the following estimate holds:*

$$\sup_{t\in[0,T]} \| u(x,t) \|^2_{L_2(\Omega)} \le C[\| \varphi \|^2_{L_2(\Omega)} + \| \psi \|^2_{L_2(\Omega)} + T\int_0^T \| f(x,\xi) \|^2_{L_2(\Omega)} d\xi], \quad (15)$$

where the constant C *does not depend on* T.

Following K.B. Sabitov we introduce the concept of a generalized solution of problem (7)-(9) according to the following definition:

Definition 1. *Let functions* $\varphi_k(x)$, $\psi_k(x)$ *and* $f_k(x,t)$ *satisfy the conditions of Theorem 3 and converge in the norm of* $L_2(\Omega)$ *(for the last function, the indicated convergence is uniformly with respect to* $t \in [0,T]$*) to the functions* $\varphi(x)$, $\psi(x)$ *and* $f(x,t)$ *respectively. A function* $u(x,t)$ *will be called generalized solution to problem (7)-(9), if there exists a sequence of classical solutions* $u_k(x,t)$ *to problem (7)-(9) with the initial data* $\varphi_k(x)$, $\psi_k(x)$ *and the right-hand side* $f_k(x,t)$, *converging uniformly with respect to* $t \in [0,T]$ *in* $L_2(\Omega)$ *to the function* $u(x,t)$.

Theorem 5. *Let functions* $\varphi(x)$, $\psi(x)$ *and* $f(x,t)$ *(for all* $t \in [0,T]$*) belong to the space* $L_2(\Omega)$. *Then there exists the unique and stable generalized solution of the problem (7)-(9), which is determined by the sum of the series (14) and the norm of the solution* $\| u(x,t) \|_{L_2(\Omega)}$ *is a continuous function with respect to* $t \in [0,T]$.

Let us now consider the initial-boundary value problem for a parabolic

equation: find a function $u(x,t) \in C(\overline{\Omega} \times [0,T])$ such that $u_t(x,t), A(x,D)u(x,t) \in C(\overline{\Omega} \times (0,T])$ and satisfying the equation

$$u_t + A(x,D)u(x,t) = f(x,t), \quad x \in \Omega, \quad t > 0; \tag{16}$$

the initial condition

$$u(x,0) = \varphi(x), \quad x \in \Omega; \tag{17}$$

and boundary conditions (9).

Theorem 6. *Let* $\tau > \frac{N}{2m}$ *and* $\varphi(x) \in D(\hat{A}^\tau)$. *Further, let* $f(x,t) \in D(\hat{A}^\tau)$ *for all* $t \in [0,T]$, *moreover the function*

$$F(t) = \| \hat{A}^\tau f(x,t) \|_{L_2(\Omega)}$$

be continuous in the variable $t \in [0,T]$. *Then there exists the unique solution to Problem (16), (17), (9) and this solution can be represented in the form of the series*

$$u(x,t) = \sum_{k=1}^{\infty} [e^{-\lambda_k t} \varphi_k + \int_0^t e^{-\lambda_k (t-\xi)} f_k(\xi) d\xi] v_k(x), \quad t \in (0,T],$$

which uniformly and absolutely converges in the closed domain $(\overline{\Omega})$, *where* φ_k *and* $f_k(t)$ *are the corresponding Fourier coefficients.*

The third chapter of the dissertation is titled **"Initial-boundary value problems for the subdiffusion equations".** In first section we studied an initial-boundary value problem for a time-fractional subdiffusion equation with an arbitrary order elliptic differential operator.

Let $\rho \in (0,1)$ be a constant number. Consider the differential equation

$$\partial_t^\rho u(x,t) + A(x,D)u(x,t) = f(x,t), \quad x \in \Omega, \quad t > 0; \tag{18}$$

with initial

$$\lim_{t \to 0} \partial_t^{\rho-1} u(x,t) = \varphi(x), \quad x \in \Omega; \tag{19}$$

and boundary

$$B_j u(x,t) = \sum_{|\alpha| \le m_j} b_{\alpha,j}(x) D^\alpha u(x,t) = 0, \tag{20}$$

$$0 \le m_j \le m-1, \quad j = 1,2,...,l; \quad x \in \partial\Omega;$$

conditions, where $f(x,t), \varphi(x)$ and coefficients $b_{\alpha,j}(x)$ are given functions. The uniqueness and existence of the classical solution of the posed problem are proved by the classical Fourier method. Sufficient conditions for the initial function and for the right-hand side of the equation are indicated, under which the corresponding Fourier series converge absolutely and uniformly.

Theorem 7. *Let* $f(x,t) \in C(\Omega \times (0,\infty))$ *and* $\varphi \in C(\Omega)$. *Then problem (18)-(20) may have only one solution.*

Theorem 8. *Let* $\tau > \frac{N}{2m}$ *and* $\varphi \in D(\hat{A}^\tau)$. *Moreover, let* $f(x,t) \in D(\hat{A}^\tau)$ *for all*

$t \in [0,T]$, *and the function*

$$F(t) = \| \hat{A}^{\tau} f(x,t) \|_{L_2(\Omega)}$$

be continuous on $t \in [0,T]$. *Then there exists a solution of initial-boundary value problem (18)-(20) and it has the form of series*

$$u(x,t) = \sum_{1}^{\infty} [\varphi_j t^{\rho-1} E_{\rho,\rho}(-\lambda_j t^{\rho}) + \int_0^t f_j(t-\xi)\xi^{\rho-1} E_{\rho,\rho}(-\lambda_j \xi^{\rho}) d\xi] v_j(x), \quad (21)$$

which absolutely and uniformly converges on $x \in \bar{\Omega}$ *and for each* $t \in (0,T]$.

In the case of an initial-boundary value problem on N -dimensional torus, one can easily see that these conditions are not only sufficient, but also necessary. Let us consider the last case. So let $A(D) = \sum_{|\alpha|=m} a_\alpha D^\alpha$ be a homogeneous symmetric nonnegative elliptic differential operator with constant coefficients. Let the differential equation and initial condition have the form ($0 < \rho < 1$)

$$\partial_t^{\rho} u(x,t) + A(D)u(x,t) = f(x,t), \quad x \in \mathbb{T}^N, \quad t > 0; \quad (22)$$

$$\lim_{t \to 0} \partial_t^{\rho-1} u(x,t) = \varphi(x), \quad x \in \mathbb{T}^N; \quad (23)$$

Instead of boundary conditions (20) we consider the 2π -periodic in each argument x_j functions and suppose that $\varphi(x)$ and $f(x,t)$ are also 2π -periodic functions in x_j.

Theorem 9. *Let* $\tau > \frac{N}{2}$ *and* $\varphi \in L_2^{\tau}(\mathbb{T}^N)$. *Moreover, let* $f(x,t) \in L_2^{\tau}(\mathbb{T}^N)$ *for any* $t \in [0,T]$ *and the function*

$$F(t) = \| f(\cdot,t) \|^2_{L_2^{\tau}(\mathbb{T}^N)}$$

be continuous on $[0,T]$. *Then there exists a solution of initial-boundary value problem (22)-(23) and it has the form*

$$u(x,t) = \sum_{n \in \mathbb{Z}^N} [\varphi_n t^{\rho-1} E_{\rho,\rho}(-A(n)t^{\rho}) + \int_0^t f_n(t-\xi)\xi^{\rho-1} E_{\rho,\rho}(-A(n)\xi^{\rho}) d\xi] e^{inx},$$

which absolutely and uniformly converges on $x \in \mathbb{T}^N$ *and for each* $t \in (0,T]$.

Remark 1. *Note, when* $\tau > \frac{N}{2}$, *according to the Sobolev embedding theorem, all functions in* $L_2^{\tau}(\mathbb{T}^N)$ *are* 2π *-periodic continuous functions. The fulfillment of the inverse inequality* $\tau \le \frac{N}{2}$, *admits the existence of unbounded functions in* $L_2^{\tau}(\mathbb{T}^N)$. *Therefore, condition* $\tau > \frac{N}{2}$ *of this theorem is not only sufficient for the statement to be hold, but it is also necessary.*

In second section we studied an initial-boundary value problem for a time-fractional subdiffusion equation with the Riemann-Liouville derivatives on N-dimensional torus. Let $\rho \in (0,1)$ be a constant number. Consider the initial-boundary value problem

$$\partial_t^\rho u(x,t) - \Delta u(x,t) = f(x,t), \quad x \in \mathbb{T}^N, \quad 0 < t \le T, \tag{24}$$

$$\lim_{t \to 0} \partial_t^{\rho-1} u(x,t) = \varphi(x), \quad x \in \mathbb{T}^N. \tag{25}$$

Instead of boundary conditions we consider the 2π-periodic in each argument x_j functions and suppose that $\varphi(x)$ and $f(x,t)$ are also 2π-periodic functions in x_j. The condition imposed on the initial function in first section has the form $\varphi \in L_2^\tau(\mathbb{T}^N)$, $\tau > \frac{N}{2}$. Inequality $\tau > \frac{N}{2}$ appears not only in this chapter, but also both in the second and fourth chapters of the dissertation. Of course, this condition cannot be replaced by $\varphi \in L_2^{\frac{N}{2}}(\mathbb{T}^N)$, since it is well known that in this case functions from the class $L_2^{\frac{N}{2}}(\mathbb{T}^N)$ may be unbounded. Naturally, this raises the question: is it possible to replace condition $\varphi \in L_2^\tau(\mathbb{T}^N)$, $\tau > \frac{N}{2}$, by $\varphi \in C(\mathbb{T}^N) \cap L_2^{\frac{N}{2}}(\mathbb{T}^N)$. In this section we prove that this is possible. Also in second section the condition on the right side of the equation found in first section is weakened by replacing the condition $\| f(\cdot,t) \|^2_{L_2^a(\mathbb{T}^N)} \in C[0,T]$ with $\| t^{1-\rho} f(\cdot,t) \|^2_{L_2^a(\mathbb{T}^N)} \in C[0,T]$.

Theorem 10. *Let* $\varphi \in C(\mathbb{T}^N) \cap L_2^{\frac{N}{2}}(\mathbb{T}^N)$. *Moreover, let* $a > \frac{N}{2}$ *and* $f(x,t) \in L_2^a(\mathbb{T}^N)$ *for* $0 < t \le T$ *and* $\| t^{1-\rho} f(\cdot,t) \|^2_{L_2^a(\mathbb{T}^N)} \in C[0,T]$. *Then there exists a solution of initial-boundary value problem (24)-(25) and it has the form*

$$u(x,t) = \sum_{n \in \mathbb{Z}^N} [\varphi_n t^{\rho-1} E_{\rho,\rho}(-|n|^2 t^\rho) + \int_0^t f_n(t-\xi) \xi^{\rho-1} E_{\rho,\rho}(-|n|^2 \xi^\rho) d\xi] e^{inx}, \tag{26}$$

which absolutely and uniformly converges on $x \in \mathbb{T}^N$ *and for each* $t \in (0,T]$, *where* φ_n *and* $f_n(t)$ *are corresponding Fourier coefficients. Moreover, the series obtained after applying term-wise the operators* ∂_t^ρ *and* A *also converge absolutely and uniformly on* $x \in \mathbb{T}^N$ *and for each* $t \in (0,T]$.

In third section we investigated an initial-boundary value problem for a time-fractional subdiffusion equation with the Caputo derivatives on N-dimensional torus by the classical Fourier method.

Let $\rho \in (0,1)$ be a constant number. Consider the initial-boundary value

problem

$$D_t^{\rho}u(x,t)+Au(x,t)=f(x,t),\quad x\in\mathbb{T}^N,\quad 0<t\le T, \tag{27}$$

$$u(x,0)=\varphi(x),\quad x\in\mathbb{T}^N, \tag{28}$$

where $A=-\Delta$-the Laplace operator, f and φ are given continuous functions. Since the method we use is based on the expansion in terms of eigenfunctions of an elliptic operator, this method can also be applied to an arbitrary elliptic operator even with variable coefficients, provided that the operator has a complete orthonormal system of eigenfunctions.

Theorem 11. *Let* $a>\frac{N}{2}$ *and* $\varphi\in L_2^a(\mathbb{T}^N)$. *Moreover, let* $f(x,t)\in L_2^a(\mathbb{T}^N)$ *for* $t\in[0,T]$ *and* $\|f(\cdot,t)\|^2_{L_2^a(\mathbb{T}^N)}\in C[0,T]$. *Then, there exists a solution of initial-boundary value problem (27)-(28) and it has the form*

$$u(x,t)=\sum_{n\in Z^N}[\varphi_n E_{\rho,1}(-|n|^2\,t^{\rho})+\int_0^t f_n(t-\xi)\xi^{\rho-1}E_{\rho,\rho}(-|n|^2\,\xi^{\rho})d\xi]e^{inx}, \tag{29}$$

which converges absolutely and uniformly on $x\in\mathbb{T}^N$ *and for each* $t\in(0,T]$. *Here,* φ_n *and* $f_n(t)$ *are corresponding Fourier coefficients. Moreover, the series obtained after applying term-wise the operators* D_t^{ρ} *and* A *also converge absolutely and uniformly on* $x\in\mathbb{T}^N$ *and for each* $t\in(0,T]$.

A result similar to Theorem 11 was obtained in second section for a subdiffusion equation with the Riemann-Liouville derivative. But the condition found for the initial function $\varphi(x)$ in second section is less restrictive. For example, in the one-dimensional case, it has been proved that it is sufficient to require $\varphi(x)\in C(T^1)\cap L_2^{\frac{1}{2}}(\mathbb{T}^N)$. At the end of this paragraph we prove that even Hölder continuous with exponent $a=\frac{1}{2}$ for the initial function is not sufficient for the validity of Theorem 11 in the one-dimensional case.

The fourth chapter of the dissertation is titled **"Inverse problems for subdiffusion equations"**. In first section we investigated an inverse problem of determining the right-hand side of a subdiffusion equation with Caputo fractional derivative whose elliptic part has the most general form and is defined on an N-dimensional torus $\mathbb{T}^N$.

Problem: Find a pair of 2π-periodic in each variable x_j functions $\{u(x,t),f(x)\}$: $f(x)\in C(\mathbb{T}^N)$ and $u(x,t)\in C(\mathbb{T}^N\times[0,T])$ with the properties $D_t^{\rho}u(x,t),A(D)u(x,t)\in C(\mathbb{T}^N\times(0,T])$ that satisfies the following differential equation ($0<\rho\le 1$)

$$D_t^\rho u(x,t) + A(D)u(x,t) = f(x), \quad x \in \mathbb{T}^N, \quad t > 0; \tag{30}$$

and the initial

$$u(x,0) = \varphi(x), \quad x \in \mathbb{T}^N; \tag{31}$$

and the additional conditions

$$u(x,T) = \Psi(x), \quad x \in \mathbb{T}^N. \tag{32}$$

We will assume that the given functions $\varphi(x)$ and $\Psi(x)$ are 2π-periodic in each variable x_j as well.

Since the Fourier method is used here, the results of this section can be generalized to more general equations.

Theorem 12. *Let* $\varphi \in L_2^\tau(\mathbb{T}^N)$ *and* $\Psi \in L_2^{\tau+m}(\mathbb{T}^N)$, *where* $\tau > \frac{N}{2}$. *Then there exists a unique periodic solution* $\{u(x,t), f(x)\}$ *of the inverse problem (30)-(32) which can be represented in the form of the series:*

$$u(x,t) = \sum_{n \in \mathbb{Z}^N} \varphi_n E_{\rho,1}(-A(n)t^\rho)e^{inx} + \sum_{n \in \mathbb{Z}^N} f_n t^\rho E_{\rho,\rho+1}(-A(n)t^\rho)e^{inx}, \tag{33}$$

and

$$f(x) = \sum_{n \in \mathbb{Z}^N} \frac{\Psi_n}{T^\rho E_{\rho,\rho+1}(-A(n)T^\rho)} e^{inx} - \sum_{n \in \mathbb{Z}^N} \frac{\varphi_n E_{\rho,1}(-A(n)T^\rho)}{T^\rho E_{\rho,\rho+1}(-A(n)T^\rho)} e^{inx}. \tag{34}$$

Theorem 13. *Let the function* $\varphi(x)$ *and* $\Psi(x)$ *be continuous in* $\mathbb{T}^N$. *Then there can exist only one classical solution* $\{u(x,t), f(x)\}$ *of the inverse problem (30)-(32)*

.

In second section we studied inverse problem of determining the right-hand side of subdiffusion equation with a fractional derivative in the sense Riemann-Liouville, whose elliptic part has the most common form and is defined in an arbitrary multidimensional domain (with sufficiently smooth boundary).

Let $0 < \rho \le 1$. Consider the subdiffusion equation

$$\partial_t^\rho u(x,t) + A(x,D)u(x,t) = f(x), \quad x \in \Omega, \quad 0 < t \le T, \tag{35}$$

in which the function $f(x)$ characterizing the action of heat sources depends only on x. If this function is known, then to find the temperature distribution $u(x,t)$ unambiguously, it is necessary to impose additional conditions (see Chapter 3). For example, one can set the initial condition

$$\lim_{t \to 0} \partial_t^{\rho-1} u(x,t) = \varphi(x), \quad x \in \overline{\Omega}, \tag{36}$$

and the boundary conditions

$$B_j u(x,t) = \sum_{|\alpha| \le m_j} b_{\alpha,j}(x) D^\alpha u(x,t) = 0, \tag{37}$$

$$0 \le m_j \le m-1, j=1,2,...,l; x \in \partial\Omega, 0 \le t \le T,$$

where $\varphi(x)$ and the coefficients $b_{\alpha,j}(x)$ are given functions. As a result, we obtain the forward initial-boundary value problem, considered in Chapter 3.

In this paragraph, we consider the following inverse problem: along with the solution $u(x,t)$ to the initial–boundary value problem, find the heat source density $f(x)$ such that the temperature distribution at time T achieves a prescribed level,

$$u(x,T) = \Psi(x), \quad x \in \overline{\Omega}. \tag{38}$$

Inverse problems like this arise when studying thermophysical and some other processes. The Fourier method is used to prove theorems on the existence and uniqueness of the classical solution of the initial-boundary value problem and on the unique reconstruction of the unknown right-hand side of the equation. The concept of generalized solution is introduced and a theorem on its existence is proved. The stability of classical and generalized solutions is proved. Requirements for the initial function and for the additional condition are established under which the classical Fourier method can be applied to the inverse problem under consideration. The results obtained are also new for the classical diffusion equation.

Theorem 14. *Let the functions $\varphi(x), \Psi(x)$ are continuous in the domain Ω. Then there can exist only one classical solution $\{u(x,t), f(x)\}$ of the inverse problem (35)-(37), (38).*

Theorem 15. *Let $\varphi \in D(\hat{A}^{\tau})$ and $\Psi \in D(\hat{A}^{\tau+1})$, where $\tau > \frac{N}{2m}$. Then there exists a solution $\{u(x,t), f(x)\}$ of the inverse problem (35)-(37), (38) and it can be represented as series*

$$u(x,t) = \sum_{k=1}^{\infty} \varphi_k t^{\rho-1} E_{\rho,\rho}(-\lambda_k t^{\rho}) v_k(x) + \sum_{k=1}^{\infty} f_k t^{\rho} E_{\rho,\rho+1}(-\lambda_k t^{\rho}) v_k(x), \tag{39}$$

and

$$f(x) = \sum_{k=1}^{\infty} \frac{\Psi_k}{T^{\rho} E_{\rho,\rho+1}(-\lambda_k T^{\rho})} v_k(x) - \sum_{k=1}^{\infty} \frac{\varphi_k E_{\rho,\rho}(-\lambda_k T^{\rho})}{T E_{\rho,\rho+1}(-\lambda_k T^{\rho})} v_k(x), \tag{40}$$

that converge absolutely and uniformly in $x \in \overline{\Omega}$ for all $t \in (0,T]$.

As an example of the application of Theorem 15, consider the inverse problem studied above in the N-dimensional torus $\mathbb{T}^N$.

Let $A(D) = \sum_{|\alpha|=m} a_{\alpha} D^{\alpha}$ -homogeneous elliptical symmetrical positive differential operator with constant coefficients. Let the differential equation, initial and additional conditions have the form $(0 < \rho \le 1)$

$$\partial_t^{\rho} u(x,t) + A(D)u(x,t) = f(x), \quad x \in \mathbb{T}^N, \quad t > 0; \tag{41}$$

$$\lim_{t \to 0} \partial_t^{\rho-1} u(x,t) = \varphi(x), \quad x \in \mathbb{T}^N; \tag{42}$$

$$u(x,T) = \Psi(x), \quad x \in \mathbb{T}^N. \tag{43}$$

Instead of boundary conditions (37), we will require that the functions $u(x,t)$ and $f(x)$ are 2π-periodic in to each variable x_j. We will also assume that the given functions $\varphi(x)$ and $\Psi(x)$ are 2π-periodic in each variable x_j.

Theorem 16. *Let $\varphi \in L_2^{\tau}(\mathbb{T}^N)$ and $\Psi \in L_2^{\tau+m}(\mathbb{T}^N)$, where $\tau > \frac{N}{2}$. Then there is a unique periodic solution $\{u(x,t), f(x)\}$ of the inverse problem (41)-(43) and it has the form*

$$u(x,t) = \sum_{n \in \mathbb{Z}^N} [\varphi_n t^{\rho-1} E_{\rho,\rho}(-A(n)t^{\rho}) + f_n t^{\rho} E_{\rho,\rho+1}(-A(n)t^{\rho})]e^{inx},$$

$$f(x) = \sum_{n \in \mathbb{Z}^N} f_n e^{inx},$$

where

$$f_n = \frac{\Psi_n}{T^{\rho} E_{\rho,\rho+1}(-A(n)T^{\rho})} - \frac{\varphi_n E_{\rho,\rho}(-A(n)T^{\rho})}{T E_{\rho,\rho+1}(-A(n)T^{\rho})},$$

and φ_n, Ψ_n are the Fourier coefficients, respectively, of the functions $\varphi(x)$ and $\Psi(x)$. Moreover, the above series converge absolutely and uniformly in $x \in \mathbb{T}^N$ for all $t \in (0,T]$.

We once again consider the inverse problem (35)-(37), (38), but this time we study the existence of generalized solutions. To this end, by $C((0,T],L_2(\Omega))$ we denote the class of functions $u(x,t)$ square integrable in Ω for which the function $U(t) = \| u(\cdot,t) \|_{L_2(\Omega)}$ is continuous on the set $(0,T]$. For such functions $u(x,t)$, condition (36) will be understood in the generalized sense,

$$\lim_{t \to 0} \| \partial_t^{\rho-1} u(x,t) - \varphi(x) \|_{L_2(\Omega)} = 0. \tag{44}$$

Definition 2. *A Generalized solution of the inverse problem (35), (44), (37), (38) is a pair of functions $\{u(x,t), f(x)\}$ such that $f \in L_2(\Omega)$, $A(x,D)u(x,t), \partial_t^{\rho} u(x,t) \in C((0,T],L_2(\Omega))$, relations (35), (37), (38) hold almost everywhere in Ω, and relation (44) is satisfied in the generalized sense.*

Theorem 17. *Let condition (A) be satisfied and let $\varphi \in L_2(\Omega)$ and $\Psi \in W_2^m(\Omega)$. Let further the function Ψ satisfies the boundary conditions (37). Then there is the only generalized solution $\{u(x,t), f(x)\}$ of the inverse problem (35) , (44), (37), (38), and one has the estimates*

$$\| f \|_{L_2(\Omega)} \le C[\| \varphi \|_{L_2(\Omega)} + \| \Psi \|_{W_2^m(\Omega)}]. \tag{45}$$

$$c \| u \|_{W_2^m(\Omega)} \le \| Au \|_{L_2(\Omega)} \le C[\frac{1}{t} \| \varphi \|_{L_2(\Omega)} + \| \Psi \|_{W_2^m(\Omega)}], \quad t > 0, \tag{46}$$

$$\| \partial_t^{\rho} u \|_{L_2(\Omega)} \leq C[\frac{1}{t} \| \varphi \|_{L_2(\Omega)} + \| \Psi \|_{W_2^m(\Omega)}], \quad t > 0. \tag{47}$$

CONCLUSION

The dissertation is devoted to the study of initial-boundary and inverse problems for classical equations of hyperbolic and parabolic types, as well as for subdiffusion equations. The main method for solving these problems is the classical Fourier method. Therefore, in the entire work it is required that the elliptic part of the equation has a complete in L_2 orthonormal system of eigenfunctions. In the previous papers, where the classical solution is studied, the authors mainly considered such equations, the eigenfunctions of the elliptic parts of which had an explicit form. For example, u_{xx} on an interval or Laplace's equation on a rectangle. Eigenfunctions in these cases are trigonometric functions or their products. The main purpose of this dissertation is to apply the Fourier method in cases where the explicit form of the eigenfunctions is not known. To achieve this goal, we rely on the fundamental results of V.A. Il'in, M.A. Krasnoselskii and Sh.A. Alimov.

The main results of the research are as follows:

1. We studied initial-boundary value problems for hyperbolic and parabolic equations, the elliptic part of which has the most general form and is defined in an arbitrary multidimensional domain (with a sufficiently smooth boundary) and N-dimensional torus $\mathbb{T}^N$. Using the Fourier method, we prove the existence and uniqueness of a classical solution and generalized solution to a mixed problem.

2. We studied an initial-boundary value problem for a time-fractional subdiffusion equation with an Riemann-Liouville and Caputo derivatives. Uniqueness and existence of the classical solution of the posed problem are proved by the classical Fourier method.

3. For the subdiffusion equations with the Riemann-Liouville derivatives, using a more accurate estimate of the Mittag-Leffler functions, we proved the existence of a solution to the mixed problem for a wider class of initial data. At the same time, it is essential that the fractional Riemann-Liouville derivative participates in the equation of subdiffusion. It should be noted that the condition on the initial function found in this section is less restrictive than the analogous condition in the case of an equation with Caputo derivatives, and we provided a corresponding example.

4. We investigated inverse problem of determining the right-hand side of subdiffusion equation with a fractional derivative in the sense Riemann-Liouville and Caputo, whose elliptic part has the most common form and is defined in an arbitrary multidimensional domain (with sufficiently smooth boundary) and N-dimensional torus $\mathbb{T}^N$. The concept of a generalized solution is introduced and a theorem on its existence is proved. The stability of classical and generalized solutions is also proved.

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ИНСТИТУТ МАТЕМАТИКИ

НАЧАЛЬНО-КРАЕВЫЕ И ОБРАТНЫЕ ЗАДАЧИ ДЛЯ УРАВНЕНИЙ ВЫСОКОГО ПОРЯДКА

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ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования найти классических решении прямых и обратных задач для параболических, гиперболических уравнений и уравнений субдиффузии с эллиптической части уравнения рассмотрим эллиптический оператор любого порядка, определенный в произвольной ограниченной области.

Объектом исследования Классические гиперболические и параболические уравнения, уравнения субдиффузии с производными Капуто и Римана-Лиувилля, эллиптическая часть которых представляет собой произвольный эллиптический оператор, заданный в произвольной N -мерной области.

Научная новизна исследования состоит в следующем:

Доказана однозначная разрешимость в классическом смысле начально-краевых задач для гиперболических и параболических уравнений, эллиптическая часть которых представляет собой произвольный эллиптический оператор, определенный в произвольной области.

Доказана однозначная разрешимость в классическом смысле начально-краевых задач для уравнений субдиффузии с производными Капуто и Римана-Лиувилля, эллиптическая часть которых представляет собой произвольный эллиптический оператор, определенный в произвольной области.

Доказана однозначная разрешимость в классическом смысле и существование обобщенном смысле обратных задач определения правой части уравнений субдиффузии с производными Капуто и Римана-Лиувилля, эллиптическая часть которых является произвольным эллиптическим оператором, определенным в произвольной области.

Внедрение результатов исследования. Результаты были использованы в следующих научных исследованиях:

найденные решения начально-краевой и обратной задач уравнений субдиффузии были использованы № AAAA-A19-119072290002-9 в рамках международных проекта «Природные катастрофы Камчатки – землетрясения и извержение вулканов». для описания массопереноса радиоактивного газа радона, оперативного предвестника землетрясений, из грунта в приземный слой атмосферы. (Справка Камчатского государственного университета имени Витуса Беринга от 24 октября 2022 г. № 44-12. Российская Федерация). Применение научных результатов позволило разработать алгоритмов решения обратных задач геофизики.

найденные решения начально-краевой и обратной задач уравнений субдиффузии с дробными производными были использованы при выполнении международных научно-исследовательских работ № НИОКТР AAAA-A19-119013190077-1, 2019-2021г. в рамках темы «Дробное исчисление и дифференциальные уравнения дробного и распределенного порядка». (Справка института прикладной математики и автоматизации Кабардино-Балкарского научного центра РАН от 20 мая 2022 г. № 01-14/30). Результаты

нашли применение при решении локальных и нелокальных краевых задач для эволюционных уравнений с дробными производными по временной переменной, которые в последние десятилетия находят эффективное применение при математическом моделировании различных физических и биологических процессов.

Структура и объем диссертации. Диссертация состоит из введения, четырех глав, заключения и списка литературы. Общий объем диссертации составляет 126 страниц.

E’LON QILINGAN ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo‘lim (part I, I часть)

1. R. R.Ashurov, O.T. Mukhiddinova. Inverse problem of determining the heat source density for the subdiffusion equation. // Differential Equations. 2020. – Vol. 56(12). –P. 1550–1563. (3. Scopus IF=1,052).
2. R. R.Ashurov, O.T. Muhiddinova. Initial-boundary value problem for a time-fractional subdiffusion equation with an arbitrary elliptic differential operator. // Lobachevski Journal of Mathematics. 2021. –Vol. 42(3). –P. 517–525. (3. Scopus IF=0.53).
3. O.T. Mukhiddinova. Initial-boundary value problem for a subdiffusion equation with the Caputo derivative. // International Journal of Applied Mathematics. 2022. –Vol. 35(1). –P. 135–146. (3. Scopus IF=0.27).
4. O.T. Mukhiddinova. Initial-boundary value problem for parabolic equations containing an elliptic operator of arbitrary order. // Uzbek Mathematical Journal. 2021. –Vol. 65(2). –P. 111–116. (01.00.00, № 6).
5. R. Ashurov, O. Muhiddinova. Initial-boundary value problem for a time-fractional subdiffusion equation on the torus. // Uzbek Mathematical Journal. 2021. –Vol. 65(3). –P. 17–24. (01.00.00, № 6).
6. O.T. Mukhiddinova. Inverse problem for a subdiffusion equation with the Caputo derivative on the torus. // Uzbek Mathematical Journal. 2022. –Vol. 66(3). –P. 101–111. (01.00.00, № 6).

II bo‘lim (part II, II часть)

7. Р.Р.Ашуров, А.Т.Мухиддинова. Начально-краевые задачи для гиперболических уравнений с эллиптическим оператором произвольного порядка. // Вестник КРАУНЦ. Физико-математические науки. 2020. –Том 30(1). –С. 8–19.
8. Р.Р.Ашуров, А.Т.Мухиддинова. Метод Фурье для решения начально-краевых задач. // Республиканская научная конференция “Современные методы математической физики и их приложения”. –Ташкент, 17–18 ноября 2020 г. –С. 40–43.
9. А.Т.Мухиддинова. Начально-краевые задачи для параболических уравнений с эллиптическим оператором произвольного порядка. // International conference “Frontier in mathematics and computer science”, – Tashkent, October 12–15, 2020. –С. 199–200.
10. O.T.Mukhiddinova. Initial-boundary value problem of the Caputo time-fractional derivative for a subdiffusion equation. // Республика илмий анжумани “Математиканинг замонавий масалалари: муаммолар ва ечимлар”. –Термиз, 21–23 октябрь 2020 й. –С. 147–149.

11. А.Т.Мухиддинова. Обратная задача по определению плотности тепловых источников для параболических уравнений с эллиптическим оператором произвольного порядка. // Республиканская научная конференция “Актуальные проблемы стохастического анализа”. – Ташкент, 20–21 февраля 2021г. –С. 318–320.
12. Р.Р. Ашуров, А.Т.Мухиддинова. Обратная задача по определению плотности тепловых источников для уравнения субдиффузии. // Международная научно-практическая конференция “Проблемы современной фундаментальной и прикладной математики”. –Нур–Султан. 4–июнь 2021г. –С. 92–95.
13. O.T. Mukhiddinova. Inverse problem for a subdiffusion equation with the Caputo derivative on the torus. // International conference “Annual International April Mathematical Conference”. –Kazagiztan, 6-8 april 2022. –P. 145–147.
14. R.R. Ashurov, O.T. Mukhiddinova. Initial-boundary value problem for a time-fractional subdiffusion equation with an arbitrary elliptic differential operator. // Халқаро илмий-амалий анжумани “Амалий математика ва ахборот технологияларининг замонавий муаммолари”. –Бухоро, 11–12 май 2022 й. –P. 142–143.
15. Р.Р. Ашуров, А.Т. Мухиддинова. Начально-краевые задачи для гиперболических уравнений с эллиптическим оператором произвольного порядка. // Республика илмий–амалий анжумани “XXI аср – интеллектуал ёшлар асри”. –Ташкент, 27–май 2022 й. –С. 3–9.
16. R.R. Ashurov, O.T. Mukhiddinova. Initial-boundary value problems for fractional order equations whith the Riemann-Liouville derivative. // International conference “The First Al-Khorezmi International Conference on Modern Problems of Mathematics”. –Urgench, July 25–28, 2022. –P. 17.
17. R.R. Ashurov, O.T. Mukhiddinova. Initial-boundary value problem for a time-fractional subdiffusion equation on the torus. // Республиканская научная конференция “Операторные алгебры, неассоциативные структуры и смежные проблемы”. –Ташкент, 14–15 сентября 2022 г. –С. 10.
18. Р.Р. Ашуров, А.Т. Мухиддинова. Обратная задача по определению плотности тепловых источников для уравнения субдиффузии. // International conference “Mathematical analysis and its applications in modern mathematical physics”. –Самарканд, 23–24 сентября 2022 г. –С. 12.

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