V. I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

MATEMATIKA INSTITUTI

XO‘JAQULOV JONIBEK RAIMNAZAROVICH

METRIK GRAFLARDA KASR TARTIBLI DIFFUZIYA-TO‘LQIN TENGLAMALARI UCHUN TO‘G‘RI VA TESKARI MASALALAR

01.01.02 – Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

TOSHKENT – 2023

UDK: 517.958

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Content of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

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Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, Fan va Innovatsiyalar Vazirligi huzuridagi Oliy attestatsiya komissiyasida № B2023.2.PhD/FM870 raqam bilan ro'yxatga olingan.

Dissertatsiya V. I. Romanovskiy nomidagi Matematika institutida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, ingliz, rus(rezyume)) Ilmiy kengash veb-sahifasi (http://kengash.mathinst.uz) va "ZiyoNet" ta'lim axborot tarmog'iga http://www.ziyonet.uz/ joylashtirilgan.

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Dissertatsiya himoyasi V. I. Romanovskiy nomidagi Matematika instituti huzuridagi DSc.02/30.12.2019.FM.86.01 raqamli Ilmiy kengashning 2023 yil " 14 " noyabr soat 17:00 dagi majlisida bo'lib o'tadi. (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko'chasi, 9-uy. Tel.: (+99871) 207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

Dissertatsiya bilan V. I. Romanovskiy nomidagi Matematika institutining Axborot-resurs markazida tanishish mumkin (169 - raqami bilan ro'yxatga olingan). (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko'chasi, 9-uy. Tel.: (+99871) 207-91-40).

Dissertatsiya avtoreferati 2023 yil " 27 " oktabr kuni tarqatildi.
(2023 yil " 27 " oktabrdagi 2-raqamli reestr bayonnomasi).

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Ma’lumki, jahon miqyosida olib borilayotgan fizika, mexaika, kimyo, biofizika, biotexnologiya va ishlab chiqarishdagi ko‘plab ilmiy izlanishlar matematik fizikaning turli tenglamalar nazariyasi masalalariga keltiriladi. Tabiiyki, bu kabi masalalarning fizik ma’nolaridan kelib chiqib, tarmoqlangan sistemalardagi ko‘plab jarayonlar metrik graflarda ushbu tenglamalarni o‘rganishni taqazo etadi. Masalan, tarmoqlangan sohalardagi jarayonlarni modellashtirishda metrik graflarda berilgan xususiy hosilali differensial tenglamalar uchun boshlang‘ich-chegaraviy masalalar qo‘llaniladi. Asosan, grafning tarmoqlanish nuqtasida oqimning saqlanish sharti, ya’ni Kirxgoff sharti qo‘llanilib, u eng sodda holatda graf uchida yechimning uzluksizligini va yechim bir tomonlama hosilalarining yig‘indisi nolga tengligini ifodalaydi. Tibbiyot sohasida ham jarayonlarni matematik modellashtirishda graflarda berilgan differensial tenglamalar keng qo‘llaniladi. Masalan, asab tizimlarida impulsning tarqalish jarayonlarini modellashtirishda metrik graflarda berilgan diffuziya tenglamasi uchun boshlang‘ich-chegaraviy masalalar ishlatiladi. Shuningdek, genlarda RNK bilan bog‘liq jarayonlarni modellashtirishda differensial tenglamalar zinapoyasimon graflarda qaraladi.

Hozirgi kunda mamlakatimizning iqtisodiy rivojlanishi uchun muhim fanlar sifatida matematika, fizika, geologiya va biologiya fanlariga alohida e’tibor berilmoqda. Matematika fanining rivojlanishi asnosida ushbu fanning asosiy yo‘nalishlaridan biri bo‘lgan, amaliy va nazariy xususiyatga ega differensial tenglamalar va matematik fizikani rivojlantirishiga ham alohida e’tibor qaratilmoqda. Shuningdek, matematik fizika fanining muhim tarmoqlaridan biri bo‘lgan kasr tartibli differensial tenglamalarni metrik graflarda o‘rganilish ham jadal rivojlanib bormoqda.

Ma’lumki, ko‘pgina tadqiqotlar subdiffuziya tenglamalarining o‘ng tomonini aniqlash bo‘yicha teskari masalalarga keltiriladi. Ba’zi amaliy masalalarda esa qaralayotgan vaqtning boshlang‘ch va oxirgi qiymatlari bilan bog‘liq optimal boshqaruv masalalariga keltiriladi. Metrik graflarda vaqt bo‘yicha kasr hosilali differensial tenglamalar uchun to‘g‘ri va teskari masalalarning yechimlarini topish maqsadli ilmiy tadqiqotlardan bittasi hisoblanadi. Bunga metrik graflarda berilgan differensial tenglamalar uchun boshlang‘ich-chegaraviy masalalarni o‘rganishga qaratilgan e’tiborni ham qo‘shish mumkin. Metrik graflarda berilgan turli tenglamalarni kvant fizikasi jarayonlariga qo‘llab, e’tiborga molik natijalar olingan. «Differensial tenglamalar va matematik fizika»[1] fanining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalar va faoliyat yo‘nalishlari etib belgilandi. Shu bois, metrik graflarda berilgan differensial tenglamalar uchun to‘g‘ri va teskari masalalarni yechish fanning muhim vazifalaridan hisoblanadi.

Ushbu dissertatsiya ishining predmeti va tadqiqot obyektlari O‘zbekiston

[1] O‘zbekiston Respublikasi Vazirlar Mahkamasining 2017 yil 18 maydagi “O‘zbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy-tadqiqot muassasalari faoliyatini tashkil etish chora-tadbirlari to‘g‘risida”gi 292-son qarori.

Respublikasi Prezidentining 2017-yil 7-fevraldagi “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi to‘g‘risida”gi PF-4947-sonli farmoni, 2017-yil 17-fevraldagi “Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi PQ-2789-sonli qarori, 2018-yil 27-apreldagi "Innovatsion g‘oyalar, texnologiyalar va loyihalarni amaliy joriy qilish tizimini yanada takomillashtirish chora-tadbirlari to‘g‘risida"gi PQ-3682-sonli qarori va 2019 -yil 9-iyuldagi “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akademiyasining V. I. Romanovskiy nomidagi matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi PQ-4387 sonli Prezident qarori hamda 2020-yil 7-maydagi "Matematika sohasidagi ta’lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida"gi PQ-4708 sonli Prezident qarori, bundan tashqari, mazkur faoliyatga tegishli barcha normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishga ushbu dissertatsiya ishi muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi. Mazkur dissertatsiya ishi respublika fan va texnologiyalar rivojlanishining IV “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Graflarda differensial tenglamalarni o‘rganish nisbatan yangi soha bo‘lib, o‘tgan asrning 80-yillarida rus olimi Merkov tomonidan graflarda berilgan issiqlik tarqalish tenglamasini o‘rganish bilan boshlangan.Graflarda differensial tenglamalar uchun boshlang‘ich-chegaraviy masalalar, Grin funksiyasi, differensial tengsizliklar kabi mavzularga bag‘ishlangan dastlabki monografiya rossiyalik matematik Y. V. Pokorniy va uning shogirdlari tomonidan chop etilgan. G. A. Sviridyuk va uning shogirdi V. V. Shemetova tomonidan graflarda differensial tenglamalarni o‘rganishga bag‘ishlangan “Graflarda Sobolevskiy tipidagi tenglamalarning bir sinfini tadqiq etish” mavzusidagi dastlabki dissertatsiya ishi himoya qilingan. Ushbu tadqiqot ishida Barenblatt-Jeltov-Kochina, Xoff, Oskolkova, Korpusov-Pletner-Sveshnikov tenglamalari uchun turli masalalar o‘rganilgan. Chex olimlari P.Eksner va P.Seba graflarda berilgan klassik tenglamalar uchun boshlang‘ich-chegaraviy masalalarni tadqiq etishgan. Bu sohadagi keyingi ilmiy natijalar U. Smilanskiy, P. Kuchmant, G. Berkolaiko R. Shreyder, A. Kostrikin va ularning shogirdlari tomonidan 2000-2010 yillarda chop etilgan. Ular asosan Shredinger tenglamasi uchun metrik graflarda spektral va sochilish masalalarini tatqiq qilishgan. Kvant grafi tushunchasi ham aynan shu mualliflar tomonidan ishlatila boshlangan. P Eksner va O. Postlar tomonidan ingichka tarmoqlangan sohalarda Shredinger operatori o‘rganilgan. Soha tarmoqlarining ko‘ndalang kesimi o‘lchovi nolga intilgandagi limit holatida kvant graflari hosil bo‘lishi ko‘rsatilgan. Joel Friedman and Jean-Pierre Tillichlar tomonidan "edge-based" Laplacian (yoki kvant grafi) grafi xos funktsiyalarining to‘liqligi o‘rganilgan.

Metrik graflarda Korteveg-de Friz va uning chiziqli qismi Eyri tenglamasi uchun bir qancha masalalar o‘rganilgan. Bu sohada hozirda mavjud ilmiy natijalar

Z. A. Sobirov, M. I. Ahmedov, N. Uesker, D. Noya, D. Magnola, K. Seyfert va boshqalar tomonidan o‘rganilgan. A. S. Volkova tomonidan oddiy yulduzsimon issiqlik tarqalish tenglamasining umumlashgan yechimlari o‘rganilgan. Shuningdek, chegaraviy qiymatlar yordamida boshqaruv masalalari tatqiq qilingan. 2021 yilda M. R. Eshimbetov tomonidan “Metrik graflarda to‘lqin tenglamalari uchun Fokasning umumlashgan almashtirish usuli” mavzusida dissertatsiya ishi yozilgan. Mazkur ishda turli metrik graflarda Shredinger, issiqlik va to‘lqin tarqalish tenglamalari boshlang‘ich-chegaraviy masalalarni yechishda Fokas usuli umumlashtirilgan. Kasr hosilali differensial tenglamalar nazariyasi so‘nggi bir necha o‘n yilliklarda, asosan, zamonaviy fan va texnologiyalarning ko‘plab sohalarida qo‘llanilishi tufayli katta ahamiyatga ega bo‘ldi. O‘z navbatida, nazariyaning yetarlicha ommalashganligi mutaxassislar e’tiborini tortdi, bu esa kasr tartibli differensial tenglamalarning matematik jihatlari va ularni yechish usullari bo‘yicha ko‘plab tadqiqotlarga sabab bo‘ldi. Vaqt bo‘yicha kasr hosilali tenglamalar uchun turli masalalar ko‘plab mutaxassislar tomonidan o‘rganilgan. Jumladan, vatandoshlarimiz Sh. Alimov, R. Ashurov, B. Qodirqulov, D. Durdiyev, E. Karimov, Z. Sobirov, O. Abdullayev, T. Yo‘ldoshev va boshqalar ham bu yo‘nalishda yangi ilmiy natijalarni qo‘lga kiritishgan. XXI asr boshlarida kasr tartibli hosilaning qo‘llanilishi haqida bir qator ishlar nashr etilgan. A. Alixanov tomonidan olingan tengsizlik kasr tartibli hosilali differensial tenglamalar uchun aprior baho olishda muhim ahamiyatga ega. Rus matematigi A.V. Psxu tomonidan kasr hosilali tenglama uchun turli boshlang‘ich, boshlang‘ich-chegaraviy masalalar Grin funksiyalar usuli yordamida atroflicha o‘rganilgan.

Shuningdek, ko‘pgina tadqiqotlar kasr hosilali tenglamalarining o‘ng tomonini aniqlash bo‘yicha teskari masalalarga bag‘ishlangan. M.Ruzhansky, N. Tokmagambetov va B. T. Torebeklarning maqolasida elliptik qismi diskret spektrga ega bo‘lgan ixtiyoriy elliptik operator bo‘lgan hol uchun Kaputo kasr hosilali subdiffuziya tenglamasining o‘ng tomonini aniqlash bo‘yicha teskari masalasi o‘rganib chiqilgan va mualliflar umumlashgan yechimning mavjudligi va yagonaligini Furye usuli yordamida isbotlagan. Xususiy hosilali differensial tenglamalarning tartibini aniqlash bo‘yicha R. Ashurov va uning shogirdlari tomonidan ko‘plab teskari masalalar o‘rganilmoqda. Bundan tashqari, D. Durdiyev va uning shogirdlari Kasr hosilali differensial tenglamalar uchun noma’lum koeffitsiyent topishga oid turli xildagi teskari masalalar o‘rganishmoqda. Muhammad Ali, Sara Aziz va Salman A. Maliklarning ishlarida birinchi o‘zgaruvchi bo‘yicha $0<\gamma<1$ va ikkinchi o‘zgaruvchi bo‘yicha $1<\alpha<2$ oraliqlardagi kasr tartibli tenglama uchun ikki turdagi teskari manba masalalari ko‘rib chiqilgan. Birinchisi, fazoga bog‘liqlikni tiklash manba funksiyasini o‘rganishga, ikkinchisi vaqtga bog‘liq manba funksiyaini aniqlashga bag‘ishlangan. Butun va kasr hosilali differensial tenglamalar uchun nolokal masalalarni o‘rganish sohada muhim ahamiyat kasb etadi. Jumladan, S. V. Pao reaksiya-diffuziya tenglamalari uchun, N. N. Shopolov issiqlik tarqalish tenglamasi uchun, J. M. Rassias va E. T. Karimovlar aralash parabolik tenglamalar uchun turli xildagi nolokal masalalarni tadqiq etishgan. R. Ashurov va Y. Fayzievlar vaqt bo‘yicha kasr hosilali subdiffuziya tenglamalari uchun, T. K. Yo‘ldoshev va B. J. Qodirqulovlar Xilfer kasr operatori bilan berilgan aralash

tipdagi to‘rtinchi tartibli differensial tenglama uchun, E. Karimov, M. Mamchuev, M. Rujanskiy, B. Toshtemirovlar qo‘sh tartibli Hilfer kasr differensial operatorlari ishtirok etgan aralash tipdagi tenglamalar uchun turli nolokal masalalar o‘rganganlar. Shuningdek, B. Toshtemirov tomonidan “Kasr tartibli integro-differensial operatorlar qatnashgan xususiy hosilali singulyar differensial tenglamalar uchun to‘g‘ri va teskari masalalar” mavzusidagi PhD dissertatsiyasi himoya qilingan. Kasr tartibli differensial tenglamalarni metrik graflarda o‘rganish zamonaviy matematikaning eng qiziqarli sohalaridan biri hisoblanib yaqin yillarda o‘rganila boshlangan. V. Mehandiratta, M. Mehra tomonidan yulduzsimon metrik grafda kasr tartibli Kaputo operatori qatnashgan differensial tenglamalar sonli usullarda o‘rganganlar. P. S. Fosting, G. Mophou va boshqalar graflarda kasr tartibli tenglamalar uchun optimal boshqaruv masalalarini o‘rganganlar. Z. A. Sobirov va K. Rahimovlar tomonidan zinapoyasimon va yulduzsimon metrik graflarda subdiffuziya va Eyri tenglamalari uchun boshlang‘ich, boshlang‘ich-chegaraviy masalalar Grin funksiyasi va potensiallar usulida tadqiq qilingan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim yoki ilmiy-tadqiqot muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti V. I. Romanovskiy nomidagi Matematika instituti ilmiy-tadqiqot ishlari rejasiga muvofiq “Differensial tenglamalar va unga turdosh matematik sohalarning dolzarb muammolari” dasturi doirasida bajarildi.

Tadqiqotning maqsadi metrik graflarda berilgan vaqt bo‘yicha kasr tartibli differensial tenglamalar uchun to‘gri va teskari masalalarni qo‘yish va bu masalalarning yechimini topishdan iborat.

Tadqiqotning vazifalari quyidagilardan iborat:

metrik graflarda berilgan vaqt bo‘yicha kasr hosilali diffuziya-to‘lqin tenglamalari uchun boshlang‘ich-chegaraviy masalalarning yechimini topish;

yulduzsimon metrik grafda berilgan vaqt bo‘yicha kasr hosilali psevdo-parabolik tenglama uchun teskari masalalarni yechish;

daraxtsimon metrik graflarda berilgan vaqt bo‘yicha kasr hosilali diffuziya-to‘lqin tenglamasi uchun nolokal masalalarning yechimini topish;

daraxtsimon metrik graflarda berilgan vaqt bo‘yicha kasr hosilali psevdo-subdiffuziya tenglamasi uchun boshlang‘ich-chegaraviy masalalarning yechimini mavjud va yagonaligini isbotlash.

Tadqiqotning obyekti. Yulduzsimon metrik graflar, chekli sondagi intervallardan tashkil topgan daraxtsimon metrik graf, chekli intervallardan tashkil topgan zinapoyasimon metrik graf.

Tadqiqotning predmeti. Vaqt bo‘yicha nolokal masala, boshlang‘ich-chegaraviy masalalar, vaqt, fazoga bog‘liq noma’lum manbani topish teskari masalasi.

Tadqiqotning usullari. Tadqiqot ishida asosan o‘zgaruvchilarni ajratish (Furye), energiya integrallari va aprior baholash usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat**:**

yulduzsimon graflarda berilgan vaqt bo‘yicha Kaputo va Prabhakar kasr tartibli hosila qatnashgan diffuziya tenglamalari uchun boshlang‘ich-chegaraviy

masalalarning yechimlari mavjud va yagonaligi isbotlangan;

yulduzsimon metrik grafda berilgan vaqt bo‘yicha kasr hosilali psevdo-parabolik tenglama uchun teskari masalalarning funksional qatorlar ko‘rinishidagi yechimlari topilgan;

daraxtsimon metrik graflarda berilgan vaqt bo‘yicha Hilfer kasr tartibli hosila ishtirok etgan diffuziya-to‘lqin tenglamasi uchun nolokal masalaning bir qiymatli yechilishi isbotlangan;

daraxtsimon metrik graflarda berilgan vaqt bo‘yicha kasr hosilali psevdo-subdiffuziya tenglamasi uchun boshlang‘ich-chegaraviy masalalarning yechimini mavjud va yagonaligi isbotlangan.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

Olingan ushbu natijalar va dissertatsiyada qo‘llanilgan usullar oliy o‘quv yurtlarida magistratura talabalari va doktorantlar uchun o‘quv kurs sifatida o‘qitilishi mumkin. Bundan tashqari, metrik graflarda berilgan vaqt bo‘yicha kasr tartibli xususiy hosilali differensial tenglamalar uchun boshlang‘ich-chegaraviy, nolokal va teskari masalalarni o‘rganishga oid dissertatsiyalarda, ilmiy maqolalarda hamda to‘lqin tarqalishi bilan bog‘liq turli jarayonlarni matematik modellarini ishlab chiqish uchun foydalanish mumkin.

Tadqiqot natijalarining ishonchliligi. Natijalar spektral nazariya va o‘zgaruvchilarni ajratish (Furye) usuli yordamida olingan. Olingan barcha natijalar matematik jihatdan to‘g‘ri.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati shundan iboratki, dissertatsiya ishida olingan ilmiy natijalar vaqt bo‘yicha turli kasr tartibli differensial tenglamalar uchun to‘g‘ri va teskari masalalarni yanada chuqurroq tadqiq qilishda qo‘llanilishi mumkin. Xususan, ushbu tadqiqot ishida olingan aprior baholar va ba’zi muhim tengsizliklardan kasr hosilali tenglamalar uchun qaralayotgan boshlang‘ich-chegaraviy va nolokal masalalarning yechimini yagonaligini isbotlashda foydalanish mumkin. Bundan tashqari, olingan natijalardan metrik graflarda issiqlik tarqalishining va ko‘plab tenglamalar uchun tarmoqlangan strukturalarda to‘lqin tarqalishining xossalarini o‘rganish uchun foydalanish mumkin.

Tadqiqotning amaliy ahamiyati shundan iboratki, olingan natijalar to‘lqin tarqalishining tarmoqlanish nuqtasidan o‘tish va qaytish koeffitsientini topishda, tarmoqlangan maydonlarda to‘lqin jarayonlarini tahlil qilishda, asab tizimlarida impuls tarqalishi jarayonini modelleshtirishda asos bo‘lib xizmat qiladi.

Tadqiqot natijalarining joriy qilinishi. Metrik graflardagi kasr hosilali differensial tenglamalar uchun boshlang‘ich-chegaraviy masalalar bo‘yicha olingan natijalar asosida:

metrik graflarning tarmoqlangan sohalarida diffuziya-to‘lqin tenglamasi uchun nolokal masalaning bir qiymatli yechimidan №15-198 RG/PHYS/AS_I-FR 3240287086 raqamli “Antizotropik suyuqliklarda relaksion jarayonlarini eksprimental o‘rganish: tebranish va yo‘nalish spektrlarini tahlil qilish” mavzusidagi xalqaro loyihada graf qirralarining uchrashish nuqtasidagi sochilish parametrlarini hisoblashda foydalanilgan (O‘zbekiston Milliy universitetining 2023 yil 30

avgustdagi 04/11-5125-sonli ma’lumotnomasi). Ilmiy natijaning qo‘llanishi tarmoqlanish nuqtasida o‘tkazuvchanlik va qaytish koeffitsientlarini topish imkonini bergan;

yulduzsimon grafda Hilfer kasr integro-differentsiallash operatori qatnashgan subdiffuziya tenglamasi uchun boshlang‘ich-chegaraviy masalalarning yechimidan AR09259394 raqamli “Musbat operatorli evolyutsiya tenglamalari uchun teskari masalalar” mavzusidagi xorijiy loyihada kasr tartibli xususiy hosilali differensial tenglamalar uchun teskari masalalarni yechishda foydalanilgan (Matematika va Matematik modellashtirish institutining 2023 yil 5-sentyabrdagi 01-06/128-sonli ma’lumotnomasi, Qozog‘iston). Ilmiy natijaning qo‘llanishi musbat operator qatnashgan kasr tartibli xususiy hosilali evolyutsiya tenglamalari uchun teskari masalalarni yechishni imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 10 ta ilmiy anjumanlarda, jumladan, 5 ta xalqaro va 5 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha jami 16 ta ilmiy ishlar chop etilgan bo‘lib, shundan 6 tasi O‘zbekiston Respublikasi Oliy attestatsiya komissiyasining falsafa doktorlik dissertatsiyalarining asosiy ilmiy natijalarini chop etish uchun tavsiya etilgan ilmiy nashrlarda chop etilgan, 2 tasi SCOPUS va WoS ma’lumotlar bazalarida indekslangan jurnallarda chop etilgan, 4 tasi respublika jurnallarida chop etilgan maqolalar va 10 tasi tezisdir.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning hajmi 126 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalarining rivojlanishini ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, mavzu bo‘yicha xorijiy ilmiy-tadqiqotlar sharhi, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning “**Dastlabki ma’lumotlar**” deb nomlangan birinchi bobida yordamchi ma’lumotlar keltirilgan bo‘lib, ushbu tadqiqot ishini tushunish qulay bo‘lishi uchun yaratilgan. Bu yerda yangi natijalar keltirilmagan. Bu bobda biz kasr tartibli hosilalar va integrallar, Mittag-Leffler va Prabhakar tipidagi funksiyalarining asosiy xossalarini keltiramiz. Bundan tashqari, vaqt bo‘yicha kasr tartibli hosila ishtirok etgan differensial tenglamalar uchun Koshi masalasi yechimlarini qaraymiz. Shuningdek, metrik graflar, metrik graflarda qaralgan spektral masalalarni keltiramiz.

Birinchi bobdagi ba’zi ta’riflar va tasdiqlarni keltirib o‘tamiz.

1-ta'rif. $f(x)$ funksiyaning Riman-Liuvill α $(\alpha>0)$ kasr tartibli integrali quyidagi ko'rinishda ifodalanadi:

$$\left(I_{at}^{\alpha}f\right)(t)=\frac{1}{\Gamma(\alpha)}\int_{a}^{t}\frac{f(\tau)d\tau}{(t-\tau)^{1-a}}\ ,\ t>a,\ a\in\mathbb{R},\ n-1<\alpha<n,\ n\in\mathbb{N}.$$

2-ta'rif. Quyidagi

$$\left(D_{at}^{\alpha}f\right)(t):=\left(\frac{d}{dt}\right)^{n}\left(I_{at}^{n-\alpha}f\right)(t)=\frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dt}\right)^{n}\int_{a}^{t}\frac{f(\tau)d\tau}{(t-\tau)^{\alpha-n+1}}\ ,t>a$$

va

$$\left({}_{C}D_{at}^{\alpha}f\right)(t):=\left(I_{at}^{n-\alpha}\left(\frac{d}{dt}\right)^{n}f\right)(t):=\frac{1}{\Gamma(n-\alpha)}\int_{a}^{t}\frac{f^{(n)}(\tau)}{\left(t-\tau\right)^{\alpha-n+1}}d\tau\ ,\ t>a$$

munosabat bilan aniqlangan operatorlar mos ravishda Riman-Liuvill va Kaputo kasr tartibli hosilalari deyiladi, bu yerda $n\in\mathbb{N}$ and $n-1<\alpha<n$.

Kaputo va Riman-Liuvill kasr hosilalari orasida quyidagi bog'lanish mavjud:

$$\left(D_{at}^{\alpha}f\right)(t)=\sum_{k=0}^{n-1}\frac{f^{(k)}(a)}{\Gamma(1+k-\alpha)}(t-a)^{k-\alpha}+\left({}_{C}D_{at}^{\alpha}f\right)(t),\ \ t>a,$$

bu yerda $n\in\mathbb{N}$ va $n-1<\alpha<n$.

3-ta'rif. $\alpha\,(n-1<\alpha\le n)$ *tartibli* $0<\mu\le 1$ *tipdagi Hilfer kasr tartibli hosilasi quyidagicha aniqlanadi:*

$$D_{at}^{\alpha,\mu}f(t)=I_{at}^{\mu(n-\alpha)}\left(\frac{d}{dt}\right)^{n}I_{at}^{(1-\mu)(n-\alpha)}f(t).$$

4-ta'rif. $f(t)$ funksiyaning Prabhakar β $(\beta>0)$ kasr tartibli integrali quyidagi ko'rinishda ifodalanadi:

$${}^{P}I_{at}^{\alpha,\beta,\gamma,\delta}f(t)=\int_{a}^{t}(t-\tau)^{\beta-1}E_{\alpha,\beta}^{\gamma}\left(\delta(t-\tau)^{\alpha}\right)f(\tau)d\tau\ ,\ t>a.$$

5-ta'rif. β tartibli regulyarizatsiyalashtirilgan Prabhakar kasr tartibli hosilasi quyidagi ko'rinishda aniqlanadi:

$${}^{PC}D_{at}^{\alpha,\beta,\gamma,\delta}f(t)={}^{P}I_{at}^{\alpha,m-\beta,-\gamma,\delta}\frac{d^{m}}{dt^{m}}f(t),$$

bu yerda $\alpha>0$, $m-1<\beta<m$, $\gamma,\delta\in R$.

1-lemma. Aytaylik, $v(t)$ funksiya $[0,T]$ kesmada absolyut uzluksiz bo'lsin. U holda quyidagi tengsizlik o'rinli:

$$v(t)\,{}_{C}D_{0t}^{\alpha}v(t)\ge\frac{1}{2}\,{}_{C}D_{0t}^{\alpha}v^{2}(t).\quad 0<\alpha<1.$$

2-lemma. Aytaylik, nomanfiy $y(t)$ funksiya deyarli barcha $t\in[0,T]$ lar uchun absolyut uzluksiz funksiya bo'lib, quyidagi

$${}_{C}D_{0t}^{\alpha}y(t)\le c_{1}y(t)+c_{2}(t),\ 0<\alpha\le 1$$

tengsizlikni qanoatlantirsin va $c_1>0$, $c_2(t)$ funksiya esa $[0,T]$ kesmada nomanfiy, integrallanuvchi funksiya bo'lsin. U holda quyidagi

$$y(t) \le y(0)E_\alpha\left(c_1 t^a\right) + \Gamma(\alpha)E_{\alpha,\alpha}\left(c_1 t^\alpha\right) I_{0t}^\alpha c_2(t)$$

tengsizlik o‘rinli bo‘ladi. Bu yerda $E_\alpha(z)$ va $E_{\alpha,\beta}(z)$ funksiyalar mos ravishda bir va ikki parametrli Mittag–Leffler funksiyalari. Ushbu funksiyalar quyidagi ko‘rinishda ifodalanadi:

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)},$$

bu yerda $\alpha > 0$, $\beta \in \mathbb{C}$ va $z \in \mathbb{C}$, agar $\beta = 1$ bo‘lsa ushbu funksiya $E_{\alpha,1}(z) = E_\alpha(z)$ ko‘rinishda yoziladi.

6-ta’rif. Quyidagi

$$E_{\alpha,\beta}^{\gamma}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\alpha n + \beta)} \frac{z^n}{n!}$$

funksiya Prabhakar funksiyasi yoki umumlashgan Mittag-Leffler funksiyasi deb ataladi, bu yerda $\alpha, \beta, \gamma \in \mathbb{C}$, $Re(\alpha), Re(\beta) > 0$ and $(\gamma)_n$ - Pohgammer belgisi.

7-ta’rif. Ikki o‘zgaruvchili Mittag-Leffler funksiyasi quyidagi ko‘rinishda ifodalanadi:

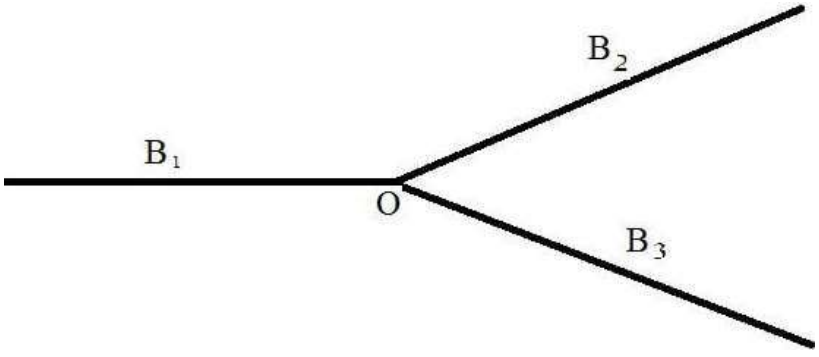
$$E_2(x,y) = \left(\begin{matrix} \gamma_1, & \alpha_1, & \beta_1; & \gamma_2, & \alpha_2, \\ \delta_1, & \alpha_3, & \beta_2; & \delta_2, & \alpha_4; & \delta_3, & \beta_3 \end{matrix} \middle| \begin{matrix} x \\ y \end{matrix} \right) =$$

$$= \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{1}{\Gamma(\delta_1 + \alpha_3 m + \beta_2 n)} \frac{\Gamma(\gamma_1 + \alpha_1 m + \beta_1 n)}{\Gamma(\gamma_1)} \frac{\Gamma(\gamma_2 + \alpha_2 m)}{\Gamma(\gamma_2)} \frac{x^m}{\Gamma(\delta_2 + \alpha_4 m)} \frac{y^n}{\Gamma(\delta_3 + \beta_3 n)},$$

bu yerda $\gamma_1,\ \gamma_2,\ \delta_1,\ \delta_2,\ \delta_3,\ x, y \in \mathbb{C}$, $min\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3\} > 0$.

Dissertatsiyaning asosiy natijalari ikkinchi bobdan boshlanadi. “**Yulduzsimon graflarda kasr tartibli differensial tenglamalar uchun to‘g‘ri va teskari masalalar**” deb nomlanuvchi ikkinchi bob uchta paragrafdan iborat bo‘lib, birinchi va ikkinchi paragraflarda yulduzsimon grafda Kaputo va Prabhakar kasr tartibli hosila bilan berilgan issiqlik tarqalish tenglamasi uchun boshlang‘ich-chegaraviy masalalar o‘rganilgan.

Aytaylik, Γ graf $O(0,0)$ nuqtadan chiquvchi uchta qirradan tashkil topgan bo‘lsin. Ushbu qirralarning koordinatalari 0 dan $l_k\ (0 < l_k < +\infty,\ k = 1,2,3)$ gacha aniqlangan bo‘lsin. Graf qirralarini $B_k = (0, l_k)$ orqali belgilaymiz.



1-rasm.

Biz $B_k \times (0,T)$ sohada Kaputo hosilasi bilan berilgan quyidagi kasr tartibli differensial tenglamani qaraymiz:

$$ {}_C D_{0t}^{\alpha} u^{(k)}(x,t) - u_{xx}^{(k)}(x,t) = f^{(k)}(x,t), \quad x \in B_k, \quad t \in (0,T), \; k = 1,2,3, \qquad (1) $$

1-masala. (1) tenglamaning

$$ u^{(1)}(0,t) = u^{(2)}(0,t) = u^{(3)}(0,t), \qquad (2) $$

$$ u_x^{(1)}(0,t) + u_x^{(2)}(0,t) + u_x^{(3)}(0,t) = 0, \; t \in [0,T], \qquad (3) $$

$$ u^{(k)}(l_k,t) = 0, \; t \in [0,T], \; k = 1,2,3, \qquad (4) $$

$$ u^{(k)}(x,0) = \varphi_k(x), \; x \in \overline{B}_k, \; k = 1,2,3 \qquad (5) $$

shartlarni qanoatlantiruvchi $u^{(k)}(x,t)$ yechimi topilsin. Bu yerda $\varphi_k(x)$ *va* $f^{(k)}(x,t)$ - berilgan funksiyalar.

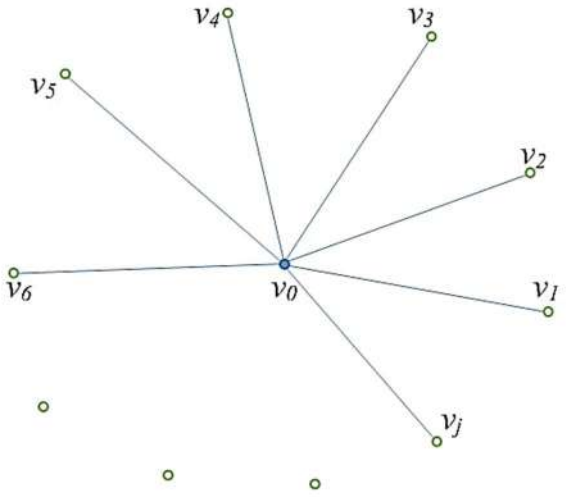
Quyidagi tasdiq o‘rinli:

1-teorema. *Agar* $\varphi_k(x)$ *va* $f^{(k)}(x,t)$ *funksiyalar* $\varphi_k(x) \in C^1[0,l_k]$, $f^{(k)}(x,t) \in C_{x,t}^{1,0}\left([0,l_k] \times (0,T)\right)$ *sinflardan olingan bo‘lsa hamda* $\varphi_k''(x)$, $\dfrac{\partial^2}{\partial x^2} f^{(k)}(\cdot,t)$ *funksiyalar mos ravishda* $(0,l_k)$ *va* $B_k \times (0,T)$ *sohalarda absolyut integrallanuvchi bo‘lsa, bundan tashqari, (2)-(4) shartlar* $\varphi_k(x)$ *va* $f^{(k)}(x,t)$ *funksiyalar uchun ham bajarilsa, u holda qaralayotgan (1)-(5) masalaning yagona yechimi mavjud bo‘ladi va uni* $\overline{B}_k \times [0,T]$ *sohada tekis yaqinlashuvchi quyidagi funksional qator ko‘rinishida yozish mumkin:*

$$ u(x,t) = \sum_{n=1}^{\infty} \left(\int_0^t (t-z)^{\alpha-1} E_{\alpha,\alpha}[-\lambda_n^2 (t-z)^{\alpha}] f_n(z) dz + \varphi_n E_{\alpha,1}(-\lambda_n^2 t^{\alpha}) \right) X_n(x). $$

Bu yerda φ_n va $f_n(t)$ mos ravishda $\varphi_k(x)$ va $f^{(k)}(x,t)$ funksiyalarning Furye koeffitsiyentlari, λ_n - xos sonlar.

Ikkinchi bobning ikkinchi paragrafida yulduzsimon grafda Prabhakar kasr tartibli hosilasi bilan berilgan issiqlik tarqalish tenglamasi uchun boshlang‘ich-chegaraviy masalalar o‘rganilgan.



2-rasm.

Berilgan Γ yulduzsimon grafda quyidagi

$$ {}^{PC}D_{0t}^{\alpha,\beta,\gamma,\delta}u^{(k)}(x,t)-u_{xx}^{(k)}(x,t)=f^{(k)}(x,t),\ x\in B_k,\ \ t\in(0,T)\ k=\overline{1,j} \qquad (6) $$

tenglamani qaraylik, bu yerda $\alpha>0$, $0<\beta<1$, $\gamma,\delta\in R$, $f^{(k)}(x,t)$ berilgan funksiyalar.

2-masala $B_k\times(0,T)$ sohada shunday $u^{(k)}(x,t)$ funksiya topilsinki, (6) tenglamani hamda quyidagi:

$$ u^{(1)}(0,t)=u^{(2)}(0,t)=...=u^{(j)}(0,t),\quad t\in[0,T], \qquad (7) $$

$$ u_x^{(1)}(0,t)+u_x^{(2)}(0,t)+...+u_x^{(j)}(0,t)=0,\quad t\in[0,T], \qquad (8) $$

$$ u^{(k)}(l_k,t)=0,\ t\in[0,T],k=\overline{1,j}, \qquad (9) $$

$$ u^{(k)}(x,0)=\varphi_k(x),\ x\in\overline{B}_k\ ,\ k=\overline{1,j}\ , \qquad (10) $$

shartlarni qanoatlantirsin.

Qaralayotgan (6)-(10) masala yechimining yagonaligi quyidagi lemma yordamida ko‘rsatiladi:

2-lemma. Shunday t_1 soni mavjudki, $[0,t_1]$ oraliqda absolyut uzluksiz har qanday $v(t)$ funksiya uchun quyidagi tengsizlik o‘rinli:

$$ v(t)\cdot{}^{P}D_{0t}^{\alpha,\beta,\gamma,\delta}v(t)\ge\frac{1}{2}{}^{P}D_{0t}^{\alpha,\beta,\gamma,\delta}v^2(t),\qquad 0<\beta<1. $$

Asosiy natija quyidagi tasdiqda keltirilgan:

2-teorema. *Berilgan* $\varphi_k(x)$ *va* $f^{(k)}(x,t)$ *funksiyalar* $\varphi_k(x)\in C^5[0,l_k]$, $f^{(k)}(x,t)\in C_{x,t}^{2,0}\left([0,l_k]\times[0,T]\right)$ *sinflarda aniqlangan bo‘lib,* $\varphi_k^{(6)}(x)$, $\frac{\partial^3}{\partial x^3}f^{(k)}(x,t)$ *funksiyalar mos ravishda* $(0,l_k)$ *va* $B_k\times(0,T)$ *sohalarda absolyut integrallanuvchi bo‘lsa, bundan tashqari (7)-(9) shartlar* $\varphi_k(x)$, $\varphi_k''(x)$, $\varphi_k^{IV}(x)$, $f^{(k)}(x,t)$ *funksiyalar uchun, (7), (9) shartlar esa* $\frac{\partial^2}{\partial x^2}f^{(k)}(x,t)$ *funksiyalar uchun bajarilsin, u holda qaralayotgan masalaning yagona yechimi mavjud bo‘ladi.*

Mazkur masalaning yechimi ikki o‘zgaruvchili Mittag-Leffler funksiyasi orqali ifodalangan, $\overline{B}_k\times[0,T]$ sohada tekis yaqinlashuvchi quyidagi funksional qator ko‘rinishida topiladi:

$$ u^{(k)}(x,t)=\sum_{n=1}^{\infty}\left(\varphi_n+\varphi_n\lambda_n^2\Gamma(\gamma)t^{\beta}E_2\left(\begin{matrix}&\gamma&\gamma&1&1&0\\ \beta+1&\beta&\alpha&\gamma&\gamma&1&1\end{matrix}\middle|\begin{matrix}\lambda^2t^{\beta}\\ \delta t^{\alpha}\end{matrix}\right)+\right. $$

$$ \left.+\Gamma(\gamma)\int_0^t(t-\tau)^{\beta-1}f_n(\tau)E_2\left(\begin{matrix}&\gamma&\gamma&1&1&0\\ \beta&\beta&\alpha&\gamma&\gamma&1&1\end{matrix}\middle|\begin{matrix}\lambda_n^2t^{\beta}\\ \delta t^{\alpha}\end{matrix}\right)d\tau\right)X_n^{(k)}(x), $$

bu yerda φ_n va $f_n(t)$ mos ravishda $\varphi_k(x)$ va $f^{(k)}(x,t)$ funksiyalarning Furye koeffitsiyntlari.

Ikkinchi bobning uchinchi paragrafida esa yulduzsimon grafda Kaputo kasr hosilasi bilan berilgan psevdoparabolik tenglama uchun ikkita teskari masala

o‘rganilgan.

Berilgan Γ grafning har bir qirrasida ushbu kasr hosilali differensial tenglama:

$$ {}_C D_{0t}^{\alpha} u^{(k)}(x,t) = a u_{xx}^{(k)}(x,t) + b\, {}_C D_{0t}^{\alpha} u_{xx}^{(k)}(x,t) + c u^{(k)}(x,t) + f^{(k)}(x,t) + h^{(k)}(x,t), \quad (11) $$

bu yerda $x \in B_k$, $0 < t < T$, $0 < \alpha < 1$, a, b, c -o‘zgarmas sonlar, hamda quyidagi

$$ u^{(1)}(0,t) = u^{(2)}(0,t) = ... = u^{(j)}(0,t), \quad t \in [0,T], \quad (12) $$

$$ u_x^{(1)}(0,t) + u_x^{(2)}(0,t) + ... + u_x^{(j)}(0,t) = 0, \quad t \in [0,T], \quad (13) $$

$$ u_x^{(k)}(l_k,t) = 0,\ t \in [0,T], k = \overline{1,j}, \quad (14) $$

$$ u^{(k)}(x,0) = \varphi_k(x), \quad x \in B_k\ ,\ k = \overline{1,j} \quad (15) $$

shartlar berilgan bo‘lsin.

(11)-(15) masalani to‘g‘ri masala sifatida o‘rganish mumkin. Biz (11)-(15) boshlang‘ich-chegaraviy masalaga bog‘liq bo‘lgan ikkita teskari masala o‘rganamiz. Birinchi teskari masala fazoviy o‘zgaruvchiga bog‘liq bo‘lgan manba funksiyasini tiklash bilan bog‘liq, ya’ni $h^{(k)}(x,t) = h^{(k)}(x) g(t)$. Bu yerda $h^{(k)}(x)$-noma’lum, $g(t)$-berilgan ma’lum funksiyalar.

Qiziqarli bo‘lgan ikkinchi teskari masala-vaqtga bog‘liq bo‘lgan manba funksiyasini tiklanishi, ya’ni $h^{(k)}(x,t) = g(t) h^{(k)}(x)$. Bu yerda $h^{(k)}(x)$-berilgan, $g(t)$-noma’lum funksiyalar.

3-masala. $B_k \times (0,T)$ sohada shunday $\left(u^{(k)}(x,t), h^{(k)}(x,t)\right)$ funksiyalar juftligi topilsinki, bu juftlik (11) tenglamani, (12)-(15) hamda quyidagi qo‘shimcha

$$ u^{(k)}(x,T) = \psi_k(x), x \in B_k, k = \overline{1,j}. \quad (16) $$

shartlarni qanoatlantirsin, bu yerda $\psi_k(x)$ - berilgan funksiyalar.

3-teorema: *Aytaylik* $f^{(k)}(x,t)$ *va* $\varphi_k(x), \psi_k(x)$ - *funksiyalar quyidagi shartlarni qanoatlantirsin:*

(1) $f^{(k)}(x,t) \in C\left([0,l_k] \times (0,T)\right)$ va $\dfrac{\partial}{\partial x} f^{(k)}(x,t)$ funksiyalar $B_k \times (0,T)$ sohada absolyut integrallanuvchi,

(2) $\varphi_k(x), \psi_k(x) \in C^2[0,l_k]$, $\varphi_k''(x)$, $\psi_k''(x)$ funksiyalar $(0,l_k)$ oraliqda absolyut integrallanuvchi,

(3) (12)-(14) shartlar $\varphi_k(x)$ *funksiyalar uchun, (12) shart* $\varphi_n''(x)$ *va* $f^{(k)}(x,t)$ *funksiyalar uchun o’rinli bo’lsin,*

(4) (12)-(14) shartlar $\psi_k(x)$ *funksiyalar uchun, (12) shart* $\psi_n''(x)$ *funksiyalar uchun o’rinli bo’lsin*.

U holda (11)-(16) masalaning yagona yechimi mavjud bo‘ladi.

(11)-(16) masalaning yechimlari $[0,l_k] \times [0,T]$ sohada tekis yaqinlashuvchi qatorlar ko‘rinishida quyidagicha topilgan:

$$ u^{(k)}(x,t) = \sum_{n=1}^{\infty} \frac{1}{1+b\lambda_n^2} \left[\frac{(1+b\lambda_n^2)[\psi_n - \varphi_n E_{\alpha}\left(-\mu_n^2 T^{\alpha}\right)] - F_n(T)}{G_n(T)} G_n(t) + \right. $$

$$+F_n(t)+\varphi_n E_\alpha\left(-\mu_n^2 t^\alpha\right)]X_n^{(k)}(x),$$

$$h^{(k)}(x,t)=\sum_{n=1}^{\infty}\left[\frac{(1+b\lambda_n^2)[\psi_n-\varphi_n E_\alpha\left(-\mu_n^2 T^\alpha\right)]-F_n(T)}{G_n(T)}g(t)X_n^{(k)}(x),\right.$$

bu yerda $\mu_n^2=\dfrac{a\lambda_n^2-c}{1+b\lambda_n^2}$,

$$F_n(t)=\int_0^t(t-\tau)^{\alpha-1}E_{\alpha,\alpha}\left[-\mu_n^2(t-\tau)^\alpha\right]f_n(\tau)d\tau,$$

$$G_n(t)=\int_0^t(t-\tau)^{\alpha-1}E_{\alpha,\alpha}\left[-\mu_n^2(t-\tau)^\alpha\right]g(\tau)d\tau.$$

4- masala. $B_k\times(0,T)$ sohada shunday $\left(u^{(k)}(x,t),h^{(k)}(x,t)\right)$ funksiyalar juftligi topilsinki, Bu juftlik (11) tenglamani, (12)-(15) hamda quyidagi integral tipdagi

$$\int_\Gamma u(x,t)dx=\Phi(t),\quad t\in[0,T]. \tag{17}$$

shartlarni qanoatlantirsin, bu yerda $\Phi(t)$- berilgan ma'lum funksiya.

4-teorema. *Aytaylik* $f^{(k)}(x,t)$ *va* $\varphi_k(x)$, $h^{(k)}(x)$ *funksiyalar quyidagi shartlarni qanoatlantirsin:*

(1) $h^{(k)}(x)\in C[0,l_k]$, $\dfrac{\partial}{\partial x}h^{(k)}(x)$*-funksiyalar* $(0,l_k)$ *oraliqda absolyut integrallanuvchi,*

(2) *(12) shart* $h^{(k)}(x)$ *funksiyalar uchun o'rinli bo'lsin,*

(3) *3-teoremaning (1)-(3) shartlarini qanoatlantirsin.*

U holda 2- teskari masalaning yagona yechimi mavjud.

4-masalaning yechimi $\overline{B}_k\times[0,T]$ sohada tekis yaqinlashuvchi funksional qatorlar ko'rinishida quyidagicha topilgan:

$$u^{(k)}(x,t)=\sum_{n=1}^{\infty}\left[\frac{h_n}{1+b\lambda_n^2}\int_0^t(t-\tau)^{\alpha-1}E_{\alpha,\alpha}\left[-\mu_n^2(t-\tau)^\alpha\right]g(\tau)d\tau+\right.$$

$$\left.+\frac{1}{1+b\lambda_n^2}\int_0^t(t-\tau)^{\alpha-1}E_{\alpha,\alpha}\left[-\mu_n^2(t-\tau)^\alpha\right]f_n(\tau)d\tau+\varphi_n E_{\alpha,1}\left(-\mu_n^2 t^\alpha\right)\right]X_n^{(k)}(x),$$

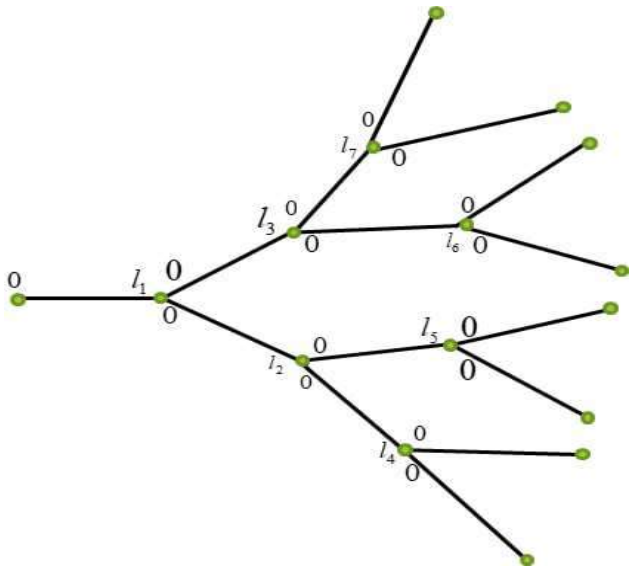
$$g(t)=\left(\int_\Gamma h(x)dx\right)^{-1}\left[{}_C D_{0t}^\alpha\Phi(t)-c\Phi(t)-\int_\Gamma f(x,t)dx\right],$$

bu yerda h_n va $f_n(t)$ -funksiyalar mos ravishda $h(x)$ va $f_n(x,t)$ funksiyalarning Furye koeffitsiyentlari.

Dissertatsiyaning "**Daraxtsimon metrik graflarda berilgan kasr hosilali differensial tenglamalar uchun uchun boshlang'ich-chegaraviy va nolokal masalalar**" deb nomlangan uchinchi bobida vaqt bo'yicha kasr hosilali to'lqin, diffuziya-to'lqin va psevdo-subdiffuziya tenglamalari uchun ikki xil daraxtsimon metrik grafda boshlang'ich-chegaraviy hamda nolokal masalalar tadqiq etilgan.

Biz ushbu bobning birinchi paragrafida Γ graf deb O nuqtadan bitta qirrasi

chiquvchi va qolgan ichki ν_k , $k=\overline{1,2^j-1}$ uchlarida bitta kiruvchi va ikkita chiquvchi qirralarga ega bo‘lgan metrik grafni qaraymiz(3-rasm). Graf qirralarini 0 dan l_k gacha va biz ularni $B_1, B_2, ..., B_{2^{j+1}-1}$ ko‘rinishlarda belgilaymiz. Shunday qilib Γ graf deganda $\Gamma = V \cup E$ bog‘langan grafni tushunamiz, bu V to‘plam-grafning ichki uchlari to‘plami $\{\nu_k\}_{k=1}^{2^j-1}$, $O(0,0)$ va $(l_k,0)$, $k=\overline{2^j,2^{j+1}-1}$ nuqtalar to‘plamidan iborat, $E=\{B_k\}_{k=1}^{2^{j+1}-1}$ graf qirralari to‘plami.



3-rasm.

Γ grafning har bir qirrasida ushbu

$$ {}_C D_{0t}^{\alpha} u^{(k)}(x,t) - u_{xx}^{(k)}(x,t) = f^{(k)}(x,t),\ (x,t) \in \left(B_k \times (0,T)\right),\ k=\overline{1,2^{j+1}-1}, \qquad (19) $$

kasr hosilali to‘lqin tenglamasini va ushbu

$$ u^{(k)}(x,0) = \varphi_k(x),\ u_t^{(k)}(x,0) = \psi_k(x),\ x \in \overline{B}_k,\ k=\overline{1,2^{j+1}-1} \qquad (20) $$

boshlang‘ich shartlarni qanoatlantiruvchi yechimini qidiraylik, bu yerda $1<\alpha<2$.

Qirralarning tutashish nuqtasida esa barcha $t \in [0,T]$ lar uchun Kirxgoff shartlari bajarilsin:

$$ u^{(k)}(l_k,t) = u^{(2k)}(0,t) = u^{(2k+1)}(0,t)\ , \qquad (21) $$

$$ -u_x^{(k)}(l_k,t) + u_x^{(2k)}(0,t) + u_x^{(2k+1)}(0,t) = 0\ ,\ ,\ k=\overline{1,2^j-1}. \qquad (22) $$

Grafning chekka nuqtalarida quyidagi bir jinsli

$$ u^{(1)}(0,t) = 0,\ u^{(k)}(l_k,t) = 0,\ t \in [0,T]\ ,\ k=\overline{2^j,2^{j+1}-1}. \qquad (23) $$

chegaraviy shartlarni qaraymiz.

5-masala. (19) tenglamaning (20)-(23) shartlarni qanoatlantiruvchi yechimlari topilsin.

5-teorema. *Aytaylik* $\varphi_k(x), \psi_k(x) \in C^1[0,l_k]$, $f^{(k)}(x,t) \in C_{x,t}^{1,0}\left([0,l_k]\times(0,T)\right)$

bo‘lsin, $\varphi_k''(x)$, $\psi_k''(x)$ va $\dfrac{\partial^2}{\partial x^2} f^{(k)}(x,t)$ *funksiyalar mos ravishda* $(0,l_k)$ *va* $B_k \times (0,T)$ *sohalarda absolyut integrallansin, shuningdek,* $\varphi_k(x)$, $\psi_k(x)$ *va* $f^{(k)}(x,t)$ *funksiyalar uchun (21)-(23) shartlar bajarilsin. U holda 5-masalaning yagona yechimini quyidagi ko‘rinishda yozish mumkin:*

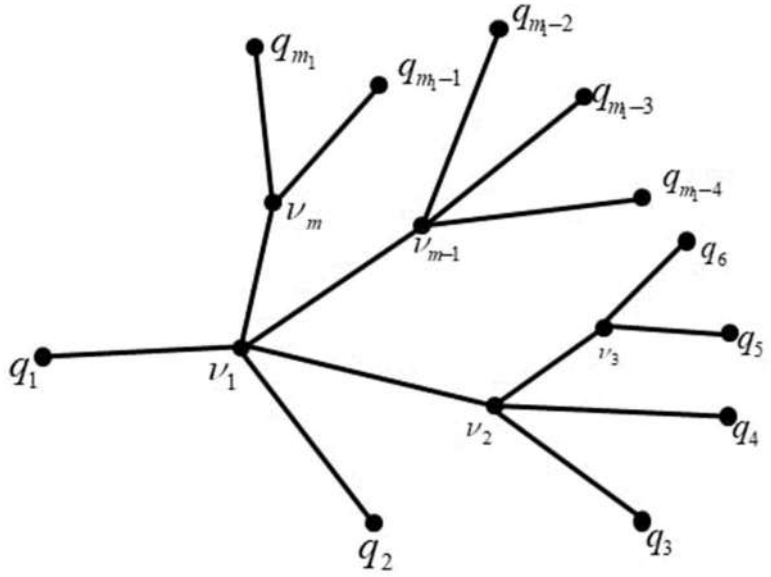
$$u^{(k)}(x,t) = \sum_{n=1}^{\infty} \Bigg[\int_0^t (t-z)^{\alpha-1} E_{\alpha,\alpha}[-\lambda_n^2 (t-z)^{\alpha}] f_n(z) dz +$$

$$+ \varphi_n E_{\alpha,1}(-\lambda_n^2 t^{\alpha}) + \psi_n t E_{\alpha,2}(-\lambda_n^2 t^{\alpha}) \Bigg] X_n^{(k)}(x), \ (k = \overline{1, 2^{j+1}-1}),$$

bu yerda φ_n va ψ_n – mos ravishda $\varphi_k(x)$ va $\psi_k(x)$ funksiyalarning Furye koeffitsiyentlari.

Uchinchi bobning ikkinchi va uchinchi paragraflarida daraxtsimon metrik grafda Hilfer ma’nosidagi kasr hosilasi bilan berilgan differensial tenglamalari uchun nolokal va boshlangich chegaraviy masalalar o‘rganilgan.

$\Gamma = E \cup V$ – bog‘lamli metrik graf bo‘lsin, bu yerda $E = \{B_k\}_{k=1}^{j}$ –to‘plam grafning qirralari to‘plami, $V = \{\nu_k\}_{k=1}^{m} \cup \{q_k\}_{k=1}^{m_1}$ – to‘plam esa grafning uchlari to‘plami. Har bir qirraga $(0,L_k)$, $k = \overline{1,j}$ intervallarni mos qo‘yamiz. Agar B_k qirralarning oxiri ν uchdan iborat bo‘lsa, u holda B_k qirra ν uch bilan uchrashadi deymiz va buni $B_k \sim \nu$ kabi belgilaymiz. $\{B : B \sim \nu, B \in E\}$ to‘plamning elementlari sonini ν uchning valentligi deymiz. Agar uchning valentligi 1 ga teng bo‘lsa, u holda u grafning chegaraviy uchlari deyiladi. $\{q_1, q_2, ..., q_{m_1}\} = \partial\Gamma \subset V$ – grafning chegaraviy uchlari to‘plami.



4-rasm.

Ta’riflangan $\Gamma = E \cup V$ grafning har bir qirrasida quyidagi Hilfer ma’nosidagi kasr hosilali differensial tenglamani qaraymiz:

$$D_{0t}^{\alpha,\mu} u^{(k)}(x,t) - u_{xx}^{(k)}(x,t) = f^{(k)}(x,t), \ (x,t) \in \left(B_k \times (0,T)\right), \qquad (24)$$

bu yerda $l-1 < \alpha < l$, $l = \{1,2\}$, $0 \le \mu \le 1$, $k = \overline{1,j}$, $f^{(k)}(x,t)$-berilgan ma’lum

funksiyalar .

6-masala. $B_k \times (0,T)$ sohada shunday $u^{(k)}(x,t)$ funksiyalar topilsinki, bu funksiyalar (24) tenglamani hamda quyidagi:
nolokal va boshlang‘ich shartlarni

$$\begin{aligned} &I_{0t}^{l-\gamma} u^{(k)}(x,t)|_{t=0} = M I_{0t}^{l-\gamma} u^{(k)}(x,t)|_{t=T}, \ \ \gamma = \alpha + l\mu - \alpha\mu, \\ &\frac{d}{dt} I_{0t}^{l-\gamma} u^{(k)}(x,t)|_{t=0} = \varphi_k(x), \ l=2, \ \ x \in B_k, \ k = \overline{1,j}, \end{aligned} \tag{25}$$

grafning ichki nuqtalarida δ *tipidagi shartlar* ni

$$\begin{cases} u^{(k)}(x,t) \ \ -\nu \in V \setminus \partial\Gamma \ \ da \ uzluksiz, \\ \ \ \ \ \sum\limits_{B_k \sim \nu} u_x^{(k)}(x,t)\Big|_{\nu} = 0, \end{cases} \tag{26}$$

hamda chegaraviy

$$u^{(k)}(x,t)|_{q_k} = 0, \qquad q_k \in \partial\Gamma, \ \ t \in (0,T], \ k = \overline{1,m_k} \tag{27}$$

shartlarni qanoatlantirsin.
Bu yerda $\varphi_k(x)$, $\psi_k(x)$ - yetarlicha silliq berilgan funksiyalar.

3-lemma. *Aytaylik* $t^{1-\gamma} f^{(k)}(x,t) \in C\left(\overline{B}_k \times [0,T]\right), \quad 0 < \alpha < 1,$

$\varphi_k(x) \in C\left(\overline{B}_k\right), \ t^{2-\gamma} f^{(k)}(x,t) \in C\left(\overline{B}_k \times [0,T]\right), 1 < \alpha < 2$, *va* $l-1 < \alpha < l, \ l = \{1,2\}$

sonlar uchun $M^2 < \dfrac{1}{l-1+T^{\beta-1} E_{\alpha+l-1,\beta}(T^\alpha)}$ *tengsizlik o‘rinli bo‘lsin. U holda agar 6-masalaning yechimi mavjud bo‘lsa, u yagona bo‘ladi,*

Bu yerda $\beta = \begin{cases} 1, \ \ agar \, l = 1, \\ \alpha - 1, \ \ agar \ \ l = 2. \end{cases}$

6-teorema. *Aytaylik* $\varphi_k(x), \in C^1[0,L_k], \quad t^{l-\gamma} f^{(k)}(x,t) \in C\left([0,L_k] \times [0,T]\right),$

$t^{l-\gamma} \dfrac{\partial}{\partial x} f^{(k)}(x,t) \in C\left([0,l_k] \times [0,T]\right)$, $(0,L_k)$ *va* $B_k \times (0,T)$ *sohalarda mos ravishda* $\varphi_k''(x)$ *va* $\dfrac{\partial^2}{\partial x^2} f^{(k)}(x,t)$ *funksiyalar absolyut integrallanuvchi bo‘lsin. Bundan tashqari* $f^{(k)}(x,t)$ *va* $\varphi_k(x)$ *funksiyalar (26)-(27) shartlarni qanoatlantirsin va* $M \neq \dfrac{1}{E_{\alpha,1}(-\lambda_n^2 T^\alpha)}$ *munosabat o‘rinli bo‘lsin. U holda 6-masalaning yechimi mavjud bo‘ladi.*

6-masalaning yechimi $[0,l_k] \times [0,T]$ sohada tekis yaqinlashuvchi funksional qator ko‘rinishida quyidagicha topilgan:

$$u^{(k)}(x,t) = \sum_{n=1}^{\infty} \left[\frac{M}{1 - M F_{1,n}(T)} F_{2,n}(T) t^{\gamma-1} E_{\alpha,\gamma}(-\lambda_n^2 t^\alpha) + \right.$$

$$+\int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}\left(-\lambda_n^2 (t-\tau)^{\alpha}\right) f_n(\tau) d\tau \Big] X_n^{(k)}(x), \;\; l=1$$

va

$$u^{(k)}(x,t) = \sum_{n=1}^{\infty} \Bigg[\frac{M}{1-MF_{1,n}(T)} \left(\varphi_n F_{4,n}(T) + F_{5,n}(T)\right) t^{\gamma-2} E_{\alpha,\gamma-1}(-\lambda_n^2 t^{\alpha}) +$$

$$+\varphi_n t^{\gamma-1} E_{\alpha,\gamma}(-\lambda_n^2 t^{\alpha}) + \int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}\left(-\lambda_n^2 (t-\tau)^{\alpha}\right) f_n(\tau) d\tau \Bigg] X_n^{(k)}(x), \; l=2$$

bu yerda

$$F_{1,n}(t) = E_{\alpha,1}(-\lambda_n^2 t^{\alpha}),$$

$$F_{2,n}(t) = \frac{1}{\Gamma(1-\gamma)} \int_0^t f_n(s) ds \int_s^t (t-\tau)^{-\gamma} (\tau - s)^{\alpha-1} E_{\alpha,\alpha}(-\lambda_n^2 (\tau-s)^{\alpha}) d\tau$$

$$F_{4,n}(t) = tE_{\alpha,2}(-\lambda_n^2 t^{\alpha}), \;\; F_{5,n}(t) = \int_0^t (t-\tau)^{1+\alpha-\gamma} E_{\alpha,2+\alpha-\gamma}(-\lambda_n^2 (t-\tau)^{\alpha}) f_n(\tau) d\tau .$$

Uchinchi bobning uchinchi paragrafida yuqorida ta'riflangan daraxtsimon Γ metrik grafning har bir qirrasida ushbu tenglamani

$$D_{0t}^{\alpha,\mu} u^{(k)}(x,t) = a u_{xx}^{(k)}(x,t) + b D_{0t}^{\alpha,\mu} u_{xx}^{(k)}(x,t) + c u^{(k)}(x,t) + f^{(k)}(x,t), \quad (28)$$

bu yerda $x \in B_k$, $0 < t < T$, $k = \overline{1,j}$, $l-1 < \alpha < l$, $l \in \{1,2\}$, $0 \le \mu \le 1$, a,b *va* c o'zgarmas sonlar, hamda quyidagi

$$\begin{aligned} & I_{0t}^{l-\gamma} u^{(k)}(x,t)\big|_{t=0} = \varphi_k(x), \\ & [\alpha] \frac{\partial}{\partial t} I_{0t}^{l-\gamma} u^{(k)}(x,t)\big|_{t=0} = [\alpha]\psi_k(x), \;\; x \in B_k, \; k = \overline{1,j}, \end{aligned} \quad (29)$$

boshlang'ich shartlarni qanoatlantiruvchi yechimini qidiraylik.
Bundan tashqari (26) ulash va (27) chegaraviy shartlarni qaraymiz.

7-masala. (28) tenglamaning (26) – (27) va (29) shartlarni qanoatlantiruvchi yechimi topilsin.

Masalaning bir qiymatli yechilishiga oid quyidagi tasdiq o'rinli:

7-teorema. *Aytaylik* $\varphi_k(x), \psi_k(x) \in C^3[0, L_k]$, $t^{1-\gamma} f^{(k)}(x,t) \in C_{x,t}^{1,0}\left([0,L_k] \times [0,T]\right)$ *hamda* $\varphi_k^{(4)}(x)$, $\psi_k^{(4)}(x)$, $\dfrac{\partial^2}{\partial x^2} f^{(k)}(x,t)$- *funksiyalar mos ravishda* $(0,l_k)$ *va* $B_k \times (0,T)$ *sohalarda absolyut integrallanuvchi bo'lsin, shuningdek* $\varphi_k(x)$, $\psi_k(x)$, $\varphi_k''(x)$, $\psi_k''(x)$ *va* $f^{(k)}(x,t)$ *funksiyalar uchun (26)-(27) shartlar bajarilsin. U holda 7-masalaning yagona yechimi mavjud.*

7-masalaning yechimi $\overline{B}_k \times [0,T]$ sohada tekis yaqinlashuvchi funksional qator ko'rinishida quyidagicha topilgan:

$$u^{(k)}(x,t) = \sum_{n=1}^{\infty} \Bigg[\varphi_n t^{\gamma-1} E_{\alpha,\gamma}\left(-\delta_n^2 t^{\alpha}\right) +$$

$$+\frac{1}{1+a_2\lambda_n^2} \int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}\left(-\delta_n^2 (t-\tau)^{\alpha}\right) f_n(\tau) d\tau \Bigg] X_n^{(k)}(x), \; l=1,$$

$$u^{(k)}(x,t)=\sum_{n=1}^{\infty}\left[\frac{1}{1+a_2\lambda_n^2}\int_0^t(t-\tau)^{\alpha-1}E_{\alpha,\alpha}\left[-\delta_n^2(t-\tau)^{\alpha}\right]f_n(\tau)d\tau+\right.$$
$$\left.+\varphi_n t^{\gamma-2}E_{\alpha,\gamma-1}\left(-\delta_n^2 t^{\alpha}\right)+\psi_n t^{\gamma-1}E_{\alpha,\gamma}\left(-\delta_n^2 t^{\alpha}\right)\right]X_n^{(k)}(x),\ l=2.$$

Ushbu dissertatsiya ishini tayyorlashda bevosita yordam bergan, o‘z maslahatlarini ayamagan f.m.-f.n., dotsent Sobirov Zarifboy Axmedovich hamda f.m.-f.n., dotsent Abdullayev Obidjon Xayrullayevichlarga o‘z minnatdorchiligimni bildiraman.

XULOSA

Dissertatsiya ishi metrik graflarda berilgan kasr tartibli hosilali turli differensial tenglamalar uchun boshlang‘ich-chegaraviy, nolokal va teskari masalalarni o‘zgaruvchilarni ajratish usuli (Furye usuli) orqali tadqiq qilishga bag‘ishlangan. Qaralgan masalalar yechimlarining yagonaligi asosan aprior baholashlar yordamida ko‘rsatilgan.

Tadqiqot ishining asosiy natijalari quyidagilardan iborat:

1. Metrik graflarda Kaputo va Prabhakar kasr hosilalari bilan berilgan diffuziya-to‘lqin tenglamalari uchun boshlang‘ich-chegaraviy masalalarning qator ko‘rinishidagi yaqqol yechimlari topilgan.

2. Yulduzsimon metrik grafda berilgan vaqt bo‘yicha kasr hosilali pseudo-parabolik tenglama uchun ikki xil teskari masalalarning funksional qatorlar ko‘rinishidagi yechimlari topilgan.

3. Daraxtsimon metrik grafda berilgan vaqt bo‘yicha kasr hosilali diffuziya-to‘lqin tenglamasi uchun nolokal masala yechimining mavjudligi va yagonaligi isbotlangan va qator ko‘rinishidagi aniq yechim topilgan.

4. Daraxtsimon metrik grafda berilgan vaqt bo‘yicha kasr hosilali psevdo-subdiffuziya tenglamasi uchun boshlang‘ich-chegaraviy masalalaning qator ko‘rinishidagi yechimi topilgan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED AFTER V. I. ROMANOVSKIY

INSTITUTE OF MATHEMATICS

KHUJAKULOV JONIBEK RAIMNAZAROVICH

DIRECT AND INVERSE PROBLEMS FOR FRACTIONAL ORDER DIFFUSION-WAVE EQUATIONS ON METRIC GRAPHS

01.01.02 – Differential Equations and Mathematical Physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT – 2023

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Ministry of Higher education, Science and Innovations of the Republic of Uzbekistan under number № B2023.2.PhD/FM870

Dissertation has been prepared at Institute of Mathematics named after V. I. Romanovskiy.

The abstract of the thesis is posted in three languages (Uzbek, English, Russian (summary)) on the website http://kengash.mathinst.uz and in the website of "ZiyoNet" Information and educational portal http://www.ziyonet.uz.

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Defense will take place " 14 " November 2023 at 17:00 at the meeting of Scientific Council number DSc.02/30.12.2019.FM.86.01 at Institute of Mathematics named after V. I. Romanovskiy (Address: University str. 9, Almazar area, Tashkent, 100174, Uzbekistan, Ph.: (+99871) 207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

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Abstract of dissertation sent out on " 27 " October 2023 year
(Mailing report № 2 on " 27 " October 2023 year)

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INTRODUCTION

Actuality and demand of the theme of the dissertation. It is known that many scientific investigations in physics, mechanics, chemistry, biophysics, biotechnology and production conducted on a global scale are related to the problems of the theory of various equations of mathematical physics. In fact, based on the physical meaning of such problems, the processes in many branched systems require the study of these equations in metric graphs. For example, in the modeling of processes in branched fields, initial-boundary problems are used for differential differential equations given on metric graphs. Basically, at the branching point of the graph, the flow conservation condition, i.e., the Kirchhoff condition, is used, which in the simplest case expresses the continuity of the solution at the end of the graph and the sum of the one-way derivatives of the solution is equal to zero.

In the field of medicine, differential equations given in graphs are widely used in mathematical modeling of processes. For example, initial-boundary problems are used for the diffusion equation given in metric graphs in the modeling of impulse propagation processes in nervous systems. Moreover, in the modeling of processes related to RNA in genes, differential equations are considered in ladder graphs.

Currently, special attention is paid to mathematics, physics, geology and biology as important sciences for the economic development of our country. In the course of the development of the mathematics, special attention is paid to the development of differential equations and mathematical physics, which are one of the main directions of mathematics and have a practical and theoretical nature.We have to note that the study of fractional differential equations on metric graphs, which is one of the important branches of mathematical physics, is rapidly developing. It is possible to add attention to the study of initial-boundary problems for differential equations given on metric graphs. Remarkable results were obtained by applying various equations given in metric graphs to the processes of quantum physics. Conducting scientific research at the level of international standards in the priority areas of “Differential Equations and Mathematical Physics”[1] has been defined as the main tasks and areas of activity of the mathematics. Therefore, solving the direct and inverse problems for differential equations given on metric graphs is one of the important tasks of modern mathematics.

The subject and object of this dissertation work has been chosen according to actual directions, mentioned in the Decree of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 “On measures to further improve of the activities of the Academy of Sciences, organization, management and financing of research activities", PP-3682 dated April 27, 2018” On measures to further improve the system of practical implementation of innovative ideas, technologies and projects" and PP-4387 from July 9, 2019 “On measures to further development of mathematical education and sciences, total improvement of the activity of the Uzbekistan Academy of Sciences V. I. Romanovsky Institute of Mathematics" and also PD-4708 dated May

[1] Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May « On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan» № 292.

7, 2020 "On measures to Improve the quality of education and research in mathematics". In addition, this dissertation work could be used to extent in the implementation of the tasks defined in all normative legal documents related to this activity.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV, «Mathematics, Mechanics and Computer Science» .

The degree of scrutiny of the problem. The study of differential equations on graphs is a relatively new field, which began in the 80s of the last century with the study of the heat distribution equation given on graphs by the Russian scientist Merkov. The first monograph devoted to topics, such as initial-boundary problems for differential equations on graphs, Green's function, differential inequalities was published by the Russian mathematician Y. V. Pokorny and his students. G. A. Sviridyuk, V. V. Shemetova defended the initial dissertation work on the topic "Research of a class of Sobolevsky-type equations on graphs" devoted to the study of differential equations in graphs. In that research work, various problems for Barenblatt-Zheltov-Kochina, Hoff, Oskolkova, Korpusov-Pletner-Sveshnikov equations were studied. Czech scientists P. Eksner and P. Seba studied initial-boundary problems for classical equations given in graphs. Further scientific results in this field were published by U. Smilansky, P. Kuchmant, G. Berkolaiko, R. Shrader, A. Kostrikin and their students in 2000-2010. They mainly investigated spectral and scattering problems in metric graphs for the Schrödinger equation. The concept of quantum graph was also used by these authors. The Schrödinger operator in thinly-branched fields was studied by P. Exner and O. Postlar. It is shown that quantum graphs are formed in the limit state when the cross-sectional dimension of field networks tends to zero. Joel Friedman and Jean-Pierre Tillich studied the completeness of "edge-based" Laplacian (or quantum graph) eigenfunctions.

Several problems for the Korteweg-de Vries equation and its linear part Airy equation in metric graphs have been studied. The currently available scientific results in this field were studied by Z. A. Sobirov, M. I. Ahmedov, N. Uesker, D. Noya, D. Magnola, K. Seyfert and others. A. S. Volkova studied the generalized solutions of the simple stellar heat distribution equation. Also, control problems using boundary values are investigated. In addition, in the research of R. Ch. Kulaev, one can find a lot of information that was studied on graphs. In 2021, M. R. Eshimbetov wrote a dissertation on the topic "Fokas' generalized substitution method for wave equations in metric graphs". In this work, the Fokas method is generalized for solving initial-boundary problems of Schrödinger, heat and wave equations on different metric graphs. The theory of fractional differential equations has gained great importance in the last few decades, mainly due to its application in many areas of modern science and technology. In turn, the sufficiently popularization of the theory attracted the attention of specialists, which led to many studies on the mathematical aspects of fractional differential equations and their solution methods. Different problems for time fractional differential equations have been studied by many specialists. In particular, our compatriots Sh.

Alimov, R. Ashurov, B. Kadirkulov, D. Durdiev, E. Karimov, Z. Sobirov, O. Abdullaev, T. Yuldashev, and others also achieved new scientific results in this direction. At the beginning of the 21st century, a number of works were published on the use of fractional derivatives. The inequality obtained by A. Alikhanov is important in obtaining an a priori estimate for fractional derivative differential equations. Russian mathematician A.V. Various initial and initial-boundary problems for the fractional differential equation were thoroughly studied by Pschu using the method of Green's functions.

Also, many studies are devoted to inverse problems of determining the right-hand side of fractional derivative equations. M. Ruzhansky, N. Tokmagambetov and B.T. Torebek's article, the inverse problem of determining the right-hand side of the Caputo fractional derivative subdiffusion equation for the case where the elliptic part is an arbitrary elliptic operator with a discrete spectrum was studied, and the authors proved the existence and uniqueness of the generalized solution using the Fourier method. Many inverse problems are being studied by R. Ashurov and his students to determine the order of differential equations with particular derivatives. Moreover D. Durdiev and his students are studing differend inverse coeffitcient problems for time-fractional differential equations In the works of Muhammad Ali, Sara Aziz, and Salman A. Malik considers two types of inverse source problems for a fractional differential equation of order $0<\gamma<1$ in the first variable and $1<\alpha<2$ in the second variable. The first one is devoted to the study of the spatially dependent recovery source term, and the second one is devoted to the determination of the time-dependent source term. The study of nonlocal problems for integral and fractional order differential equations is of great importance in the field. In particular, S. V. Pao for reaction-diffusion equations, N. N. Shopolov for heat diffusion equation, J. M. Rassias and E. T. Karimov studied various nonlocal problems for mixed parabolic equations. R. Ashurov and Y. Fayziev studied fractional derivative subdiffusion equations with respect to time, T. K. Yuldashev and B. J. Kadirkulov studied various nonlocal problems for mixed-type differential equations with Hilfer fractional differential operators, E. Karimov, M. Mamchuev, M. Ruzhanskiy, B. Toshtemirov for mixed-type differential equations with double-order Hilfer fractional differential operators. . Also, B. Toshtemirov defended his PhD thesis on the topic “Direct and inverse problems for singular partial differential equations with fractional integral-differential operators”. The study of fractional differential equations in metric graphs is considered one of the most interesting fields of modern mathematics and has been studied in recent years. V. Mehandiratta, M. Mehra studied the differential equations involving the fractional Caputo operator on star graph by numerical methods. P. S. Fosting, G. Mophou and others studied optimal control problems for fractional equations on graphs. Z. A. Sobirov and K. U. Rahimov studied subdiffusion and Airy equations on ladder and star metric graphs, initial and initial-boundary problems using Green's function method and potential methods. A. Turab, Juan J. Nieto at all. were investigated different problems for fractional differential equations on hexasilinane, chemical and cyclohexane graphs.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. This PhD thesis was carried out as

part of the program "Current problems of differential equations and related mathematical fields" in accordance with the research plan of V. I. Romanovskiy Institute of Mathematics.

The aim of research work consists of setting the direct and inverse problems for fractional differential equations with respect to time given on metric graphs and founding the solution of these problems.

Research problems:

to find the solution of boundary-value problems for time-fractional derivative diffusion-wave equations on metric graphs;

to solve inverse problems for pseudoparabolic equation with time-fractional derivative on star graph;

to find solution of nonlocal problems for the time-fractional derivative diffusion-wave equation on tree metric graph;

to prove the existence and uniqueness of the solution of initial-boundary problems for the time-fractional pseudo-subdiffusion equation on tree metric graphs.

The research object. Star graphs, tree metric graph consisting of a finite number intervals, ladder-type metric graph consisting of a finite interval.

The research subject. Time-dependent nonlocal problem, initial-boundary problem, inverse problem addresses the recovery of a space dependent source term.

Research methods. In the research work the method of separation of variables (Fourier method), the energy integral method, and a priori estimates are used.

Scientific novelty of the research work consists of the following:

The exist ence and a uniqueness of solutions of initial-boundary problems for time -fractional diffusion equations involving Kaputo and Prabhakar on the star graphs have been proved;

the unique solutions to inverse problems for the pseudoparabolic equation with the fractional derivative with respect to the given time on the star graph are found in the form of functional series;

it is proved that the solution of the nonlocal problem for the time -fractional diffusion-wave equation involving the Hilfer derivative on tree metric graphs exists and unique;

a unique solvability of initial-boundary boundary problems for the time-fractional pseudo-subdiffusion equation on tree metric graph.

Practical results of the research consists of the following:

The obtained results and the methods used in the dissertation can be taught as a scecial course for master's students and doctoral students. In addition, obtained results can be applied in developing mathematical models of various processes related to wave generation on the study of initial-boundary, nonlocal and inverse problems for time-fractional partial differential equations, with respect to time given on metric graphs.

The reliability of the results of the study. The results were obtained using the methods of spectral theory and method of separation of variables (the Fourier method). All obtained results are mathematically correct.

Scientific and practical significance of the research results. The scientific significance of the research results is that the scientific results obtained in the

dissertation work can be used in more in-depth research of direct and inverse problems for fractional order partial differential equations. In particular, a priori estimates and some important inequalities obtained in this research work can be used to prove the uniqueness of the solution of the considered initial-boundary and non-local problems for fractional differential equations. In addition, the obtained results can be used to study the properties of heat propagation in metric graphs and wave propagation in branched structures for many equations. The practical significance of the research is that the obtained results serve as a basis for finding the coefficient of passage and return from the branching point of wave propagation, analyzing wave processes in branched fields, and modeling the process of impulse propagation in nervous systems.

Implementation of the research results. The results obtained on initial-boundary problems for fractional differential equations on metric graphs were applied in the following research projects:

the solution of the diffusion-wave equation given on the branched regions of metric graphs was used to calculate the scattering parameters at the meeting point of the graph edges in grant №15-198 RG/PHYS/AS_I-FR 3240287086 according to "Experimental Study of Relaxation Prosseses in anisotropic liquids: Analysis of oscillation and orientation spectra" (Reference from National University of Uzbekistan № 04/11-5125, August 30, 2023). The results of the dissertation made it possible to find the reflectivity and permeability coefficients at the branch point;

the solution of the subdiffusion equation with the Hilfer fractional integro-differentiation operator on a star graph in project № AP09259394 "Inverse problems for evolution equations with positive operators" (Reference from Institute of Mathematics and Mathematical Modeling. (Kazakhstan) № 01-06/128 September 5, 2023) was used in the study of inverse problems for fractional differential equations. Using the scientific result allowed them to solve inverse problems for the partial differential evolution equation with a positive fractional derivative.

Approbation of the research results. The results of this research were discussed at 10 scientific conferences, including 6 international and 4 national scientific conferences.

Publications of the research results. On the topic of the dissertation 16 research papers have been published in the scientific journals, 6 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis and 4 of them were published in international journals, 2 of them are included the SCOPUS and WoS information database, 4 papers published in national mathematical journals and 10 theses.

The structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The total volume of the thesis is 126 pages.

MAIN CONTENT OF THE DISSERTATION

The introduction substantiates the actuality and demand of the theme of the dissertation, determines the correspondence of the study to priority areas of development of science and technology, provides an overview of foreign scientific research on the dissertation topic and the degree of study of the problem, formulates goals and objectives, identifies the object and subject of the study, outlines the scientific novelty and practical results of the study, the theoretical and practical significance of the results obtained is disclosed, information is given on the implementation of the research results, on published works and information on the structure of the dissertation.

The first chapter of the dissertation titled "Preliminary information" provides auxiliary information and is designed to facilitate the understanding of this research work. No new results are presented here. In this chapter, we present the basic properties of fractional derivatives and integrals, Mittag-Leffler and Prabhakar-type functions. In addition, we consider the solutions of the Cauchy problem for differential equations involving the fractional derivative with respect to time. We also present metric graphs, spectral problems considered in metric graphs.

Let's go over some definitions and assertions from the first chapter.

Definition 1. The Riemann-Liouville fractional integrals of order α of a function $f(x)$ is defined the following form:

$$\left(I_{at}^{\alpha} f\right)(t) = \frac{1}{\Gamma(\alpha)} \int_a^t \frac{f(\tau)d\tau}{(t-\tau)^{1-a}}\ ,\ t > a,\ a \in \mathbb{R}, n-1 < \alpha < n\,.$$

Definition 2. Operators defined by relations

$$\left(D_{at}^{\alpha} f\right)(t) := \left(\frac{d}{dt}\right)^n \left(I_{at}^{n-\alpha} f\right)(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)d\tau}{(t-\tau)^{\alpha-n+1}}\ , t > a$$

and

$$\left({}_C D_{at}^{\alpha} f\right)(t) := \left(I_{at}^{n-\alpha} \left(\frac{d}{dt}\right)^n f\right)(t) := \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau\ ,\ t > a\,,$$

respectively, are called fractional derivatives of Caputo and Riemann-Liouville. Here $n \in \mathbb{N}$ and $n-1 < \alpha < n$.

Caputo and Riemann-Liouville fractional derivatives have the following relation:

$$\left(D_{at}^{\alpha} f\right)(t) = \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{\Gamma(1+k-\alpha)} (t-a)^{k-\alpha} + \left({}_C D_{at}^{\alpha} f\right)(t),\ \ t > a\,,$$

where $n \in \mathbb{N}$ and $n-1 < \alpha < n$.

Definition 3. The Hilfer fractional derivative of order $\alpha\,(n-1 < \alpha \le n)$ type $\mu \in [0,1]$ are defined as follows:

$$D_{at}^{\alpha,\mu} f(t) = I_{at}^{\mu(n-\alpha)} \left(\frac{d}{dt}\right)^n I_{at}^{(1-\mu)(n-\alpha)} f(t)\,.$$

Definition 4. The Prabhakar integral of order $\beta\ \ (\beta > 0)$ is defined by

$${}^{P} I_{at}^{\alpha,\beta,\gamma,\delta} f(t) = \int_a^t (t-\tau)^{\beta-1} E_{\alpha,\beta}^{\gamma} \left(\delta (t-\tau)^{\alpha}\right) f(\tau) d\tau, t > a,$$

where $E_{\alpha,\beta}^{\gamma}(z)$ is the Prabhakar function.

Definition 5. The regularized Prabhakar derivative of order β is defined as follows

$$^{PC}D_{at}^{\alpha,\beta,\gamma,\delta}f(t)={}^{P}I_{at}^{\alpha,m-\beta,-\gamma,\delta}\frac{d^m}{dt^m}f(t),$$

where $\alpha>0$, $0<\beta<1$, $\gamma,\delta\in\mathbb{R}$.

Lemma 1. Let function $v(t)$ be absolutely continuous on $[0,T]$, one has the inequality

$$v(t)\,{}_{C}D_{0t}^{\alpha}v(t)\geq\frac{1}{2}\,{}_{C}D_{0t}^{\alpha}v^2(t).\ 0<\alpha<1.$$

Lemma 2. Let a nonnegative absolutely continuous function $y(t)$ satisfy the inequality

$${}_{C}D_{0t}^{\alpha}y(t)\leq c_1y(t)+c_2(t),\ \ 0<\alpha\leq1,$$

for almost all $t\in[0,T]$. Then

$$y(t)\leq y(0)E_{\alpha}(c_1t^{a})+\Gamma(\alpha)E_{\alpha,\alpha}(c_1t^{\alpha})I_{0t}^{\alpha}c_2(t).$$

Here $c_1>0$ and $c_2(t)$is an integrable nonnegative function on $[0,T]$, $E_{\alpha}(z)$ and $E_{\alpha,\beta}(z)$ are the Mittag-Leffler functions, of one and two parameter, respectively.

These functions are defined as follows:

$$E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)},$$

where $\alpha>0$, $\beta,z\in\mathbb{C}$. If $\beta=1$ this function can be written as $E_{\alpha,1}(z)=E_{\alpha}(z)$.

Definition 6. The function

$$E_{\alpha,\beta}^{\gamma}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\alpha n+\beta)}\frac{z^n}{n!}$$

is called as generalization of Mittag-Leffler function or Prabhakar function, where $\alpha,\beta,\gamma\in\mathbb{C}$, $Re(\alpha),Re(\beta)>0$ and $(\gamma)_n$ is the Pochammer symbol.

Definition 7. A Mittag-Leffler type function of two variables is defined as:

$$E_2(x,y)=\left(\begin{matrix}\gamma_1, & \alpha_1, & \beta_1; & \gamma_2, & \alpha_2, & \\ \delta_1, & \alpha_3, & \beta_2; & \delta_2, & \alpha_4; & \delta_3, & \beta_3\end{matrix}\middle|\begin{matrix}x\\y\end{matrix}\right)=$$

$$=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{1}{\Gamma(\delta_1+\alpha_3m+\beta_2n)}\frac{\Gamma(\gamma_1+\alpha_1m+\beta_1n)}{\Gamma(\gamma_1)}\frac{\Gamma(\gamma_2+\alpha_2m)}{\Gamma(\gamma_2)}\frac{x^m}{\Gamma(\delta_2+\alpha_4m)}\frac{y^n}{\Gamma(\delta_3+\beta_3n)},$$

where $\gamma_1,\ \gamma_2,\ \delta_1,\ \delta_2,\ \delta_3,\ x,y\in C$, $min\{\alpha_1,\alpha_2,\alpha_3,\alpha_4,\beta_1,\beta_2,\beta_3\}>0$.

The main results of the dissertation begin with the second chapter. The second chapter, titled **"Direct and inverse problems for fractional differential equations on star graphs"**, consists of three paragraphs, in the first paragraph we consider initial-boundary problem for the heat equation with Caputo fractional derivative on simple star graph.

Let that the graph Γ consists of three edges from point $O(0,0)$. Let the coordinates of these edges be defined from 0 to $l_k\,(0<l_k<+\infty,\ k=1,2,3.)$. We define the edges of the graph by $B_k=(0,l_k)$, $k=1,2,3.$

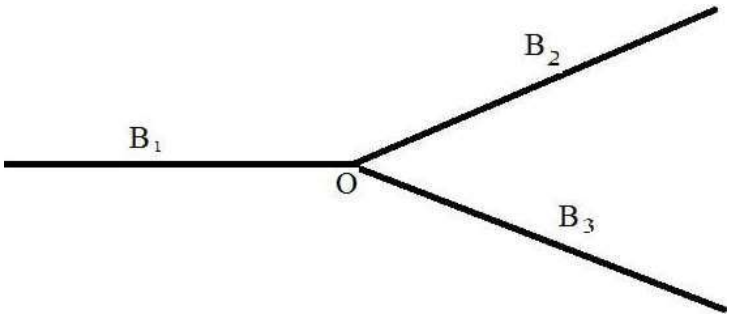

figure 1.

We consider the following fractional differential equations involving the Caputo derivative in $B_k\times(0,T)$:

$$ {}_C D_{0t}^{\alpha} u^{(k)}(x,t) - u_{xx}^{(k)}(x,t) = f^{(k)}(x,t),\ x\in B_k,\ k=1,2,3,\ 0<t<T, \tag{1}$$

Problem 1. To Find functions $u^{(k)}(x,t)\,(k=1,2,3)$, which satisfy the equation (1) in the domain $\Gamma\times(0,T)$, and the following

vertex conditions

$$u^{(1)}(0,t)=u^{(2)}(0,t)=u^{(3)}(0,t), \tag{2}$$

$$u_x^{(1)}(0,t)+u_x^{(2)}(0,t)+u_x^{(3)}(0,t)=0,\ t\in[0,T], \tag{3}$$

boundary conditions

$$u^{(k)}(L_k,t)=0,\ t\in[0,T],\ k=1,2,3, \tag{4}$$

and initial conditions

$$u^{(k)}(x,0)=\varphi_k(x),\ x\in\overline{B}_k,\ k=1,2,3, \tag{5}$$

here $f^{(k)}(x,t)$ and $\varphi_k(x)$ are given functions.

Theorem 1. *Let* $\varphi_k(x)\in C^1[0,l_k]$, $f^{(k)}(x,t)\in C_{x,t}^{1,0}\left([0,l_k]\times(0,T)\right)$ *besides* $\varphi_k''(x)$ *and* $\dfrac{\partial^2}{\partial x^2}f^{(k)}(\cdot,t)$ are absolutely integrable functions in $(0,l_k)$ and $B_k\times(0,T)$ respectively. Additionally, conditions (2)-(4) are valid for functions $\varphi_k(x)$ and $f^{(k)}(x,t)$. Then the unique solution of the investigated problem (1)-(5) exists and it can be written as the following uniformly convergent functional series in the domain $[0,l_k]\times[0,T]$:

$$u(x,t)=\sum_{n=1}^{\infty}\left(\int_0^t (t-z)^{\alpha-1}E_{\alpha,\alpha}[-\lambda_n^2(t-z)^{\alpha}]f_n(z)dz+\varphi_n E_{\alpha,1}(-\lambda_n^2 t^{\alpha})\right)X_n(x),$$

where φ_n and $f_n(t)$ are the Fourier coefficents of functions $\varphi_k(x)$ and $f^{(k)}(x,t)$, respectively, λ_n are eigenvalues.

In the second paragraph of the second chapter, we consider initial-boundary problem for the heat equation with the Prabhakar fractional derivative on star graph.

We consider a graph $\Gamma = V \cup E$ consisting of a finite set of vertices (nodes) $V = \{v_k\}_{k=0}^{j}$ and a finite set of edges $E = \{B_k\}_{k=1}^{j}$ connecting these nodes.

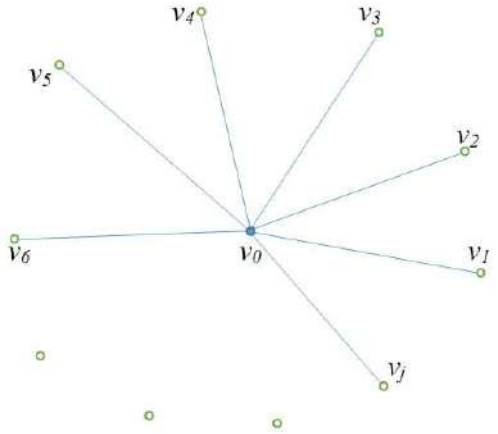

figure 2.

We consider the following fractional differential equations involving the Prabhakar derivative in $B_k \times (0,T)$:

$$ {}^{PC}D_{0t}^{\alpha,\beta,\gamma,\delta} u^{(k)}(x,t) - u_{xx}^{(k)}(x,t) = f^{(k)}(x,t),\ x \in B_k,\ k = \overline{1,j},\ 0 < t < T, \qquad (6) $$

where $\alpha > 0$, $0 < \beta < 1$, $\gamma, \delta \in R$, $f^{(k)}(x,t)$ are given functions.

Problem 2. To find a solution of (6) in the domain $B_k \times (0,T)$, satisfying the following

- vertex conditions

$$ u^{(1)}(0,t) = u^{(2)}(0,t) = \ldots = u^{(j)}(0,t), \quad t \in [0,T], \qquad (7) $$

$$ u_x^{(1)}(0,t) + u_x^{(2)}(0,t) + \ldots + u_x^{(j)}(0,t) = 0, \quad t \in [0,T], \qquad (8) $$

- boundary conditions

$$ u^{(k)}(l_k,t) = 0,\ t \in [0,T],\ k = \overline{1,j}, \qquad (9) $$

- and initial condition

$$ u^{(k)}(x,0) = \varphi_k(x),\ x \in \overline{B}_k,\ k = \overline{1,j}. \qquad (10) $$

Here $f^{(k)}(x,t)$ and $\varphi_k(x)$ are given functions.

The uniqueness of the solution of the considered problem (6)-(10) is proved using the following lemma.

Lemma 2. *There exist t_1 depending on $\alpha, \beta, \gamma, \delta$ such that for any function $v(t)$ absolutely continuous on $[0,t_1]$, one has the inequality*

$$ v(t) \cdot {}^{P}D_{0t}^{\alpha,\beta,\gamma,\delta} v(t) \geq \frac{1}{2} {}^{P}D_{0t}^{\alpha,\beta,\gamma,\delta} v^2(t),\ 0 < \beta < 1. $$

Theorem 2. *Let $\varphi_k(x) \in C^5[0,L_k]$, $f^{(k)}(x,t) \in C_{x,t}^{2,0}\left([0,L_k] \times [0,T]\right)$, besides $\varphi_k^{(6)}(x)$ and $\frac{\partial^3}{\partial x^3} f^{(k)}(x,t)$ are absolutely integrable functions in $(0,l_k)$ and $B_k \times (0,T)$ respectively. Furthermore, conditions (7)-(9) are also valid for the functions $\varphi_k''(x), \varphi_k^{IV}(x)$, $f^{(k)}(x,t)$, and conditions (7),(9) are also valid for the*

functions $\frac{\partial^2}{\partial x^2} f^{(k)}(x,t)$. *Then the problem has a unique solution.*

The solution of this problem is expressed by the two-variable Mittag-Leffler function, and is found in the form of the following functional series, which is a uniformly convergent in the domain $\Gamma \times [0,T]$:

$$u^{(k)}(x,t) = \sum_{n=1}^{\infty} \left(\varphi_n + \varphi_n \lambda_n^2 \Gamma(\gamma) t^{\beta} E_2 \left(\begin{matrix} & \gamma & \gamma & 1 & 1 & 0 \\ \beta+1 & \beta & \alpha & \gamma & \gamma & 1 & 1 \end{matrix} \middle| \begin{matrix} \lambda^2 t^{\beta} \\ \delta t^{\alpha} \end{matrix} \right) + \right.$$

$$\left. + \Gamma(\gamma) \int_0^t (t-\tau)^{\beta-1} f_n(\tau) E_2 \left(\begin{matrix} & \gamma & \gamma & 1 & 1 & 0 \\ \beta & \beta & \alpha & \gamma & \gamma & 1 & 1 \end{matrix} \middle| \begin{matrix} \lambda_n^2 t^{\beta} \\ \delta t^{\alpha} \end{matrix} \right) d\tau \right) X_n^{(k)}(x).$$

In the third paragraph of the second chapter, two inverse problems for the pseudoparabolic equation given by the Caputo fractional derivative on the star graph are studied.

On the each edge of the above defined graph Γ we consider fractional differential equations:

$$ {}_C D_{0t}^{\alpha} u^{(k)}(x,t) = a u_{xx}^{(k)}(x,t) + b\, {}_C D_{0t}^{\alpha} u_{xx}^{(k)}(x,t) + c u^{(k)}(x,t) + f^{(k)}(x,t) + h^{(k)}(x,t), \quad (11)$$

where $k = \overline{1,j}$, $0 < \alpha < 1$, $\varphi_k(x)$ are given functions, a,b,c are constants, we also consider the following conditions

$$u^{(1)}(0,t) = u^{(2)}(0,t) = \ldots = u^{(j)}(0,t), \quad t \in [0,T], \quad (12)$$

$$u_x^{(1)}(0,t) + u_x^{(2)}(0,t) + \ldots + u_x^{(j)}(0,t) = 0, \quad t \in [0,T], \quad (13)$$

$$u_x^{(k)}(l_k,t) = 0,\ t \in [0,T], k = \overline{1,j}, \quad (14)$$

$$u^{(k)}(x,0) = \varphi_k(x), \quad x \in \overline{B}_k,\ k = \overline{1,j}. \quad (15)$$

The problem (11)-(15) one can consider as a direct problem.

We will also discuss two inverse problems related to initial boundary value problem (11)-(15). The first inverse problem addressees the recovery of a space dependent source term, i.e. $h^{(k)}(x,t) = h^{(k)}(x) g(t)$, where $h^{(k)}(x)$ are unknown, $g(t)$ is given functions.

Second inverse problem we are interested in, is the recovery of time dependent source term i.e. $h^{(k)}(x,t) = g(t) h^{(k)}(x)$, where $h^{(k)}(x)$ are given, $g(t)$ is unknown functions.

Problem 3. Find a couples of functions $\left(u^{(k)}(x,t), h^{(k)}(x)\right)$ in the domain $B_k \times (0,T)$, satisfy (11)-(15) and under over-specified conditions

$$u^{(k)}(x,T) = \psi_k(x),\ x \in \overline{B}_k,\ k = \overline{1,j}, \quad (16)$$

where $\psi_k(x)$ are given functions.

Theorem 3. *If functions* $f^{(k)}(x,t)$ *and* $\varphi_k(x)$, $\psi_k(x)$ *satisfy the following conditions:*

(1) $f^{(k)}(x,t) \in C\left([0,l_k] \times (0,T)\right)$ *and* $\frac{\partial}{\partial x} f^{(k)}(x,t)$ *are absolutely integrable in*

$B_k \times (0,T)$,

(2) *$\varphi_k(x), \psi_k(x) \in C^2[0,l_k]$, $\varphi_k'''(x)$, $\psi_k'''(x)$ are absolutely integrable functions in $(0,l_k)$*

,

(3) *conditions (12)-(14) are valid for functions $\varphi_k(x)$ and condition (12) is valid for functions $\varphi_n''(x)$ and $f^{(k)}(x,t)$;*

(4) *conditions (12)-(14) are valid for functions $\psi_k(x)$ and condition (12) is valid for functions $\psi_n''(x)$.*

then the Problem 3. has a unique solution.

The solutions of the Problem 3, are found in the form of uniformly convergent series in the domain $[0,l_k]\times[0,T]$ as follows:

$$u^{(k)}(x,t) = \sum_{n=1}^{\infty} \frac{1}{1+b\lambda_n^2} \left[\frac{\left(1+b\lambda_n^2\right)\left[\psi_n - \varphi_n E_\alpha\left(-\mu_n^2 T^\alpha\right)\right] - F_n(T)}{G_n(T)} G_n(t) + \right.$$

$$\left. + F_n(t) + \varphi_n E_\alpha\left(-\mu_n^2 t^\alpha\right)\right] X_n^{(k)}(x),$$

$$h^{(k)}(x,t) = \sum_{n=1}^{\infty} \frac{(1+b\lambda_n^2)\left[\psi_n - \varphi_n E_\alpha\left(-\mu_n^2 T^\alpha\right)\right] - F_n(T)}{G_n(T)} g(t) X_n^{(k)}(x),$$

where $\mu_n^2 = \dfrac{a\lambda_n^2 - c}{1+b\lambda_n^2}$,

$$F_n(t) = \int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}\left[-\mu_n^2 (t-\tau)^\alpha\right] f_n(\tau) d\tau,$$

$$G_n(t) = \int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}\left[-\mu_n^2 (t-\tau)^\alpha\right] g(\tau) d\tau.$$

Problem 4. Find a couples of functions $\left(u^{(k)}(x,t), g(t)\right)$ in the domain $B_k \times \left(0,T\right)$, satisfy (11)-(15) and an over-specified condition of integral type

$$\int_\Gamma u(x,t)dx = \Phi(t), \quad t \in [0,T], \tag{17}$$

where $\Phi(t)$ is a given function.

Theorem 4. *Let functions $f^{(k)}(x,t)$ and $\varphi^{(k)}(x)$, $h^{(k)}(x)$ satisfy the following condition:*

(1) $h^{(k)}(x) \in C[0,l_k]$, $\dfrac{\partial}{\partial x} h^{(k)}(x)$ are absolute integrable functions in $(0,l_k)$,

(2) condition (12) is valid for functions $h^{(k)}(x)$,

(3) satisfying conditions (1)-(3) of the Theorem 3. are hold.

Then the Problem 4 has a unique solution.

The solutions of the Problem 4 are found in the form of uniformly convergent series in the domain $\Gamma \times [0,T]$ as follows:

$$u^{(k)}(x,t) = \sum_{n=1}^{\infty} \left[\frac{h_n}{1+b\lambda_n^2} \int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}\left[-\mu_n^2 (t-\tau)^\alpha\right] g(\tau) d\tau \right] +$$

$$+\frac{1}{1+b\lambda_n^2}\int_0^t (t-\tau)^{\alpha-1}E_{\alpha,\alpha}\left[-\mu_n^2(t-\tau)^\alpha\right]f_n(\tau)d\tau+\varphi_n E_{\alpha,1}\left(-\mu_n^2 t^\alpha\right)\Bigg]X_n^{(k)}(x),$$

$$g(t)=\left(\int_\Gamma h(x)dx\right)^{-1}\left[{}_C D_{0t}^\alpha\Phi(t)-c\Phi(t)-\int_\Gamma f(x,t)dx\right],$$

where h_n and $f_n(t)$ are the Fourier coefficents of functions $h_k(x)$ and $f^{(k)}(x,t)$, respectively.

The third chapter of the dissertation entitled **"Initial-boundary and nonlocal problems for fractional derivative differential equations on tree metric graphs"** considers initial-boundary and nonlocal problems for time-fractional derivative wave, diffusion-wave and pseudo-subdiffusion equations on tree metric graphs.

We consider finite regular tree with root vertex O. In the root vertex O we have one incident bond. In each interior vertex we have one incoming and two outgoing bonds (figure 3.). We denote the bonds as $B_1, B_2, ..., B_{2^{j+1}-1}$. We notice that the bonds B_k, B_{2k} and B_{2k+1}, $k=\overline{1,2^j-1}$ meet at one inner vertex point, and we denote connected graph Γ, as $\Gamma=V\cup E$, where $V=\{\nu_k\}_{k=1}^{2^{j+1}}$ is set of vertices and $E=\{B_k\}_{k=1}^{2^{j+1}-1}$ set of edges. The vertices with coordinate $x=0$ of the bond B_1 and with coordinates $x=L_k$ of the bonds B_k, $k=\overline{2^j,2^{j+1}-1}$, are called to be boundary vertices (figure 3.).

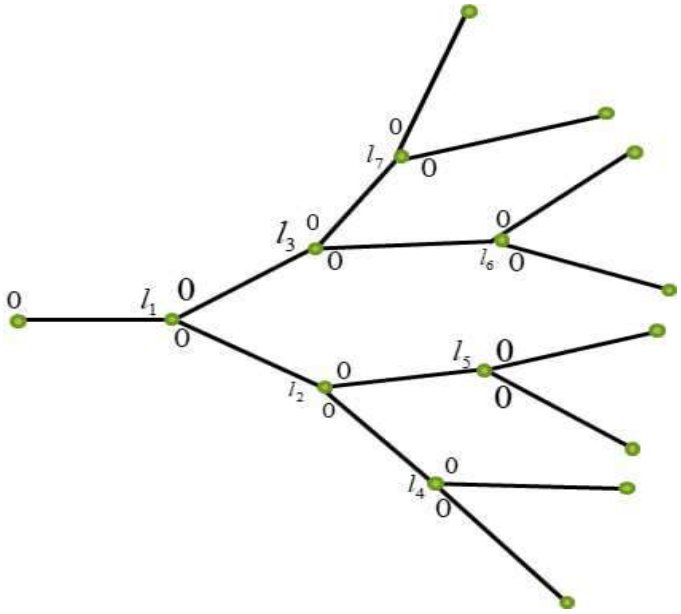

figure 3.

On the each bond of the graph Γ, we consider fractional differential equation

$${}_C D_{0t}^\alpha u^{(k)}(x,t)-u_{xx}^{(k)}(x,t)=f^{(k)}(x,t),\ (x,t)\in\left(B_k\times(0,T)\right),\ k=\overline{1,2^{j+1}-1}, \qquad (19)$$

where $1<\alpha<2$, and we look solution which satisfies the folloving initial conditions:

$$u^{(k)}(x,0)=\varphi_k(x),\ u_t^{(k)}(x,0)=\psi_k(x),\ x\in B_k,\ k=\overline{1,2^{j+1}-1}, \qquad (20)$$

Let Kirchhoff's conditions be valid for all $t\in[0,T]$ at the point where the edges meet:

$$u^{(k)}(l_k,t)=u^{(2k)}(0,t)=u^{(2k+1)}(0,t), \quad (21)$$

$$-u_x^{(k)}(l_k,t)+u_x^{(2k)}(0,t)+u_x^{(2k+1)}(0,t)=0,\ k=\overline{1,2^j-1}. \quad (22)$$

We consider homogeneous boundary conditions at the edge points of the graph

$$u^{(1)}(0,t)=0,\ u^{(k)}(l_k,0)=0,\ t\in[0,T]\ ,\ k=\overline{2^j,2^{j+1}-1}, \quad (23)$$

Problem 5. Find solution of equation (19) satisfying conditions (20)-(23).

Theorem 5. *Let* $\varphi_k(x),\ \psi_k(x)\in C^1[0,l_k],\ f^{(k)}(x,t)\in C^{1,0}_{x,t}\left([0,l_k]\times[0,T]\right)$ *besides* $\varphi_k''(x),\ \psi_k''(x)$ *and* $\frac{\partial^2}{\partial x^2}f^{(k)}(x,t)$ *are absolutely integrable functions in* $(0,L_k)$ *and* $B_k\times(0,T)$ *respectively. Additionally, conditions (21)-(23) are also valid for the functions* $\psi^{(k)}(x)$ *and* $f^{(k)}(x,t)$.

Then the unique solution of the problem 5 can be written in fhe form:

$$u^{(k)}(x,t)=\sum_{n=1}^{\infty}\Big[\int_0^t(t-z)^{\alpha-1}E_{\alpha,\alpha}[-\lambda_n^2(t-z)^{\alpha}]f_n(z)dz+$$

$$+\varphi_n E_{\alpha,1}(-\lambda_n^2t^{\alpha})+\psi_n tE_{\alpha,2}(-\lambda_n^2t^{\alpha})\Big]X_n^{(k)}(x),\ (k=\overline{1,2^{j+1}-1}),$$

where φ_n and ψ_n are the Fourier coefficents of functions $\varphi_k(x)$ and $\psi_k(x)$, respectively.

In the second and third sections of the third chapter, we studied nonlocal and initial-boundary problems for fractional differential equations with the Hilfer derivative on tree metric graph.

Let us consider a graph $\Gamma=V\cup E$ be a connected metric graph, where $E=\{B_k\}_{k=1}^{j}$ is the set of its edges, and $V=\{\nu_k\}_{j=1}^{m}$ is the set of vertices. Let us determine the coordinates x_k on the edges of the graph using isometric mapping of these edges onto the intervals $(0,l_k)$, $k=1,2,...,m.$

We will say that a vertex ν is in contact with an edge B_k if it is the end of this edge, and denote this as $B_k\sim\nu$. The number of elements of the set $\{B:B\sim\nu,B\in E\}$ is called vertex valency ν. If the valency of a vertex is equal to one, then it is called boundary. Let $\{q_1,q_2,...,q_{m_1}\}=\partial\Gamma\subset V$ be the boundary vertices of the graph .

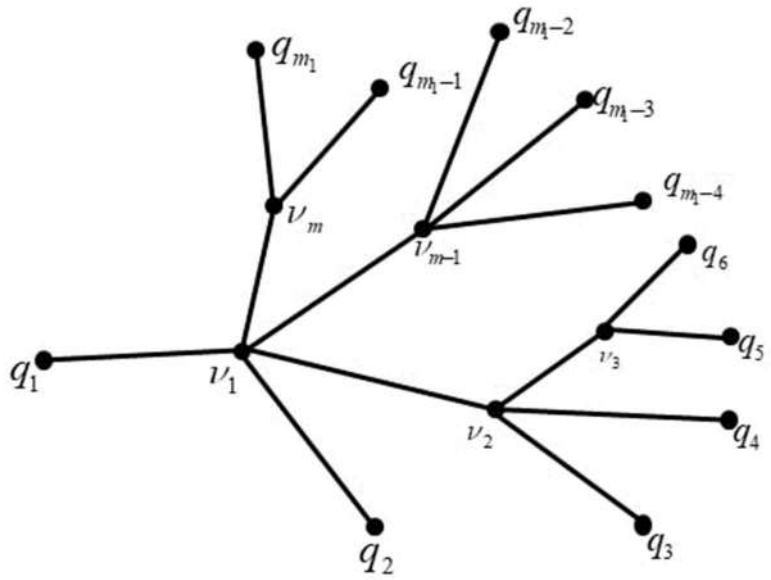

figure 4.

On the each edges of the over defined graph, we consider fractional differential equations

$$D_{0t}^{\alpha,\mu}u^{(k)}(x,t)-u_{xx}^{(k)}(x,t)=f^{(k)}(x,t),\ (x,t)\in\left(B_k\times(0,T)\right), \tag{24}$$

where $l-1<\alpha<l,\ l=\{1,2\},\ 0\le\mu\le 1,\ k=\overline{1,j},\ f^{(k)}(x,t)$ are given functions.

Problem 6. To find functions $u^{(k)}(x,t)$ in the domain $B_k\times\left(0,T\right)$, satisfy an equation (24) with the following properties:

non-local and initial conditions:

$$\begin{aligned} &I_{0t}^{l-\gamma}u^{(k)}(x,t)|_{t=0}=MI_{0t}^{l-\gamma}u^{(k)}(x,t)|_{t=T},\ \gamma=\alpha+l\mu-\alpha\mu,\\ &\frac{d}{dt}I_{0t}^{l-\gamma}u^{(k)}(x,t)|_{t=0}=\varphi_k(x),\ l=2,\ x\in B_k,\ k=\overline{1,j} \end{aligned}\quad . \tag{25}$$

At branching points the following δ *type conditions* which are at a vertex ν as follows:

$$\begin{cases} u^{(k)}(x,t)\ \textit{are continuous at}\ \nu\in V\setminus\partial\Gamma,\\ \displaystyle\sum_{B_k\sim\nu}u_x^{(k)}(x,t)\Big|_{\nu}=0, \end{cases} \tag{26}$$

boundary conditions

$$u^{(k)}(x,t)|_{q_k}=0,\qquad q_k\in\partial\Gamma,\ t\in(0,T],\ k=\overline{1,m_1}. \tag{27}$$

Here $\varphi^{(k)}(x)$, $\psi^{(k)}(x)$ are sufficiently smooth given functions.

Lemma 3. *Let*

$t^{1-\gamma}f^{(k)}(x,t)\in C\left(\overline{B}_k\times[0,T]\right),\qquad 0<\alpha<1,$

$\varphi_k(x)\in C\left(\overline{B}_k\right),\ t^{2-\gamma}f^{(k)}(x,t)\in C\left(\overline{B}_k\times[0,T]\right),1<\alpha<2,$

and

$M^2<\dfrac{1}{l-1+T^{\beta-1}E_{\alpha+l-1,\beta}(T^{\alpha})}$ *for* $l-1<\alpha<l,\ l=\{1,2\}$, *then the solution of*

the Problem 6 will be unique if it exist.

Here $\beta = \begin{cases} 1, \text{if } l = 1, \\ \alpha - 1, \text{ if } l = 2. \end{cases}$

Theorem 6. *Let* $\varphi_k(x), \in C^1[0,l_k], t^{l-\gamma} f^{(k)}(x,t) \in C\left([0,L_k]\times[0,T]\right)$, $t^{l-\gamma}\dfrac{\partial}{\partial x} f^{(k)}(x,t) \in C\left([0,l_k]\times[0,T]\right)$, *besides* $\varphi_k''(x)$ *and* $\dfrac{\partial^2}{\partial x^2} f^{(k)}(x,t)$ *are absolutly integrable functions in* $(0,l_k)$ *and* $B_k \times (0,T)$, *respectively. Furthermore, conditions (26)-(27) are also valid for functions* $f^{(k)}(x,t)$, $\varphi^{(k)}(x)$, *and* $M \neq \dfrac{1}{E_{\alpha,1}(-\lambda_n^2 T^\alpha)}$. *then the solution of the the Problem exists.*

The solution to the Problem 6. is found in the form of a uniformly convergent functional series in the domain $[0,l_k]\times[0,T]$, given as follows:

$$u^{(k)}(x,t) = \sum_{n=1}^{\infty}\left[\frac{M}{1-MF_{1,n}(T)} F_{2,n}(T) t^{\gamma-1} E_{\alpha,\gamma}(-\lambda_n^2 t^\alpha) + \right.$$

$$\left. + \int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}\left(-\lambda_n^2 (t-\tau)^\alpha\right) f_n(\tau) d\tau \right] X_n^{(k)}(x), \quad l = 1$$

and

$$u^{(k)}(x,t) = \sum_{n=1}^{\infty}\left[\frac{M}{1-MF_{1,n}(T)}\left(\varphi_n F_{4,n}(T) + F_{5,n}(T)\right) t^{\gamma-2} E_{\alpha,\gamma-1}(-\lambda_n^2 t^\alpha) + \right.$$

$$\left. + \varphi_n t^{\gamma-1} E_{\alpha,\gamma}(-\lambda_n^2 t^\alpha) + \int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}\left(-\lambda_n^2 (t-\tau)^\alpha\right) f_n(\tau) d\tau \right] X_n^{(k)}(x), \; l = 2.$$

Here

$$F_{1,n}(t) = E_{\alpha,1}(-\lambda_n^2 t^\alpha),$$

$$F_{2,n}(t) = \frac{1}{\Gamma(1-\gamma)} \int_0^t f_n(s) ds \int_s^t (t-\tau)^{-\gamma} (\tau - s)^{\alpha-1} E_{\alpha,\alpha}(-\lambda_n^2 (\tau - s)^\alpha) d\tau$$

$$F_{4,n}(t) = t E_{\alpha,2}(-\lambda_n^2 t^\alpha), \; F_{5,n}(t) = \int_0^t (t-\tau)^{1+\alpha-\gamma} E_{\alpha,2+\alpha-\gamma}(-\lambda_n^2 (t-\tau)^\alpha) f_n(\tau) d\tau .$$

Each edge of the tree-like metric graph Γ defined above in the third paragraph of the third chapter we look solution of the following equation:

$$D_{0t}^{\alpha,\mu} u^{(k)}(x,t) = a u_{xx}^{(k)}(x,t) + b D_{0t}^{\alpha,\mu} u_{xx}^{(k)}(x,t) + c u^{(k)}(x,t) + f^{(k)}(x,t), \quad (28)$$

where $x \in B_k$, $0 < t < T$, $k = \overline{1,j}$, $l - 1 < \alpha < l$, $l \in \{1,2\}$, $0 \le \mu \le 1$, a, b, c *are constants,* and satisfies the following initial conditions:

$$\begin{aligned} & I_{0t}^{l-\gamma} u^{(k)}(x,t)\big|_{t=0} = \varphi_k(x), \\ & [\alpha] \frac{\partial}{\partial t} I_{0t}^{l-\gamma} u^{(k)}(x,t)\big|_{t=0} = [\alpha] \psi_k(x), \; x \in B_k, \; k = \overline{1,j}. \end{aligned} \quad (29)$$

In addition, we consider the connection (26) and the boundary conditions (27).

Problem 7. Find a solution of equation (28) that satisfies conditions (26) -(27) and (29).

Theorem 7. *Let* $\varphi_k(x), \psi_k(x) \in C^3[0, L_k]$, $t^{1-\gamma} f^{(k)}(x,t) \in C^{1,0}_{x,t}\left([0,l_k]\times[0,T]\right)$ *besides* $\varphi_k^{IV}(x)$, $\psi_k^{IV}(x)$ *and* $\dfrac{\partial^2}{\partial x^2} f^{(k)}(x,t)$ *are absolutely integrable functions in* $(0,l_k)$ *and* $(B_k \times (0,T))$ *respectively. Furthermore, conditions (3.3.2)-(3.3.4) are also valid for the functions* $\varphi_k(x)$, $\psi_k(x)$, $\varphi_k''(x)$, $\psi_k''(x)$ *and* $f^{(k)}(x,t)$.

Then the problem 7 has a unique solution.

The solution to the problem 7 is found in the form of a uniformly convergent functional series in the domain $[0,l_k]\times[0,T]$ presented as follows:

$$u^{(k)}(x,t) = \sum_{n=1}^{\infty}\Big[\varphi_n t^{\gamma-1} E_{\alpha,\gamma}\left(-\mu_n^2 t^{\alpha}\right) +$$

$$+\frac{1}{1+b\lambda_n^2}\int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}\left(-\mu_n^2 (t-\tau)^{\alpha}\right) f_n(\tau)d\tau\Big] X_n^{(k)}(x),\ l=1,$$

$$u^{(k)}(x,t) = \sum_{n=1}^{\infty}\Bigg[\frac{1}{1+b\lambda_n^2}\int_0^t (t-\tau)^{\alpha-1} E_{\alpha,\alpha}\left[-\mu_n^2 (t-\tau)^{\alpha}\right] f_n(\tau)d\tau +$$

$$+\varphi_n t^{\gamma-2} E_{\alpha,\gamma-1}(-\mu_n^2 t^{\alpha}) + \psi_n t^{\gamma-1} E_{\alpha,\gamma}(-\mu_n^2 t^{\alpha})\Big] X_n^{(k)}(x),\ l=2.$$

I am grateful to Sobirov Z. A. and Abdullaev O. Kh. for their valuable suggestions during preparation my PhD thesis.

CONCLUSION

The dissertation is devoted to the study of initial-boundary, nonlocal and inverse problems for different fractional differential equations on metric graphs usingthe method of separation of variables (the Fourier method). The uniqueness of the solution of the considered problems are shown using apriori estimates.

The main results of the research are as follows:

1. The Solutions of initial-boundary problems for diffusion-wave equations involving the Caputo and the Prabhakar fractional derivatives on metric graphs were found in the series form.

2. For the pseudo-parabolic equation with the fractional derivative on the star graph, the solutions of two different inverse problems in functional series form are found.

3. The existence and uniqueness of solution of the nonlocal problem for the time-fractional diffusion-wave equation given on tree metric graph is proved, and an exact solution in the form of a series is found.

4. The solution of the initial-boundary problem for the pseudo-subdiffusion equation with time-fractional derivative on a tree metric graph is found in the series form.

НАУЧНЫЙ СОВЕТ DSc.02/30.12.2019.FM.86.01
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ
ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В. И. РОМАНОВСКОГО

ИНСТИТУТ МАТЕМАТИКИ

ХУЖАКУЛОВ ЖОНИБЕК РАИМНАЗАРОВИЧ

ПРЯМЫЕ И ОБРАТНЫЕ ЗАДАЧИ ДЛЯ ДИФФУЗИОННО-ВОЛНОВЫХ УРАВНЕНИЙ ДРОБНОГО ПОРЯДКА НА МЕТРИЧЕСКИХ ГРАФАХ

01.01.02 – Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)
ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

ТАШКЕНТ – 2023

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве Висшего образования, Науки и Инноваций Республики Узбекистан за № B2023.2.PhD/FM870.

Диссертация выполнена в институте Математики имена В. И. Романовского.

Автореферат диссертации на трех языках (узбекский, русский, английский (резюме)) размещен на веб-странице по адресу http://kengash.mathinst.uz и на информационно-образовательном портале «ZiyoNet» по адресу http://www.ziyonet.uz.

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Защита диссертации состоится « 14 » ноября 2023 года в 17:00 на заседании Научного совета DSc.02/30.12.2019.FM.86.01 при Институте Математики имени В. И. Романовского. (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9. Тел.: (+99871) 207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Цель исследования состоит из постановок прямых и обратных задач для дифференциальных уравнений дробного порядка относительно времени, заданных на метрических графах и нахождения решений этих задач.

Объектом исследования Звездообразные графы, древообразный граф, состоящий из конечного числа интервалов, метрический граф лестничного типа, состоящий из конечного интервала.

Научная новизна исследования состоит в следующем:

Доказаны существование и единственность решения начально-краевых задач для уравнений диффузии с дробной производной по времени с оператором Капуто и Прабхакара на звездообразных графах;

найдены решения обратных задач для псевдопараболического уравнения с дробной производной по заданному времени на звездообразном графе в виде функциональных рядов;

доказаны существование и единственность решения нелокальной задачи для диффузионно-волнового уравнения с дробной производной Хилфера на древообразый метрических графах;

доказана единственность и существование решения начально-краевых задач для уравнения псевдосубдиффузии с дробной производной по времени на древо метрическом графе.

Внедрение результатов исследования. Результаты, полученные в диссертационной работе, внедрены в практику по следующим направлениям:

решение диффузионно-волнового уравнения, заданного на разветвленных областях метрических графов было использовано для вычисления параметров разбрасывания в точке встречи ребер графа в гранте №15-198 RG/PHYS/AS_I-FR 3240287086 по «Экспериментальное исследование релаксационных процессов в анизотропных жидкостях: анализ колебательных и ориентационных спектров» (Справка № 04/11-5125 от 30 августа 2023 года, Национальный Университет Узбекистана). Результаты диссертации позволили найти коэффициенты отражаемости и проходимости в точке разветвления;

решение уравнения субдиффузии с оператором дробного интегро-дифференцирования Хильфера на звездообразном графе использовался при исследовании обратных задач для уравнений в частных производных дробного порядка в проекте №AP09259394 «Обратный задачи для эволюционных уравнений с положительными операторами» (Справка № 01-06/128 от 5 сентября 2023 года, Институт математики и математического моделировария (Казахстан)). Использование научного результата позволило им решать обратных задач для уравнения эволюции в частных производных с положительным дробным производным.

Структура и объем диссертации. Диссертация состоит из введения, четырех глав, заключения и списка литературы. Общий объем диссертации составляет 126 страниц.

E’LON QILINGAN ISHLAR RO‘YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

I bo‘lim (1 часть; part 1)

1. Abdullaev O. Kh., Khujakulov J. R. On a problem for the time-fractional diffusion equation on a metric graphs. // Uzbek Mathematical Journal. 2017. – №4. –P. 3-12. (01.00.00, №6).

2. Khujakulov J. R., On inverse source problem for time fractional diffusion equation on simple metric graphs. // Uzbek Mathematical Journal. 2020. –№2. –P. 99-108. (01.00.00, №6).

3. Karimov E. T., Sobirov Z. A., Khujakulov J. R., Solvability of a problem for a time fractional differential equation with the Hilfer operator on metric graphs. // Bulletin of the Institute of Mathematics. 2021, –Vol.4(4). –P. 9-18. (01.00.00, №17).

4. Sobirov Z. A., Abdullaev O. Kh., Khujakulov J. R. Initial Boundary Value Problem For a Time Fractional Wave Equation on a metric Graph. // Differential Equations & Application. 2023. –Vol. 15(1). –P. 13–27. (WoS)

5. Khujakulov J. R. Initial-boundary value problem for a time fractional differential equation with the Prabhakar derivative on a star graph. // Bulletin of the Institute of Mathematics. 2023. –Vol. 6(2). –P. 20-30. (01.00.00, №17).

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II bo‘lim (part 2; 2 часть)

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Avtoreferat “O‘zbekiston matematika jurnali” tahririyatida 2023 yil 16 oktabrda tahrirdan o‘tkazilib, o’zbek, rus va ingliz tillaridagi matnlar o’zaro muvofiqlashtirildi.

Bosmaxona litsenziyasi:

9338

Bichimi: 84x60 $^{1}/_{16}$. «Times New Roman» garniturasi.
Raqamli bosma usulda bosildi.
Shartli bosma tabog‘i: 3. Adadi 100 dona. Buyurtma № 51/23.

Guvohnoma № 851684.
«Tipograff» MCHJ bosmaxonasida chop etilgan.
Bosmaxona manzili: 100011, Toshkent sh., Beruniy ko‘chasi, 83-uy.