V.I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

V.I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI

BOLTAYEV ASLIDDIN ASKAR O‘G‘LI

BIRINCHI TARTIBLI XOTIRALI ELASTIKLIK TENGLAMALAR SISTEMASI UCHUN TESKARI MASALALAR

01.01.02 –Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI
bo‘yicha falsafa doktori (PhD) dissertatsiyasi
AVTOREFERATI

Toshkent – 2024

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V.I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

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Dissertatsiya V.I. Romanovskiy nomidagi Matematika instituti va RFA Vladikavkaz ilmiy markazida bajarilgan.

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Dissertatsiya himoyasi V.I. Romanovskiy nomidagi Matematika instituti huzuridagi DSc.02/30.12.2019.FM.86.01 raqamli Ilmiy kengashning 2024 yil “28” may soat 16:00 dagi majlisida bo‘lib o‘tadi. (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko‘chasi, 9-uy. Tel.: (+99871) 207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

Dissertatsiya bilan V.I. Romanovskiy nomidagi Matematika institutining Axborot-resurs markazida tanishish mumkin (183-raqami bilan ro‘yxatga olingan). Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko‘chasi, 9-uy. Tel.: (+99871) 207-91-40.

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida olib borilayotgan aksariyat ilmiy-amaliy tadqiqotlar ko‘p hollarda matematik fizikaning to‘g‘ri va teskari masalalarini o‘rganishga olib kelinadi. Teskari masalalar astronomiya, kvantlarning tarqalish nazariyasi, geofizika, issiqlik fizikasi, tibbiyotga hamda EHM paydo bo‘lishi bilan zamonaviy ilm-fanning barcha sohalariga kirib bordi. Qovushqoq-elastik muhitda elastik to‘lqinlarning tarqalish jarayonini matematik usullardan foydalangan holda aniqroq o‘rganishda jarayonning xotirasini hisobga olish talab qilinadi. Matematik nuqtai nazardan, hodisaning “o‘tmishini” hisobga olish, qaralayotgan elastiklik nazariyasining matematik modelida xotiraga javob beruvchi yadroga ega bo‘lgan o‘rama ko‘rinishdagi integral operatorini kiritish zarurligiga olib keladi. To‘g‘ri masala yechimi haqida qo‘shimcha ma’lumotlarga asoslanib, to‘g‘ri masala yechimini va integro-differensial tenglama yadrosini aniqlashga doir tadqiqotlar olib borish matematik fizikaning muhim vazifalaridan biri bo‘lib qolmoqda.

Hozirgi kunda jahon miqyosida matematik fizikaning eng tez rivojlanayotgan sohasi – teskari masalalarni tadqiq qilish usullariga alohida e’tibor qaratilmoqda. Bu soha fizika va texnika fanlaridagi eng muhim matematik muammolardan biriga aylandi. Qovushqoq-elastiklik differensial tenglamalar sistemalari seysmik qidiruvda, seysmik to‘lqinlar yordamida muhitning xususiyatlarini o‘rganishda paydo bo‘ladi. To‘lqin maydoni haqidagi ma’lumotlardan foydalanib integral operator yadrosini aniqlash teskari masalasi geologik muhitning tuzilishi va xossalarini o‘rganishda muhim o‘rin tutadi. Shunday qilib, qovushqoq-elastik muhitda integro-differensial tenglamalar uchun qo‘yilgan to‘g‘ri va teskari masalalarning korrektligini o‘rganish zarurati paydo bo‘ldi. Qovushqoq-elastik muhitlarda to‘lqin tarqalish jarayonlari nazariyasining teskari masalalarini yechish usullarini ishlab chiqish zarurati dissertatsiya ishi mavzusining dolzarbligini belgilaydi.

Mamlakatimizda fundamental fanlarning ilmiy va amaliy tatbiqiga ega bo‘lgan integro-differensial tenglamalar va matematik fizikaning dolzarb yo‘nalishlariga e’tibor qaratilmoqda. Jumladan, so‘nggi yillarda, elastik to‘lqinlarning xotirali muhitlarda tarqalishi jarayonlarini ifodalovchi integro-differensial tenglamalari uchun boshlang‘ich-chegaraviy masalalarni tadqiq etish, ular uchun teskari masalalarni yechish orqali amaliy muammolarni hal etishga alohida e’tibor qaratildi. Elastik to‘lqinlarning xotirali muhitlardagi integro-differensial tenglamalari uchun teskari masalalar yechimining mavjudligini, yagonaligini isbotlashga va muhitning xotira funksiyasini aniqlashga oid salmoqli natijalarga erishildi. Differensial tenglamalar, matematik fizika va funksional analiz fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalari va faoliyat yo‘nalishlari qilib belgilandi[1]. Bu borada matematik fizikaning integro-differensial

[1] O‘zbekiston Respublikasi Vazirlar Mahkamasining 2017-yil 18-maydagi “O‘zbekiston Respublikasi Fanlar akademiyasining yangi tashkil etilayotgan ilmiy-tadqiqot muassasalari faoliyatini tashkil etish chora-tadbirlari to‘g‘risida”gi 292-son qarori va tuzilmaviy masalalar

tenglamalari uchun teskari masalalar nazariyasini rivojlantirish muhim ahamiyatga ega hisoblanadi.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevral PF-4947-sonli «O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida»gi, 2022-yil 28-yanvar PF-60-sonli «2022-2026 yillarga mo‘ljallangan Yangi O‘zbekistonning taraqqiyot strategiyasi to‘g‘risida»gi farmonlari, 2017-yil 17-fevral PQ-2789-sonli «Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida»gi, 2017-yil 20-aprel PQ-2909-sonli «Oliy ta’lim tizimini yanada rivojlantirish chora-tadbirlari to‘g‘risida»gi, 2018-yil 27-aprel PQ-3682-sonli «Innovatsion g‘oyalar, texnologiyalar va loyihalarni amaliyotga joriy qilish tizimini yanada takomillashtirish chora-tadbirlari to‘g‘risida»gi, 2020-yil 7-may PQ-4708-sonli «Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida»gi qarorlari, hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV «Matematika, mexanika va informatika» ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Teskari masalalar bo‘yicha dastlabki nashrlar fizika, geofizika, astronomiya va boshqa fanlar bilan bog‘liq. Qidiruv geofizikasining matematik-fizik muammolari teskari masalalar nazariyasining muhim yo‘nalishidir. Bu yo‘nalishda A.S. Alekseyev va S.V. Goldinning nashrlarini eslatib o‘tish mumkin. Xususan, A.S. Alekseyev birinchilardan bo‘lib yarim fazoda izotrop elastiklik tenglamalar sistemasi uchun zichlikni aniqlashning bir o‘lchamli teskari masalasini o‘rgandi. Ushbu yo‘nalishni M.M. Lavrentyev, V.G. Romanov, A.S. Blagoveshchensky, S.P. Belinsky, V.G. Yaxno va boshqalar rivojlantirib, giperbolik tipdagi tenglamalar uchun turli xil teskari masalalarni o‘rganish usullarini ishlab chiqdilar.

Giperbolik tipdagi integro-differensial tenglamalardan integral operatorning yadrosini aniqlash masalalari teskari masalalar nazariyasida yosh, tez sur’atlarda rivojlanayotgan yo‘nalish bo‘lib, bu yo‘nalishda dastlabki olingan ilmiy natijalar M. Grasselli, V.G. Romanov, A. Lorenzi, Ye. Paparoni, D.Q. Durdiyev kabi olimlarning nomlari bilan bog‘liq bo‘lgan bo‘lsa, keyinchalik A.L. Buxgeym, D.Q. Durdiyev, V.G. Yaxnolarning ishlarida giperbolik tenglamalar uchun xotira funksiyasini aniqlash teskari masalalari ham tadqiq etildi. A.L. Buxgeym, V.G. Romanov, D.Q. Durdiyev, J.Sh. Safarov, Y. Yannolarning ishlarida faqat vaqt o‘zgaruvchisiga bog‘liq bo‘lgan yadroni aniqlash masalalari ko‘rib chiqilgan bo‘lib, masala noma’lum funksiyalarga nisbatan Volterra tipidagi integral tenglamalarni yechishga keltirilgan va yechimning mavjudlik, yagonalik teoremalari, yechimning berilgan funksiyalarga uzluksiz bog‘liqlik baholari olingan.

Giperbolik tenglamalar sistemasi uchun teskari masalalar nazariyasi L.P. Nijnik, S.P. Belinsky, V.G. Romanov, T.P. Puxnachova, D.Q. Durdiyev, J.D. Totiyeva va boshqalar tomonidan keng tadqiq qilingan. V.G. Romanov[2] $u = (u_1, u_2, \dots, u_m)$, $m \geq 2$ vektor funksiya uchun

$$u_t + \sum_{j=1}^{n} A_j u_{x_j} + Bu_z + Du = F \qquad (1)$$

tenglamalar sistemasini $\Omega = \{(x, z, t): x \in \mathbb{R}^n, 0 < z < H < \infty, t > 0\}$ sohada qaragan. $(A_j, B, D)(z) - m \times m$ o‘lchamli matritsalar, $F = (F_1, \dots, F_m)(x, z, t)$, $x = (x_1, \dots, x_n)$. (1) tenglamalar sistemasi t- giperbolik, $F(x, z, t) = \Phi(t) f(x, z)$ deb faraz qilinib, Ω soha chegarasida berilgan shartlar asosida $\Phi(t)$ ma’lum bo‘lganda $f(x, z)$ funksiyani topish masalasi o‘rganilgan. S.P. Belinsky va V.G. Romanovlar[3] (1) tenglamalar sistemasi yechimi Ω sohaning chegarasida ma’lum bo‘lganda $(A_j, B, D)(z)$ matritsalarni aniqlash masalasini tadqiq qilgan. Qovushqoq-elastiklik integro-differensial tenglamalar sistemasidan o‘rama ko‘rinishidagi yadrolarni aniqlash teskari masalasini D.Q. Durdiyev, J.D. Totiyeva va boshqalarning ishlarida uchratishimiz mumkin. Bu ishlarda lokal mavjudlik, global yagonalik teoremalari va turg‘unlik baholari olingan. D.Q. Durdiyev va H.H. Turdiyevning ishlarida esa, Maksvell integro-differensial tenglamalari sistemasidan diagonal relaksatsiya matritsasini aniqlash teskari masalasi o‘rganilgan. Ushbu ishlarda yechimning global mavjudlik va yagonalik teoremalari olingan.

Ushbu dissertatsiya ishida

$$F = F(x, z, t; u) = \int_0^t R(t - \tau) u(x, z, \tau) d\tau + f(x, z, t) \qquad (2)$$

bo‘lib, (1) tenglama yechimi Ω soha chegarasida berilganda $R(t)$ matritsani topish masalasi o‘rganilgan. (1) tenglamaning fizik modeli sifatida qovushqoq-elastiklik nazariyasining kuchlanishlar tenzori va deformatsiyalar orasidagi Bolsano-Volterra integral munosabatlari qabul qilingan. Izotropik, anizotropik muhitlar qaralib, muhitning qovushqoqligini ifodalovchi $R(t)$ yadro matritsasining turli hollari uchun har xil masalalar tadqiq qilingan. Masalalar yechimining mavjudligi va yagonaligi haqida natijalar olingan.

Dissertatsiya tadqiqotining dissertatsiya bajarilayotgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti O‘zbekiston Respublikasi Fanlar Akademiyasi V.I. Romanovskiy nomidagi Matematika instituti Buxoro bo‘linmasining “Matematik fizikaning teskari masalalari” mavzusidagi kalendar rejasi, Rossiya Fanlar akademiyasi

[2] В.Г. Романов, К задаче определения правой части гиперболических систем, Диф. уравнения, 1977, 13:3, с. 509-515.

[3] С.П. Белинский, В.Г. Романов, К задаче определения коэффициентов t-гиперболической системы, Матем. заметки, 1980, 28:4, с. 525-532.

Vladikavkaz ilmiy markazi Janubiy matematika institutining №075-02-2022-896 sonli loyihasi doirasida bajarilgan.

Tadqiqotning maqsadi ikki va uch o‘lchamli integro-differensial qovushqoq-elastiklik tenglamalar sistemasidan integral hadlar yadrolarini aniqlash usullarini qurish va bu teskari masala yechimining yagonaligi, mavjudligini isbotlashdan iborat.

Tadqiqotning vazifalari quyidagilardan iborat:

ikki o‘lchamli qovushqoq-elastiklik sistemasi uchun teskari masala yechimining bir qiymatli yechiluvchanligini isbotlash;

vertikal qatlamli muhitda uch o‘lchamli qovushqoq-elastiklik tenglamalar sistemasi uchun boshlang‘ich-chegaraviy masala yechimining mavjudligi va yagonaligini ko‘rsatish;

izotrop muhitda qovushqoq-elastiklik tenglamalar sistemasi uchun xotirani aniqlashning teskari masalasini tadqiq qilish;

anizotrop muhitda tetragonal ko‘rinishli elastiklik moduliga ega qovushqoq-elastiklik tenglamalar sistemasi uchun yadroni topish teskari masalalasi yechimi global mavjudligi va yagonaligini o‘rganish.

Tadqiqotning obyekti birinchi tartibli giperbolik integro-differensial tenglamalar sistemasi uchun bir o‘lchamli teskari masalalardan iborat.

Tadqiqotning predmeti ikki va uch o‘lchamli qovushqoq-elastiklik tenglamalar sistemasi uchun teskari masalalardan iborat.

Tadqiqotning usullari. Ilmiy tadqiqot ishida funksional analiz, xususiy hosilali differensial tenglamalar, integral tenglamalar nazariyasi usullaridan foydalanilgan. Teskari masalalarning bir qiymatli yechiluvchanligini ko‘rsatish uchun masalalar Volterra tipidagi ikkinchi tur chiziqli bo‘lmagan integral tenglamalar sistemasiga olib kelinib, ularga siqiluvchan akslantirishlar prinsipi qo‘llanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

ikki o‘lchamli qovushqoq-elastiklik tenglamalar sistemasi uchun teskari masala yechimining mavjudligi va yagonaligi isbotlangan;

vertikal qatlamli muhitda uch o‘lchamli qovushqoq-elastiklik tenglamalar sistemasi uchun boshlang‘ich-chegaraviy masala yechimining mavjudligi va yagonaligi ko‘rsatilgan;

izotrop muhitda qovushqoq-elastiklik tenglamalar sistemasi uchun xotirani aniqlash teskari masalasi bir qiymatli yechilishi isbotlangan;

anizotrop muhitda tetragonal ko‘rinishli elastiklik moduliga ega qovushqoq-elastiklik tenglamalar sistemasi uchun yadroni topish teskari masalalasi yechimining global mavjudligi va yagonaligi isbotlangan.

Tadqiqotning amaliy natijalari. Tadqiqot natijalaridan seysmologiyada, neft va gaz konlarini qidirishda, hamda yuqori bosqich bakalavriat va magistratura talabalari uchun matematik fizika fanidan maxsus kurslarni o‘qishda foydalanish mumkin.

Tadqiqot natijalarining ishonchliligi. Natijalar differensial tenglamalar nazariyasi, teskari masalalar nazariyasi, matematik va funksional analiz usullarini qo‘llash orqali olingan. Olingan natijalar matematik jihatdan qat’iy isbotlangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati matematik fizikaning qovushqoq-elastiklik integro-differensial tenglamalar sistemasidan bir o‘lchamli yadrolarni aniqlash usullari qurilganligi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati qovushqoq-elastik xotirali muhitlarda to‘lqin tarqalish jarayonlarini tekshirishda tatbiq etilishi bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Birinchi tartibli xotirali giperbolik tenglamalar sistemasi uchun teskari masalalarga oid olingan ilmiy natijalar asosida:

qovushqoq-elastiklik integro-differensial tenglamalar sistemasi uchun yadroni aniqlash usullaridan 122041100096-4 raqamli “Sotsiologiyada, geofizikada va muhandislik fanlarida matematik modellashtirish” mavzusidagi xorijiy fundamental loyihada vertikal qatlamli izotrop muhitlardagi qovushqoq-elastiklik tenglamalar sistemasidan o‘rama ko‘rinishidagi yadroni aniqlash teskari masalalarida foydalanilgan (Rossiya Fanlar akademiyasining Vladikavkaz ilmiy markazi Federal davlat ilmiy muassasasi Federal ilmiy markazi filali Janubiy matematika institutining, 2023 yil 13 oktabrdagi 81-sonli ma’lumotnomasi, Rossiya Federatsiyasi). Ilmiy natijaning qo‘llanilganligi vertikal qatlamli izotrop muhitlardagi qovushqoq-elastiklik tenglamalar sistemasidan o‘rama ko‘rinishidagi yadroni aniqlash va uning global mavjudlik va yagonaligini isbotlash imkoni bergan;

izotrop muhitda qovushqoq-elastiklik tenglamalar sistemasi uchun ko‘chish vektorini va xotira funksiyasini aniqlash usullaridan AAAA-A19-119072290002-9 raqamli “Kamchatkadagi tabiiy ofatlar - zilzilalar va vulqon otilishi” mavzusidagi xorijiy loyihada ikki va uch o‘lchamli qovushqoq-elastiklik tenglamalar sistemasidan yadro matritsasini aniqlashda foydalanilgan (Vitus Bering nomidagi Kamchatka davlat universitetining 2023 yil 13 noyabrdagi 54-12-sonli ma’lumotnomasi, Rossiya federatsiyasi). Ilmiy natijaning qo‘llanilishi yechim mavjudligi va yagonaligini isbotlash imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqotning asosiy natijalari 3 ta xalqaro va 2 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinishi. Dissertatsiya mavzusi bo‘yicha jami 13 ta ilmiy ish chop etilgan, shulardan, O‘zbekiston Respublikasi Oliy Attestatsiya komissiyasining dissertatsiyalar asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 8 ta maqola, jumladan, 4 tasi xorijiy va 4 tasi respublika jurnallarida nashr qilingan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning umumiy hajmi 95 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zaruriyligi asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor

yo‘nalishlariga mosligi ko‘rsatilgan, muammoning o‘rganilganlik darajasi, mavzu bo‘yicha dunyo miqyosidagi ilmiy-tadqiqotlar sharhi keltirilgan, tadqiqot maqsadi, vazifalari, obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning “**Ikki o‘lchamli qovushqoq-elastiklik tenglamalar sistemasidan yadroni aniqlash”** deb nomlangan birinchi bobning birinchi paragrafida dissertatsiyada qo‘llaniladigan asosiy tushuncha va ta’riflar keltirilgan. Shuningdek, ushbu ilmiy tadqiqot mavzusi doirasida tadqiq etilgan ilmiy izlanishlar va olingan natijalar bayon qilingan. Ikkinchi paragrafda ikki o‘lchamli qovushqoq-elastiklik tenglamalar sistemasi uchun boshlang‘ich-chegaraviy masala o‘rganilgan.

Faraz qilaylik, $\overline{x} = (x_1, x_2) \in \mathbb{R}^2$ bo‘lsin. x_j, $j = 1{,}2$ o‘qiga normal parallel bo‘lgan maydonga ta’sir qiluvchi kuchlanishning x_i, $i = 1{,}2$ o‘qiga proyeksiyasini σ_{ij}, $i, j = 1{,}2$ bilan va zarracha siljish vektorining x_i o‘qiga proyeksiyasini $\bar{u}_i$, $i = 1{,}2$ bilan belgilaylik. Qovushqoq-elastik muhitlar uchun Guk qonuni quyidagi ko‘rinishda ifodalanadi[4]:

$$\sigma_{ij}(\overline{x}, t) = \mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) + \delta_{ij}\lambda \mathrm{div}\overline{u} +$$

$$+ \int_0^t K_{ij}(t-\tau)\left[\mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) + \delta_{ij}\lambda \mathrm{div}\overline{u}\right](\overline{x}, t-\tau)d\tau, \qquad i, j = 1{,}2, \quad (3)$$

bunda $\bar{u}(\bar{x}) = (\bar{u}_1, \bar{u}_2)$ – siljish vektori, $\mu = \mu(x_2) > 0$, $\lambda = \lambda(x_2) > 0$ – Lame koeffitsiyentlari, δ_{ij} –Kroneker simvoli, $K_{ij}(t)$ –muhitning yopishqoqligi uchun javob beradigan funksiyalar va ular uchun $K_{ij} = K_{ji}$, $i, j = 1{,}2$ munosabatlar o‘rinli bo‘lsin.

Tashqi kuchlar bo‘lmaganda qovushqoq-elastik muhitdagi zarralarning harakat tenglamasi quyidagi ko‘rinishda bo‘ladi:

$$\rho \frac{\partial^2 \overline{u}_i}{\partial t^2} = \sum_{j=1}^{2} \frac{\partial \sigma_{ij}}{\partial x_j}, \qquad i = 1{,}2, \qquad (4)$$

bunda $\rho = \rho(x_2) > 0$ –muhitning zichligi.

(3) tenglamalar sistemasini $\mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) + \delta_{ij}\lambda \mathrm{div}\overline{u}$ funksiyaga nisbatan rezolventa usuli yordamidan yechib, quyidagi Volterra ikkinchi tur integral tenglamasiga ega bo‘lamiz:

$$\sigma_{ij}(\overline{x}, t) = \mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) + \delta_{ij}\lambda \mathrm{div}\overline{u} + \int_0^t r_{ij}(t-\tau)\sigma_{ij}(\overline{x}, \tau)d\tau, \quad i, j = 1{,}2. \quad (5)$$

[4]Л.А. Галин, Контактные задачи теории упругости и вязкоупругости, М. : Наука, 1980. 242 с.

(5) tenglamalar sistemasidagi $r_{ij}(t)$ funksiyalar $K_{ij}(t)$ yadrolarning rezolventasi bo‘lib, ular

$$r_{ij}(t) = -K_{ij}(t) - \int_0^t K_{ij}(t-\tau) r_{ij}(\tau) d\tau, \quad i,j = 1,2 \tag{6}$$

integral tenglamalar yordamida aniqlanadi. $K_{ij} = K_{ji}$ munosabatdan $r_{ij} = r_{ji}$ bo‘lishi kelib chiqadi.

$u_i = \frac{\partial}{\partial t}\overline{u}_i, (i = 1,2)$ belgilash yordamida (5) tenglamalar sistemasini t o‘zgaruvchi bo‘yicha differensiallab,

$$\frac{\partial}{\partial t}\sigma_{ij}(\overline{x}, t) = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \delta_{ij}\lambda \mathrm{div} u +$$

$$+ r_{ij}(0)\sigma_{ij}(\overline{x}, t) + \int_0^t r'_{ij}(t-\tau)\sigma_{ij}(\overline{x}, \tau) d\tau, \quad i,j = 1,2, \tag{7}$$

σ_{ij} ga nisbatan integral tenglamalarni olamiz, bunda $r_{ij}(0)$ larni berilgan deb faraz qilamiz.

Qulaylik uchun $x_1 := x,\ x_2 := y$ ko‘rinishida belgilash olaylik. (4) va (7) tenglamalardan foydalanib, u_i va σ_{ij} funksiyalarga nisbatan quyidagi birinchi tartibli giperbolik integro-differensial tenglamalar sistemasiga ega bo‘lamiz:

$$A\frac{\partial \boldsymbol{U}}{\partial t} - B\frac{\partial \boldsymbol{U}}{\partial x} - C\frac{\partial \boldsymbol{U}}{\partial y} - D\boldsymbol{U} = \int_0^t R(t-\tau)\boldsymbol{U}(x,y,\tau) d\tau. \tag{8}$$

Bunda $\boldsymbol{U} = (u_1, u_2, \sigma_{11}, \sigma_{22}, \sigma_{12})^*$, $*$ −transponirlash belgisi,

$$A = \mathrm{diag}(\rho, \rho, 1,1,1),$$

$$B = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \lambda+2\mu & 0 & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & \lambda & 0 & 0 & 0 \\ 0 & \lambda+2\mu & 0 & 0 & 0 \\ \mu & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$D = \mathrm{diag}(0,0, r_{11}(0), r_{22}(0), r_{12}(0)), \quad R(t) = \mathrm{diag}(0,0, r'_{11}, r'_{22}, r'_{12})$$

tengliklar bilan aniqlanadi. (8) sistemani y va t o‘zgaruvchiga nisbatan normal tenglamalar sistemasiga keltirish mumkin. Buning uchun sistemaning ikkala tomoniga chapdan A^{-1} matritsani ko‘paytiramiz va

$$|A^{-1}C - \nu I| = 0, \tag{9}$$

xarakteristik tenglamani tuzib olamiz. (9) tenglamanining yechimi quyidagi ko‘rinishda bo‘ladi:

$$\nu_1 = -\nu_2 = \sqrt{\frac{\mu}{\rho}}, \qquad \nu_3 = -\nu_4 = \sqrt{\frac{\lambda+2\mu}{\rho}}, \qquad \nu_5 = 0. \tag{10}$$

Endi

$$\Upsilon^{-1}A^{-1}C\Upsilon = \Lambda \tag{11}$$

tenglik bilan aniqlanadigan $\Upsilon(y)$ matritsani topamiz[5]. (11) tenglikda keltirilgan Λ dioganal matritsa va uning diagonalida $A^{-1}C$ matritsaning xos sonlari joylashadi. (11) formuladan foydalanib,

$$A^{-1}C\Upsilon = \Upsilon\Lambda$$

tenglikka ega bo‘lamiz. Yuqoridagi tenglikdan ko‘rishimiz mumkinki, Υ matritsaning i-ustuni $A^{-1}C\Upsilon$ matritsaning ν_i xos qiymatiga mos keluvchi xos vektor bo‘ladi. Matematik hisoblashlar natijasida Υ (yagona emas) matritsani ushbu

$$\Upsilon(y) = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\dfrac{1}{\sqrt{\rho(\lambda+2\mu)}} & \dfrac{1}{\sqrt{\rho(\lambda+2\mu)}} & 0 \\ 0 & 0 & \dfrac{\lambda}{\lambda+2\mu} & \dfrac{\lambda}{\lambda+2\mu} & 1 \\ 0 & 0 & 1 & 1 & 0 \\ -\sqrt{\mu\rho} & \sqrt{\mu\rho} & 0 & 0 & 0 \end{pmatrix}$$

ko‘rinishda aniqlaymiz. $\boldsymbol{U} = \Upsilon\boldsymbol{\vartheta}$ belgilash yordamida yangi funksiya kiritaylik. (8) tenglamani yangi funksiyaga nisbatan yozib, hosil bo‘lgan tenglamani $\Upsilon^{-1}A^{-1}$ga chapdan ko‘paytiramiz. Natijada $\boldsymbol{\vartheta} = (\vartheta_1, \vartheta_2, \dots, \vartheta_5)^*$ vektor funksiyaga nisbatan

$$\frac{\partial\boldsymbol{\vartheta}}{\partial t} + \Lambda\frac{\partial\boldsymbol{\vartheta}}{\partial y} + B_1\frac{\partial\boldsymbol{\vartheta}}{\partial x} + C_1\boldsymbol{\vartheta} = \int_0^t R_1(y, t-\tau)\boldsymbol{\vartheta}(x, y, \tau)d\tau \qquad (12)$$

tenglamalar sistemasini olamiz, bu yerda

$$B_1 = \Upsilon^{-1}A^{-1}B\Upsilon = \left(b_{ij}\right)_{5\times5},\quad C_1(y,t) = \Upsilon^{-1}A^{-1}C\frac{\partial\Upsilon}{\partial y} + \Upsilon^{-1}D\Upsilon = \left(c_{ij}\right)_{5\times5},$$
$$R_1(y,t) = \Upsilon^{-1}A^{-1}R\Upsilon = \left(\tilde{r}_{ij}\right)_{5\times5}.$$

(12) sistemani quyidagi $\Gamma = \Gamma_0 \cup \Gamma_1 \cup \Gamma_2$, $\Gamma_0 = \{(x,y,t): x \in \mathbb{R}, 0 \le y \le H, t = 0\}$,

$\Gamma_1 = \{(x,y,t): x \in \mathbb{R}, y = 0, t > 0\}$, $\Gamma_2 = \{(x,y,t): x \in \mathbb{R}, y = H, t > 0\}$ chegarali $\Omega = \{(x,y,t): x \in \mathbb{R}, 0 < y < H, t > 0\}$, $H = const$

sohada qaraylik.

$\boldsymbol{\vartheta}(x,y,t)$ vektor funksiya uchun sohaning chegaralarida quyidagi boshlang‘ich va chegaraviy shartlarni beramiz:

$$\vartheta_i|_{\Gamma_0} = \varphi_i(x,y), \qquad i = \overline{1,5}, \qquad (13)$$

$$\vartheta_i|_{\Gamma_1} = \psi_i(x,t), \qquad i = 1,3, \quad \vartheta_i|_{\Gamma_2} = \psi_i(x,t), \qquad i = 2,4. \qquad (14)$$

(12)-(14) masala korrekt qo‘yilgan [6]. Dissertatsiya ishining birinchi bobida $\varphi_i(\cdot,y), \psi_j(\cdot,t) \in C_0(\mathbb{R})$, $i = \overline{1,5}, j = \overline{1,4}$ larni talab qilamiz. (12)-(14) masalada berilganlar x o‘zgaruvchi bo‘yicha finit bo‘lganligi sababli, bu masalaning yechimi

[5] Ф.Р. Гантмахер, Теория матриц, Наука, Гл. ред. физ.-мат. лит., М, 1988.

[6] В. Г. Романов, Обратные задачи математической физики. М.: Наука. 1984. с. 264.

ham finit bo‘ladi. Demak, biz x o‘zgaruvchi bo‘yicha Furye almashtirishini qo‘llay olamiz. Natijada (12)-(14) masala yechimining Furye almashtirishi yordamida teskari masalani tadqiq qilamiz.

Teskari masalada (12)-(14) masala yechimi haqida

$$\tilde{\vartheta}_1|_{\tilde{\Gamma}_2,\xi=0} = \tilde{h}_1(t), \quad \tilde{\vartheta}_3|_{\tilde{\Gamma}_2,\xi=0} = \tilde{h}_3(t), \quad \tilde{\vartheta}_5|_{\tilde{\Gamma}_1,\xi=0} = \tilde{h}_5(t), \tag{15}$$

qo‘shimcha shartlardan foydalanib, R_1 matritsaga kiruvchi $r_{ij}(t)$ funksiyalarni aniqlash talab etiladi. Bu yerda $\widetilde{\boldsymbol{\vartheta}}(\xi,y,t)$—$\boldsymbol{\vartheta}$ funksiya Furye almashtirishining ξ parametri bo‘yicha tasviri $\left(\widetilde{\boldsymbol{\vartheta}}(\xi,y,t) = \int_{\mathbb{R}} e^{i\xi x}\boldsymbol{\vartheta}(x,y,t)dx\right)$, $\tilde{h}_i(t) -$ berilgan yetarlicha silliq funksiya va $\tilde{\Gamma}$—Γ chegaraning (y,t) tekislikdagi proyeksiyasi. $\boldsymbol{V}(y,t) := \widetilde{\boldsymbol{\vartheta}}(\xi,y,t)|_{\xi=0}$ ko‘rinishida belgilash olib, $\boldsymbol{V}(y,t) = (\mathrm{V}_1,\dots,\mathrm{V}_5)^*$ vektor funksiyaga nisbatan

$$\frac{\partial \boldsymbol{V}}{\partial t} + \Lambda\frac{\partial \boldsymbol{V}}{\partial y} + C_1\boldsymbol{V} = \int_0^t R_1(y,\tau)\boldsymbol{V}(y,t-\tau)d\tau, \tag{16}$$

$$V_i|_{\tilde{\Gamma}_0} = \tilde{\varphi}_i(y), \qquad i = \overline{1,5}, \tag{17}$$

$$V_i|_{\tilde{\Gamma}_1} = \tilde{\psi}_i(t), \qquad i = 1,3, \quad V_i|_{\tilde{\Gamma}_2} = \tilde{\psi}_i(t), \qquad i = 2,4, \tag{18}$$

masalaga ega bo‘lamiz. Bu tengliklarda $\tilde{\varphi}_i(y)$, $i = \overline{1,5}$, $\tilde{\psi}_i(t)$, $i = \overline{1,4}$ lar mos ravishda $\varphi_i(x,y)$ va $\psi_i(x,t)$ funksiyalar Furye tasvirlarining $\xi = 0$ dagi qiymati.

Faraz qilaylik, $\tilde{\varphi}_i(y)$ va $\tilde{\psi}_i(t)$ funksiyalar Ω sohaning burchak nuqtalarida

$$\tilde{\varphi}_i(0) = \tilde{\psi}_i(0), \quad i = 1,3, \qquad \tilde{\varphi}_i(0) = \tilde{\psi}_i(H), \quad i = 2,4 \tag{19}$$

shartlarni qanoatlantirsin. (16)-(18) masalaning yechimi haqida quyidagi lemma o‘rinli:

1-lemma. *Faraz qilaylik,* $\{\rho(y),\mu(y),\lambda(y)\} \in C^1[0,H]$, $\tilde{\varphi}_i(y) \in C[0,H]$, $i = \overline{1,5}$, $\tilde{\psi}_i(t) \in C[0,T]$, $i = \overline{1,4}$, $r_{ij}(t) \in C^1[0,T]$, $i,j = 1,2$ *bo‘lib, (19) kelishuvchanlik sharti o‘rinli bo‘lsin. U holda (16)-(18) masalaning* $C^1(\Omega_{HT}) \cap C(\overline{\Omega}_{HT})$ *funksional sinfga tegishli yagona yechimi mavjud, bu yerda* $\Omega_{HT} = \{(y,t): 0 < y < H,\ 0 < t < T\}$.

$\boldsymbol{\omega}(y,t) = \frac{\partial \boldsymbol{V}}{\partial t}(y,t)$ tenglik yordamida yangi vektor funksiyasini kiritib, (16) tenglama va (18) chegaraviy shartlarini t o‘zgaruvchiga nisbatan differensiallaymiz. (16) tenglamalar sistemasi va (17) boshlang‘ich shartlar yordamida $\boldsymbol{\omega}(y,t)$ funksiyaning $t = 0$ dagi qiymatini olamiz. Natijada $\boldsymbol{\omega}(y,t)$ vektor funksiyaga nisbatan ushbu masalaga ega bo‘lamiz:

$$\frac{\partial \omega_i}{\partial t} + \nu_i\frac{\partial \omega_i}{\partial y} + \sum_{j=1}^{4} c_{ij}(y)\omega_j(y,t) = \int_0^t \sum_{j=1}^{4} \tilde{r}_{ij}(y,\tau)\omega_j(y,t-\tau)d\tau + \sum_{j=1}^{4} \tilde{r}_{ij}(y,t)\tilde{\varphi}_j(y), \quad i = \overline{1,5}, \tag{20}$$

$$\omega_i(y,t)|_{t=0} = -\nu_i\frac{d\tilde{\varphi}_i(y)}{dy} - \sum_{j=1}^{5} c_{ij}(y)\tilde{\varphi}_i(y) := \Phi_i(y), \quad i = \overline{1,5}, \tag{21}$$

$$\omega_i(y,t)|_{y=0} = \frac{d\tilde{\psi}_i(t)}{dt}, i = 1,3, \quad \omega_i|_{y=H} = \frac{d\tilde{\psi}_i(t)}{dt}, i = 2,4. \qquad (22)$$

(15) shartdan foydalanib, ω_i funksiyalar uchun

$$\omega_1|_{y=H} = \frac{d\tilde{h}_1(t)}{dt}, \quad \omega_3|_{y=H} = \frac{d\tilde{h}_3(t)}{dt}, \qquad \omega_5|_{y=0} = \frac{d\tilde{h}_5(t)}{dt} \qquad (23)$$

qo‘shimcha shart olinadi. Quyidagi kelishuvchanlik shartlari bajarilishini talab qilamiz:

$$-\nu_i \frac{d\tilde{\varphi}_i(y)}{dy}\bigg|_{y=0} - \sum_{j=1}^{5} c_{ij}(0)\tilde{\varphi}_i(0) = \frac{d\tilde{\psi}_i(t)}{dt}\bigg|_{t=0}, \quad i = 1,3, \qquad (24)$$

$$-\nu_i \frac{d\tilde{\varphi}_i(y)}{dy}\bigg|_{y=H} - \sum_{j=1}^{5} c_{ij}(H)\tilde{\varphi}_i(H) = \frac{d\tilde{\psi}_i(t)}{dt}\bigg|_{t=0}, \quad i = 2,4. \qquad (25)$$

1-teorema. *Faraz qilaylik, 1-lemmadagi shartlar bajarilsin, bundan tashqari berilganlar* $\tilde{\varphi}_i(y) \in C^1[0,H], \quad i = \overline{1,5}, \quad \tilde{\psi}_i(t) \in C^1[0,T], \quad i = \overline{1,4}$ *munosabatlarni qanoatlantirsin va (24), (25) kelishuvchanlik shartlari bajarilsin. U holda (20)-(22) masalaning* $C^1(\Omega_{HT}) \cap C(\bar{\Omega}_{HT})$ *funksional sinfga qarashli yagona yechimi mavjud.*

Dissertatsiyaning birinchi bobi uchinchi paragrafida xotirali birinchi tartibli giperbolik tenglamalar sistemasidan o‘rama ko‘rinishidagi yadroni aniqlash teskari masalasi tadqiq qilingan.

Faraz qilaylik, ushbu munosabatlar o‘rinli bo‘lsin:

$$\tilde{\varphi}_1(0) \neq \tilde{\varphi}_2(0), \quad \tilde{\varphi}_3(0) \neq -\tilde{\varphi}_4(0),$$
$$\lambda[\tilde{\varphi}_3(0) + \tilde{\varphi}_4(0) + \tilde{\varphi}_5(0)] + 2\mu(0)\tilde{\varphi}_5(0) \neq 0. \qquad (26)$$

Bundan tashqari quyidagi kelishuvchanlik shartlarini bajarilishini talab qilaylik:

$$\frac{d}{dt}\tilde{h}_i(t)\bigg|_{t=0} = -\nu_i \frac{d}{dy}\tilde{\varphi}_i(y)\bigg|_{y=0} - \sum_{j=1}^{5} c_{ij}(0)\tilde{\varphi}_i(0), \quad i = 1,3, \qquad (27)$$

$$\frac{d}{dt}\tilde{h}_5(t)\bigg|_{t=0} = -\sum_{j=1}^{5} c_{5j}(0)\tilde{\varphi}_j(0). \qquad (28)$$

Quyidagi teorema o‘rinli:

2-teorema. *Faraz qilaylik, 1-teoremadagi shartlar o‘rinli bo‘lsin. Bundan tashqari* $\tilde{\varphi}_i(y) \in C^2[0,H]$, $i = \overline{1,5}$, $\tilde{\psi}_i(t) \in C^2[0,\mu_0]$, $\tilde{h}_i(t) \in C^2[0,\mu_0]$, $i = \overline{1,3}$ *bo‘lib, (26) munosabatlar va (27), (28) kelishuvchanlik shartlari bajarilsin. U holda ixtiyoriy tayinlangan* $H > 0$ *soni uchun (20)-(23) masalaning* $[0,\mu_0]$ *kesmada aniqlangan* $C^1[0,\mu_0]$ *sinfga tegishli yagona yechimi mavjud. Bu yerda* $\mu_i(y) = \int_0^y \frac{1}{\nu_i(s)} ds$, $\mu_0 = \max\{\mu_1(H), \mu_3(H)\}$.

$r'_{11}(t), r'_{22}(t), r'_{12}(t)$ funksiyalar orqali $r_{11}(t), r_{22}(t), r_{12}(t)$ lar

$$r_{ij}(t) = r_{ij}(0) + \int_0^t r'_{ij}(\tau)d\tau, \quad i,j = 1,2,$$

formula orqali aniqlanadi. (6) integral tenglamani yechib, $K_{11}(t)$, $K_{22}(t)$, $K_{12}(t)$ funksiyalar topiladi.

Dissertatsiyaning ikkinchi bobi "**Vertikal qatlamli muhitdagi uch o'lchamli qovushqoq-elastiklik sistemalari uchun teskari masala**" deb nomlangan bo'lib, unda integral hadning bir o'lchamli yadrosini aniqlash masalasi o'rganilgan. Bobning birinchi paragrafida boshlang'ich-chegaraviy va teskari masalalarning qo'yilishi hamda boshlang'ich-chegaraviy masala yechimining mavjudlik va yagonalik teoremalari keltirilgan. Ikkinchi paragrafda teskari masalaga ekvivalent integral tenglamalarning olinishi keltirilgan. Uchinchi paragrafda teskari masala yechimining mavjudlik va yagonalik teoremasi hamda uning isboti keltirilgan.

Tashqi kuchlarning ta'siri bo'lmaganda fazoning tebranishlarini quyidagi tenglamalar sistemasi orqali tasvirlash mumkin:

$$\begin{cases} \rho \dfrac{\partial u_i}{\partial t} = \sum\limits_{j=1}^{3} \dfrac{\partial \sigma_{ij}}{\partial x_j}, \quad i = 1,2,3, \\ \dfrac{\partial \sigma}{\partial t} = \mathrm{K}\varepsilon + Q\sigma + \int\limits_0^t P(t-\tau)\,\sigma(\tau)\,d\tau, \end{cases} \quad (x_1, x_2) \in \mathbb{R}^2, x_3 \in [0, H], t > 0. \quad (29)$$

Bunda $u = (u_1, u_2, u_3)^* -$ tezlik vektori, $\sigma\left(\sigma_{ij} = \sigma_{ji}, i, j = 1,2,3\right) -$ kuchlanish tenzori va $= (\sigma_{11},\ \sigma_{12},\ \sigma_{13}, \sigma_{22},\ \sigma_{23}, \sigma_{33})^*$, $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6)^*$:

$$\varepsilon = \begin{cases} \varepsilon_1 = \dfrac{\partial u_1}{\partial x_1}, \quad \varepsilon_4 = \dfrac{\partial u_2}{\partial x_2}, \quad \varepsilon_6 = \dfrac{\partial u_3}{\partial x_3}, \\ \varepsilon_2 = \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_2} + \dfrac{\partial u_2}{\partial x_1}\right), \varepsilon_3 = \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_3} + \dfrac{\partial u_3}{\partial x_1}\right), \varepsilon_5 = \dfrac{1}{2}\left(\dfrac{\partial u_2}{\partial x_3} + \dfrac{\partial u_3}{\partial x_2}\right), \end{cases}$$

$$K = \begin{pmatrix} K_{11} & \mathbf{O}_{3\times3} \\ \mathbf{O}_{3\times3} & K_{22} \end{pmatrix}, \quad K_{11} = (k_{ij}) = \begin{cases} \lambda + 2\mu, & i = j, \\ \lambda, & i \neq j, \end{cases} \quad K_{22} = \mathrm{diag}(\mu, \mu, \mu),$$

$$Q = \mathrm{diag}\,(r_{11}(0),0,0, r_{22}(0),0, r_{33}(0)), P = \mathrm{diag}\,(r'_{11}, 0,0, r'_{22}, 0, r'_{33}),$$

$\lambda(x_3) > 0, \mu(x_3) > 0 -$ Lame koeffitsiyentlari, $\rho(x_3) > 0 -$ muhitning zichligi, $r_{11}(t), r_{22}(t), r_{33}(t) -$muhitning yopishqoqligi uchun javob beradigan funksiyalar.

(29) giperbolik integro-differensial tenglamalar sistemasini kanonik ko'rinishga keltiramiz:

$$\frac{\partial \boldsymbol{\vartheta}}{\partial t} + \Lambda \frac{\partial \boldsymbol{\vartheta}}{\partial x_3} + B_1 \frac{\partial \boldsymbol{\vartheta}}{\partial x_1} + C_1 \frac{\partial \boldsymbol{\vartheta}}{\partial x_2} + F_1 \boldsymbol{\vartheta} = \int\limits_0^t R_1(t-\tau, x_3)\,\boldsymbol{\vartheta}(x_1, x_2, x_3, \tau) d\tau. \quad (30)$$

Bunda $\boldsymbol{\vartheta} = (\vartheta_1, \vartheta_2, \ldots, \vartheta_9)$,

$$\Lambda = \mathrm{diag}(\nu_i) = \mathrm{diag}\left(-\sqrt{\frac{\lambda+2\mu}{\rho}}, -\sqrt{\frac{\mu}{\rho}}, -\sqrt{\frac{\mu}{\rho}}, 0,0,0, \sqrt{\frac{\mu}{\rho}}, \sqrt{\frac{\mu}{\rho}}, \sqrt{\frac{\lambda+2\mu}{\rho}}\right),$$

$$B_1(x_3) = \Upsilon^{-1}A^{-1}B\Upsilon = \left(b_{\mathrm{ij}}\right)_{9\times 9},$$
$$C_1(x_3) = \Upsilon^{-1}A^{-1}C\Upsilon = \left(c_{\mathrm{ij}}\right)_{9\times 9},$$
$$F_1(x_3) = \Upsilon^{-1}A^{-1}D\frac{\partial \Upsilon}{\partial x_3} + \Upsilon^{-1}A^{-1}F\Upsilon = \left(p_{\mathrm{ij}}\right)_{9\times 9},$$
$$R_1(x_3,t) = \Upsilon^{-1}A^{-1}R\Upsilon = \left(\tilde{r}_{ij}\right)_{9\times 9},$$

$$\Upsilon(x_3) = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{\lambda}{\nu_9} & 0 & 0 & 1 & 0 & 1 & 0 & 0 & -\frac{\lambda}{\nu_9} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho\nu_8 & 0 & 0 & 0 & 0 & -\rho\nu_8 & 0 \\ \frac{\lambda}{\nu_9} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\frac{\lambda}{\nu_9} \\ 0 & \rho\nu_8 & 0 & 0 & 0 & 0 & -\rho\nu_8 & 0 & 0 \\ \rho\nu_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho\nu_9 \end{pmatrix}.$$

$\boldsymbol{\vartheta}(x_1, x_2, x_3, t)$ vektor funksiya (30) tenglamalar sistemasini $\Omega = \{(x_1, x_2, x_3, t)\colon (x_1, x_2) \in \mathbb{R}^2, \quad x_3 \in (0, H), \quad t > 0\}, \quad H = \text{const}$ sohada sohada qanoatlantirsin. Bundan tashqari sohaning chegarasida noma'lum funksiya quyidagi boshlang'ich

$$\vartheta_i|_{t=0} = \varphi_i(x_1, x_2, x_3), \qquad i = \overline{1,9} \tag{31}$$

va chegaraviy

$$\vartheta_i\rfloor_{x_3=H} = g_i(x_1, x_2, t), i = 1,2,3, \ \ \vartheta_i|_{x_3=0} = g_i(x_1, x_2, t), \ \ i = 7,8,9 \tag{32}$$

qiymatlarni qabul qilsin. Bu yerda $\varphi_i(x_1, x_2, x_3), g_i(x_1, x_2, t)$ – berilgan yetarlicha silliq funksiyalar.

Teskari masalada (30)-(32) masala yechimi haqida

$$\vartheta_i\rfloor_{x_3=0} = h_i(x_1, x_2, t), i = 1,4,6, \tag{33}$$

shartlardan foydalanib, R_1 matritsaga kiruvchi $r_{11}(t), r_{22}(t), r_{33}(t)$ elementlarni topish talab etiladi. Bu yerda $h_i(x_1, x_2, t), i = 1,4,6$ lar oldindan ma'lum, yetarlicha silliq funksiyalar.

Faraz qilaylik, (31), (32) shartlarning o'ng tomonida berilgan $\varphi_i(x_1, x_2, x_3)$, $g_i(x_1, x_2, t)$ va $h_i(x_1, x_2, t)$ vektor funksiyalar har bir fiksirlangan x_3, t larda x_1, x_2 o'zgaruvchilar bo'yicha finit bo'lsin. U holda (30) tenglamalar sistemasi giperbolik tipga tegishli ekanligidan (30)-(32) masalaning biz qidirayotgan yechimining ham finitligi kelib chiqadi.

Qulaylik uchun $x_3 = z$ belgilash olaylik. $\widetilde{\boldsymbol{\vartheta}}(\eta_1, \eta_2, z, t)$ orqali $\boldsymbol{\vartheta}(x_1, x_2, z, t)$ funksiyaning x_1 va x_2 o'zgaruvchilar bo'yicha Furye almashtirishini belgilaymiz:

$$\widetilde{\boldsymbol{\vartheta}}(\eta_1, \eta_2, z, t) = \int_{\mathbb{R}^2} \boldsymbol{\vartheta}(x_1, x_2, z, t) e^{i[\eta_1 x_1 + \eta_2 x_2]} dx_1 dx_2, \tag{34}$$

bu yerda η_1 va η_2 –Furye almashtirishining parametrlari. $\eta_1 = 0, \eta_2 = 0$ da $\widetilde{\boldsymbol{\vartheta}}(\eta_1, \eta_2, z, t) = \widehat{\boldsymbol{\vartheta}}(z, t)$ belgilash olamiz. Natijada $\widehat{\boldsymbol{\vartheta}}$ funksiyaga nisbatan (30)-(33) masala quyidagi ko‘rinishni oladi:

$$\left(\frac{\partial}{\partial t} + \nu_j \frac{\partial}{\partial z}\right)\hat{\vartheta}_j(z,t) =$$

$$= \sum_{k=1}^{9} p_{jk}(z)\hat{\vartheta}_k(z,t) + \int_0^t \sum_{k=1}^{9} \hat{r}_{jk}(z,\tau)\hat{\vartheta}_k(z,t-\tau)\,d\tau,\ j = \overline{1,9}, \quad (35)$$

$$\hat{\vartheta}_i\big|_{t=0} = \hat{\varphi}_i(z), \qquad i = \overline{1,9}, \quad (36)$$

$$\hat{\vartheta}_i\big|_{z=H} = \hat{g}_i(t),\ \ i = 12{,}3, \qquad \hat{\vartheta}_i\big|_{z=0} = \hat{g}_i(t),\ \ i = 7{,}8{,}9, \quad (37)$$

$$\hat{\vartheta}_i\big|_{z=0} = \hat{h}_i(t), \quad i = 1{,}4{,}6. \quad (38)$$

$\boldsymbol{\omega}(z,t) = \frac{\partial \widehat{\boldsymbol{\vartheta}}}{\partial t}(z,t)$ belgilash yordamida yangi funksiya kiritib, (35)-(38) masaladan $\boldsymbol{\omega}(z,t) = (\omega_1, \dots, \omega_9)^*$ funksiyaga nisbatan quyidagi masalaga ega bo‘lamiz:

$$\left(\frac{\partial}{\partial t} + \nu_j \frac{\partial}{\partial z}\right)\omega_j(z,t) = \sum_{k=1}^{9} p_{jk}(z)\omega_k(z,t) + \sum_{k=1}^{9} \tilde{r}_{jk}(z,\tau)\hat{\varphi}_k(z) +$$

$$+ \int_0^t \sum_{k=1}^{9} \tilde{r}_{jk}(z,\tau)\omega_k(z,t-\tau)\,d\tau, \quad j = \overline{1,9}, \quad (39)$$

$$\omega_i|_{t=0} = -\nu_j \frac{d\hat{\varphi}_i(z)}{dz} + \sum_{j=1}^{9} p_{ij}(z)\hat{\varphi}_j(z) := \Phi_i(z), \quad i = \overline{1,9}, \quad (40)$$

$$\omega_i|_{z=H} = \frac{d}{dt}\hat{g}_i(t),\ \ i = 1{,}2{,}3, \qquad \omega_i|_{z=0} = \frac{d}{dt}\hat{g}_i(t),\ \ i = 7{,}8{,}9, \quad (41)$$

$$\omega_i|_{z=0} = \frac{d}{dt}\hat{h}_i(t), \quad i = 1{,}4{,}6. \quad (42)$$

Qaralayotgan masala yechimi uzluksizligi uchun quyidagi tengliklarni talab qilaylik:

$$\hat{\varphi}_i(H) = \hat{g}_i(0), i = 1{,}2{,}3, \qquad \hat{\varphi}_i(0) = \hat{g}_i(0),\ \ i = 7{,}8{,}9,$$

$$\left.\frac{d\hat{g}_i(t)}{dt}\right|_{t=0} = -\nu_j \left.\frac{d\hat{\varphi}_i(z)}{dz}\right|_{z=H} + \sum_{j=1}^{9} p_{ij}(H)\hat{\varphi}_j(H), \qquad i = 1{,}2{,}3, \quad (43)$$

$$\left.\frac{d\hat{g}_i(t)}{dt}\right|_{t=0} = -\nu_j \left.\frac{d\hat{\varphi}_i(z)}{dz}\right|_{z=0} + \sum_{j=1}^{9} p_{ij}(0)\hat{\varphi}_j(0), \qquad i = 7{,}8{,}9.$$

Quyidagi teorema o‘rinli:

3-teorema. *Faraz qilaylik,* $\{\rho(z),\ \mu(z),\ \lambda(z),\ \hat{\varphi}_i(z)\} \in C^1[0,H]$, $\hat{g}_i(t) \in C^1[0,T], i = 1{,}2{,}3{,}7{,}8{,}9, r_{ii}(t) \in C^1[0,T],\ i = 1{,}2{,}3$ *bo‘lib, (43) shartlar bajarilsin. U holda (39)-(41) masalaning* $\Omega_{HT} = \{(z,t): 0 < z < H, 0 < t < T\}$ *sohada aniqlangan yagona yechimi mavjud.*

Quyidagi munosabatlar bajarilsin:

$$\lambda[\hat{\varphi}_1(0)-\hat{\varphi}_9(0)]+\nu_p\hat{\varphi}_6(0)+\nu_p\hat{\varphi}_4(0)\neq 0,\quad \hat{\varphi}_5(0)\neq 0,$$
$$\hat{\varphi}_3(0)-\hat{\varphi}_8(0)\neq 0,\quad \lambda[\hat{\varphi}_1(0)-\hat{\varphi}_9(0)]+\nu_p\hat{\varphi}_6(0)\neq 0, \qquad (44)$$
$$\hat{\varphi}_2(0)-\hat{\varphi}_7(0)\neq 0,\quad \hat{\varphi}_1(0)-\hat{\varphi}_9(0)\neq 0.$$

Ushbu kelishuvchanlik shartlari o‘rinli bo‘lsin:

$$-\nu_i\frac{d\hat{\varphi}_i(z)}{dz}\Big|_{z=0}+\sum_{j=1}^{9}p_{ij}(0)\omega_j(0)=\frac{d}{dt}\hat{h}_i(t)\Big|_{t=0},\quad i=1,4,6. \qquad (45)$$

Quyidagiga ega bo‘lamiz:

4-teorema. *Faraz qilaylik, berilgan funksiyalar 3-teorema shartlarini qanoatlantirsin. Bundan tashqari* $\hat{\varphi}_i(z)\in C^2[0,H]$, $i=\overline{1,9}$, $\hat{g}_i(t)\in C^2[0,\mu_0]$, $i=1,2,3,7,8,9$, $\hat{h}_i(t)\in C^2[0,\mu_0]$, $i=1,4,6$ *bo‘lib, (44) munosabat va (45) kelishuvchanlik shartlari bajarilsin. U holda ixtiyoriy tayinlangan* $H>0$ *son uchun (39)-(42) masalaning* $[0,\mu_0]$ *segmentda aniqlangan* $C^1[0,\mu_0]$ *sinfga tegishli yagona yechimi mavjud. Bunda* $\mu_0=\int_0^H\frac{1}{\nu_1(s)}ds$.

Dissertatsiyaning “**Anizotropik muhitdagi qovushqoq-elastiklik sistemasi uchun teskari masalalar**” deb nomlanuvchi uchinchi bobida qovushqoq-elastiklik tenglamalar sistemasi uchun boshlang‘ich chegaraviy masala va bir o‘lchamli xotira yadrosini aniqlash teskari masalalari qaralgan.

Anizotrop muhitda kuchlanish tenzori σ va tezlik u quyidagi integro-differensial tenglamalar sistemasi orqali bog‘langan:

$$\begin{cases}\rho\dfrac{\partial u_i}{\partial t}=\sum\limits_{j=1}^{3}\dfrac{\partial\sigma_{ij}}{\partial x_j}, \qquad i=1,2,3,\\ \dfrac{\partial}{\partial t}\sigma_{ij}=c_{ijkl}\left(\dfrac{\partial u_k}{\partial x_l}+\dfrac{\partial u_l}{\partial x_k}\right)+r_{ij}(0)\sigma_{ij}(x,t)+\int\limits_0^t r'_{ij}(t-\tau)\sigma_{ij}(x,\tau)d\tau,\end{cases} \qquad (46)$$

bu yerda, $x=(x_1,x_2,x_3)$, $c_{ijkl}=c_{ijkl}(x_3)$ —elastiklik moduli, $\rho=\rho(x_3)>0$ −muhitning zichligi, $r_{ij}(t)$—muhitning yopishqoqligi uchun javob beradigan funksiyalar. Ushbu bob davomida $r_{ij}=r_{ji}$, hamda $r_{11}=r_{12}$ tengliklar bajarilishini talab qilamiz.

Faraz qilaylik, elastiklik modullari uchun $c_{ijkl}=c_{jikl}=c_{ijlk}=c_{klji}$ tengliklar o‘rinli bo‘lsin. Bu munosabatlar mustaqil elastiklik moduli sonini 81 tadan 21 tagacha qisqartiradi. $c_{\alpha\beta}=c_{ijkl},\big(\alpha=(ij),\beta=(kl)\big)$ va $(11)\to 1,(22)\to 2,\ (33)\to 3\ \ (23)=(32)\to 4,\ (13)=(31)\to 5,\ (12)=(21)\to 6$ yozuvlarni o‘rinli deb faraz qilsak, mustaqil elastiklik modul matritsasiga 6×6 simmetrik matritsa shaklini berish mumkin. Ushbu bobda quyidagi tetragonal elastiklik moduli matritsasi bilan anizotrop muhit ko‘rib chiqiladi[7]:

[7] Э. Дьелесан, Д. Руайе, Упругие волны в твердых телах, М.:Наука, 1982.

$$c_{\alpha\beta} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{11} & c_{13} & 0 & 0 & -c_{16} \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ c_{16} & -c_{16} & 0 & 0 & 0 & c_{66} \end{pmatrix}.$$

(46) tenglamalar sistemasini $\boldsymbol{U} = (u, \sigma)^*$ vektor funksiyaga nisbatan quyidagicha yozish mumkin:

$$\left(A\frac{\partial}{\partial t} + B\frac{\partial}{\partial x_1} + C\frac{\partial}{\partial x_2} + D\frac{\partial}{\partial x_3} + F\right)\boldsymbol{U}(x,t) = \int_0^t R(t-\tau)\boldsymbol{U}(x,\tau)d\tau, \quad (47)$$

bu yerda $A = \text{diag}\,(\rho, \rho, \rho, 1,1,1,1,1,1)$,

$$B = \begin{pmatrix} & & & -1 & 0 & 0 & 0 & 0 & 0 \\ & \mathbf{O}_{3\times3} & & 0 & 0 & 0 & -1 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & -1 & 0 \\ -c_{11} & 0 & 0 & & & & & & \\ -c_{12} & 0 & 0 & & & & & & \\ -c_{13} & 0 & 0 & & & & & & \\ 0 & -c_{44} & 0 & & & \mathbf{O}_{6\times6} & & & \\ 0 & 0 & -c_{44} & & & & & & \\ -c_{16} & 0 & 0 & & & & & & \end{pmatrix},$$

$$C = \begin{pmatrix} & & & 0 & 0 & 0 & -1 & 0 & 0 \\ & \mathbf{O}_{3\times3} & & 0 & -1 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & -c_{12} & -c_{16} & & & & & & \\ 0 & -c_{11} & -c_{16} & & & & & & \\ 0 & -c_{13} & 0 & & & & & & \\ -c_{44} & 0 & 0 & & & \mathbf{O}_{6\times6} & & & \\ 0 & 0 & 0 & & & & & & \\ 0 & c_{16} & -c_{66} & & & & & & \end{pmatrix},$$

$$D = \begin{pmatrix} & & & 0 & 0 & 0 & 0 & -1 & 0 \\ & \mathbf{O}_{3\times3} & & 0 & 0 & 0 & 0 & 0 & -1 \\ & & & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -c_{16} & -c_{13} & & & & & & \\ 0 & c_{16} & -c_{13} & & & & & & \\ 0 & 0 & -c_{33} & & & & & & \\ 0 & 0 & 0 & & & \mathbf{O}_{6\times6} & & & \\ -c_{44} & 0 & 0 & & & & & & \\ 0 & -c_{66} & 0 & & & & & & \end{pmatrix},$$

$F = \text{diag}\,\big(0,0,0, r_{11}(0), r_{22}(0), r_{33}(0), r_{12}(0), r_{13}(0), r_{23}(0)\big)$,

$R = R(t) = \text{diag}\,(0,0,0, r'_{11}, r'_{22}, r'_{33}, r'_{11}, r'_{13}, r'_{23})$, $\mathbf{O_{n\times n}}$ – nol vektor.

(47) tenglamalar sistemasini kanonik ko‘rinishga keltirish uchun $A^{-1}D$ matritsaning xos vektorlaridan tuzilgan o‘tish matritsasini quyidagicha olamiz:

$$\Upsilon(x_3)=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -c_{13}\sqrt{\frac{\rho}{c_{33}}} & 0 & -c_{16}\sqrt{\frac{\rho}{c_{66}}} & 1 & 0 & 1 & c_{16}\sqrt{\frac{\rho}{c_{66}}} & 0 & c_{13}\sqrt{\frac{\rho}{c_{33}}} \\ -c_{13}\sqrt{\frac{\rho}{c_{33}}} & 0 & c_{16}\sqrt{\frac{\rho}{c_{66}}} & 0 & 1 & 0 & -c_{16}\sqrt{\frac{\rho}{c_{66}}} & 0 & c_{13}\sqrt{\frac{\rho}{c_{33}}} \\ -\sqrt{c_{33}\rho} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{c_{33}\rho} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\sqrt{c_{44}\rho} & 0 & 0 & 0 & 0 & 0 & \sqrt{c_{44}\rho} & 0 \\ 0 & 0 & -\sqrt{c_{66}\rho} & 0 & 0 & 0 & \sqrt{c_{66}\rho} & 0 & 0 \end{pmatrix}.$$

$\boldsymbol{U}=\Upsilon\boldsymbol{\vartheta}$ tenglik yordamida yangi funksiya kiritamiz, so‘ngra (47) tenglamalar sistemasini chapdan $\Upsilon^{-1}A^{-1}$ matritsaga ko‘paytirib,

$$\left(\frac{\partial}{\partial t}+\Lambda\frac{\partial}{\partial x_3}+B_1\frac{\partial}{\partial x_1}+C_1\frac{\partial}{\partial x_2}+F_1\right)\boldsymbol{\vartheta}(x,t)=\int_0^t R_1(t-\tau,x_3)\boldsymbol{\vartheta}(x,\tau)d\tau \quad (48)$$

tenglamalar sistemasini olamiz, bu yerda $\boldsymbol{\vartheta}=(\vartheta_1,\dots,\vartheta_9)^*$,

$$\Lambda=\mathrm{diag}(\nu_i)=\mathrm{diag}\left(-\sqrt{\frac{\rho}{c_{33}}},-\sqrt{\frac{\rho}{c_{44}}},-\sqrt{\frac{\rho}{c_{66}}},0,0,0,\sqrt{\frac{\rho}{c_{66}}},\sqrt{\frac{\rho}{c_{44}}},\sqrt{\frac{\rho}{c_{33}}}\right),$$

$$B_1(x_3)=\Upsilon^{-1}A^{-1}B\Upsilon=\left(b_{ij}\right)_{9\times9},\qquad C_1(x_3)=\Upsilon^{-1}A^{-1}C\Upsilon=\left(c_{ij}\right)_{9\times9},$$

$$F_1(x_3)=\Upsilon^{-1}A^{-1}D\frac{\partial\Upsilon}{\partial x_3}+\Upsilon^{-1}A^{-1}F\Upsilon=\left(p_{ij}\right)_{9\times9},$$

$$R_1(x_3,t)=\Upsilon^{-1}A^{-1}R\Upsilon=\left(\tilde{r}_{ij}\right)_{9\times9}.$$

(48) tenglamalar sistemasini

$$\Omega=\{(x,t):(x_1,x_2)\in\mathbb{R}^2,\ x_3\in(0,H),t>0\},\qquad H=\mathrm{const}$$

sohada quyidagi boshlang‘ich va chegaraviy shartlar bilan birga qaraylik:

$$\vartheta_i|_{t=0}=\varphi_i(x),\qquad i=\overline{1,9}, \quad (49)$$

$$\vartheta_i|_{x_3=H}=g_i(x_1,x_2,t),\ \ i=\overline{1,3},\ \ \vartheta_i|_{x_3=0}=g_i(x_1,x_2,t),\ \ i=\overline{7,9}. \quad (50)$$

Bunda $\varphi_i(x),\ g_i(x_1,\ x_2,\ t)$ – berilgan yetarlicha silliq funksiyalar.

Teskari masalada (48)-(50) masala yechimi haqida

$$\vartheta_i|_{x_3=0}=h_i(x_1,x_2,t),i=\overline{1,5}, \quad (51)$$

qo‘shimcha shartlardan foydalanib, R_1 matritsaga kiruvchi $r_{ij}(t)$ elementlarni topish bo‘yicha masala tadqiq etilgan. Bunda $h_i(x_1,x_2,t),i=\overline{1,5}$ lar oldindan ma’lum va yetarlicha silliq funksiyalar.

Faraz qilaylik, (49), (50) shartlarning o‘ng tomonida berilgan $\varphi_i(x),\ i=\overline{1,9}$, $g_i(x_1,x_2,t),\ \ i=1,2,3,7,8,9$ va $h_i(x_1,x_2,t),i=\overline{1,5}$ vektor funksiyalar har bir fiksirlangan x_3,t larda x_1,x_2 o‘zgaruvchilar bo‘yicha finit bo‘lsin. (34) integral almashtirishdan foydalanib, (48)-(51) masalani quyidagi ko‘rinishda yozib olamiz:

$$\left(\frac{\partial}{\partial t}+\nu_i\frac{\partial}{\partial z}\right)\hat{\vartheta}_i(z,t)=\sum_{k=1}^{9}p_{ik}(z)\,\hat{\vartheta}_k(z,t)+\int_0^t\sum_{k=1}^{9}\tilde{r}_{ik}(z,\tau)\hat{\vartheta}_i(z,t-\tau)d\tau, \quad (52)$$

$$\hat{\vartheta}_i\big|_{t=0}=\hat{\varphi}_i(z), \qquad i=\overline{1,9}, \quad (53)$$

$$\hat{\vartheta}_i\big|_{z=H}=\hat{g}_i(t), i=\overline{1,3}, \qquad \hat{\vartheta}_i\big|_{z=0}=\hat{g}_i(z), i=\overline{7,9}, \quad (54)$$

$$\hat{\vartheta}_i\big|_{z=0}=\hat{h}_i(t), \ \ i=\overline{1,5}, \quad (55)$$

bunda $\hat{\varphi}_i(z),\hat{g}_i(t),\hat{h}_i(t)$ —mos ravishda $\varphi_i(x),g_i(x_1,x_2,t),h_i(x_1,x_2,t)$ funksiyalar Furye tasvirlarining $\eta_1=\eta_2=0$ dagi qiymati.

Qaralayotgan masala yechimi uzluksiz bo‘lishi uchun

$$\hat{\varphi}_i(0)=\hat{g}_i(H), i=1,2,3, \qquad \hat{\varphi}_i(0)=\hat{g}_i(0), i=7,8,9 \quad (56)$$

kelishuvchanlik shartlarini talab qilamiz.

$\boldsymbol{\omega}(z,t)=\frac{\partial\hat{\boldsymbol{\vartheta}}}{\partial t}(z,t)$ vektor funksiyani kiritish orqali, $\boldsymbol{\omega}(z,t)=(\omega_1,\dots,\omega_9)^*$ funksiyaga nisbatan quyidagi masalaga ega bo‘lamiz:

$$\left(\frac{\partial}{\partial t}+\nu_i\frac{\partial}{\partial z}\right)\omega_i(z,t)=\sum_{k=1}^{9}p_{ik}(z)\omega_k(z,t)+$$

$$+\sum_{k=1}^{9}\tilde{r}_{ik}(z,t)\hat{\varphi}_i(z)+\int_0^t\sum_{k=1}^{9}\tilde{r}_{ik}(z,t)\omega_k(z,t-\tau)d\tau, \quad i=\overline{1,9}, \quad (57)$$

$$\omega_i|_{t=0}=\Phi_i(z)=\frac{d\hat{\varphi}_i(z)}{dz}+\sum_{j=1}^{9}p_{ji}(z)\hat{\varphi}_i(z), \ \ i=\overline{1,9}, \quad (58)$$

$$\omega_i|_{z=H}=\frac{d}{dt}\hat{g}_i(t), \ \ i=1,2,3, \ \ \omega_i|_{z=0}=\frac{d}{dt}\hat{g}_i(t), \ \ i=7,8,9, \quad (59)$$

$$\omega_i|_{z=0}=\frac{d}{dt}\hat{h}_i(t), \ \ i=\overline{1,5}. \quad (60)$$

Faraz qilaylik, (57)-(59) masala yechimi uchun quyidagi shartlar o‘rinli bo‘lsin:

$$\frac{d\hat{g}_i(t)}{dt}\bigg|_{t=0}=-\nu_i\frac{d\hat{\varphi}_i(z)}{dz}\bigg|_{z=H}+\sum_{j=1}^{9}p_{ij}(H)\hat{\varphi}_j(H), \ \ i=\overline{1,3}, \quad (61)$$

$$\frac{d\hat{g}_i(t)}{dt}\bigg|_{t=0}=-\nu_i\frac{d\hat{\varphi}_i(z)}{dz}\bigg|_{z=0}+\sum_{j=1}^{9}\hat{p}_{ij}(0)\hat{\varphi}_j(0), \ \ i=\overline{7,9}. \quad (62)$$

(57)-(59) masala yechimi haqida quyidagi teorema o‘rinli:

5-teorema. *Faraz qilaylik,* $\{\rho(z),c_{33}(z),c_{44}(z),c_{66}(z),\hat{\varphi}(z)\}\in C^1[0,H]$, $\hat{g}(t)\in C^1[0,T],r_{ij}(t)\in C^1[0,T],i,j=\overline{1,3}$ *bo‘lib, (56),(61),(62) kelishuvchanlik shartlari o‘rinli bo‘lsin. U holda (57)-(59) masalaning* $\Omega_{HT}=\{(z,t):0<z<H,0<t<T\}$ *sohada aniqlangan yagona yechimi mavjud.*

Teskari masalani tadqiq qilishda quyidagi munosabatlar

$$\frac{c_{13}}{\nu_1}\left(\hat{\varphi}_1(0)-\hat{\varphi}_9(0)\right)+\frac{c_{16}}{\nu_3}\left(\hat{\varphi}_3(0)-\hat{\varphi}_7(0)\right)+\hat{\varphi}_4(0)+\hat{\varphi}_6(0)\neq 0,$$
$$\frac{c_{13}}{\nu_1}\left(\hat{\varphi}_9(0)-\hat{\varphi}_1(0)\right)+\frac{c_{16}}{\nu_3}\left(\hat{\varphi}_7(0)-\hat{\varphi}_3(0)\right)+\hat{\varphi}_5(0)\neq 0, \qquad (63)$$
$$\hat{\varphi}_2(0)-\hat{\varphi}_8(0)\neq 0, \qquad \hat{\varphi}_3(0)-\hat{\varphi}_7(0)\neq 0, \qquad \hat{\varphi}_1(0)-\hat{\varphi}_9(0)\neq 0$$

va

$$-\nu_i\frac{d\hat{\varphi}_i(z)}{dz}\Big|_{z=0}+\sum_{j=1}^{9}p_{ij}(0)\,\hat{\varphi}_j(0)=\frac{d\hat{h}_i(t)}{dt}\Big|_{t=0}, \qquad i=\overline{1,5} \qquad (64)$$

kelishuvchanlik shartlari bajarilishini talab qilamiz.

6-teorema. *Faraz qilaylik, berilgan funksiyalar 5-teorema shartlarini qanoatlantirsin. Bundan tashqari* $\hat{\varphi}_i(z)\in C^2[0,H]$, $i=\overline{1,9}$, $\hat{g}_i(t)\in C^2[0,\mu_0]$, $i=1,2,3,7,8,9$, $\hat{h}_i(t)\in C^2[0,\mu_0]$, $i=\overline{1,5}$, *bo‘lib, (63) munosabatlar va (64) kelishuvchanlik shartlari bajarilsin. U holda ixtiyoriy tayinlangan* $H>0$ *son uchun (57)-(60) masalaning* $[0,\mu_0]$ *segmentda aniqlangan yagona yechimi mavjud va bu yechim* $C^1[0,\mu_0]$ *sinfga tegishli bo‘ladi. Bu yerda* $\mu_i(H)=\int_0^H\frac{1}{\nu_i(s)}ds$, $\mu_0=\max\{\mu_1(H),\mu_2(H),\mu_3(H)\}$.

XULOSA

Dissertatsiyada giperbolik qovushqoq-elastik integro-differensial tenglamalar sistemasi uchun boshlang‘ich-chegaraviy masalalar va integral hadning yadrosini aniqlash teskari masalalari ko‘rib chiqildi. Teskari masalani yechish uchun uni Volterra ikkinchi turdagi integral tenglamalar sistemasiga keltirildi. O‘ng tomonida o‘rama ko‘rinishidagi integral operatori bo‘lgan giperbolik integro-differensial tenglamalar sistemasining keng sinfi uchun teskari masalalar ko‘rib chiqildi.

Asosiy natijalar quyidagilardan iborat:

vertikal bir jinsli bo‘lmagan muhitda ikki o‘lchamli qovushqoq-elastiklik tenglamalar sistemasining bir o‘lchamli yadrosini aniqlash usuli ishlab chiqildi. Global yagona yechluvchanlik shartlari olindi;

uch o‘lchamli qovushqoq-elastiklik tenglamalar sistemasining boshlang‘ich-chegaraviy masalasi uchunb mavjudlik va yagonalik teoremasi isbotlandi;

eksponensial vaznli uzluksiz funksiyalar sinfida qisqartirib akslantirish prinsipi yordamida normal shakldagi qovushqoq-elastiklikning uch o‘lchamli integro-differensial tenglamalar sistemasidan xotirani aniqlash teskari masalasining global yagona yechilishi haqidagi teorema isbotlandi;

ma’lum elastiklik modullari bilan qovushqoq-elastiklik tenglamalar sistemasidan xotira matritsasini aniqlash bir o‘lchamli masalasining yagona yechilishi shartlari aniqlandi.

Bu natijalar giperbolik tenglamalar sistemasi uchun teskari masalalardagi nochiziqlilik o‘ramasimon xususiyatga ega bo‘lganda, integral operatorlar yadrolarini aniqlashning bir o‘lchamli masalalari global yechilishini hal qilish imkonini beradi.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 AT V.I. ROMANOVSKIY INSTITUTE OF MATHEMATICS

V.I. ROMANOVSKIY INSTITUTE OF MATHEMATICS

BOLTAEV ASLIDDIN ASKAR UGLI

INVERSE PROBLEMS FOR A SYSTEM OF ELASTICITY EQUATIONS OF THE FIRST ORDER WITH MEMORY

01.01.02 – Differential equations and mathematical physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT – 2024

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Ministry of Higher Education, Science and Innovation of the Republic of Uzbekistan under number B2023.4PhD/FM946.

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INTRODUCTION (Annotation of Philosophy (PhD) dissertation)

Actuality and demand of the theme of dissertation. In the world, many scientific and practical studies, in most cases, come down to the study of direct and inverse problems of mathematical physics. Inverse problems have permeated astronomy, quantum propagation theory, geophysics, thermal physics, medicine, and all branches of modern science with the advent of computers. A more accurate study using mathematical methods of propagation of elastic waves in viscoelastic media requires taking into account the process's memory. The mean of mathematical, taking into account the prehistory leads to the need to introduce into the considered mathematical model of elasticity an additional kernel responsible for the memory of the medium such as a convolution operator. Based on additional condition about solving a direct problem, conducting research on solving direct problems and determining the kernels of integro-differential equations remains one of the important tasks in mathematical physics.

Nowadays, special attention is paid to the most rapidly developing field of mathematical physics in the world - the methods of studying inverse problems. This field has become one of the most important mathematical problems in physics and engineering. Systems of differential equations of viscoelasticity arise, for example, in seismic exploration, seismic waves are studied using the properties of the medium. Problems of determining the kernel of an integral operator from some information about the wave field play an important role in studying the structure and properties of the geological medium. Thus, the need arose for research correctness of direct and inverse problems for integro-differential equations in viscoelastic media. The need to develop methods for solving inverse problems of the theory of wave processes in viscoelastic media determines the relevance of the topic of the dissertation work.

In our country, attention has been paid to the current trends in mathematical physics, which have a scientific and practical application of fundamental sciences. In particular, in recent years, special attention has been paid to the research of initial-boundary problems for integro-differential equations representing the propagation processes of elastic waves in memory media, and solving practical problems by solving inverse problems for them. Significant results have been achieved to prove the existence and uniqueness of the solution of inverse problems for the integro-differential equations of elastic waves in memory media and to determine the memory function of the media.The main tasks and directions of activity of mathematical science are to conduct research at the level of international standards in the priority areas of differential equations, mathematical physics and functional analysis[8]. In this regard, it seems essential to develop the theory of inverse problems for integro-differential equations of mathematical physics.

This dissertation work is intended to solve the problems outlined in the

[8]Resolution of the Cabinet of Ministers of the Republic of Uzbekistan No. 292 "On measures to organize the activities of the newly created research institutions of the Academy of Sciences of the Republic of Uzbekistan" dated May 18, 2017.

Decree of the President of the Republic of Uzbekistan No.DP - 4947 dated February 7, 2017 “About the Action Strategy for the further development of the Republic of Uzbekistan”, in resolutions No.RP - 2789 dated February 17, 2017 “On measures to further improve the activities of the Academy of Sciences, organization, management and financing of research activities”, No.RP - 2909 dated April 20, 2017 “On measures for the further development of the higher education system”, No.RP - 3682 dated April 27, 2018 “On measures to further improve the system for the practical implementation of innovative ideas, technologies and projects”, №RP - 4708 of May 07, 2020 “On measures to improve the quality of education and the development of scientific research in the field of mathematics”, as well as in other regulatory legal acts related to this area activities.

Connection of research to priority directions of development of science and technologies of the Republic. This work was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV “Mathematics, Mechanics and Computer Science”.

The degree of scrutiny of the problem. The first publications on inverse problems were related to physics, geophysics, astronomy and so on. A significant direction in the theory of inverse problems is the inverse problems of exploration geophysics. Here we should mention the first publications of A.S. Alekseev and S.V. Goldin. In particular, A.S. Alekseev was the first to study the one-dimensional inverse problem of determining density for the equations of isotropic elasticity in a half-space. Developing this direction, M.M. Lavrentiev, V.G. Romanov, A.S. Blagoveshchensky, S.P. Belinsky, V.G. Yakhno and others developed methods for studying various formulations of inverse problems for a system of equations.

The problem of determining the kernel of the integral operator in integro-differential equations and system of hyperbolic type is a young, intensively developing direction in the theory of inverse problems. The first publications in this area found in the literature are associated with M. Graselli, V.G. Romanov A. Lorenzi, E. Paparoni D.K. Durdiev. In A.L. Bukhgeym, D.K. Durdiev, V.G. Yakhno’s researches the inverse problems with memory for hyperbolic equations and systems are studied. In A.L. Bukhgeym, V.G. Romanov, D.K. Durdiyev,
J.Sh. Safarov, Y. Yanno’s works, considered the problem of determining a kernel that depends only on a time variable for the case of investigated. The problems are reduced to solving Volterra-type integral equations for unknown functions. Existence and uniqueness theorems are obtained, as well as estimates for the continuous dependence of the solution on given functions.

The theory of inverse problems for hyperbolic systems was developed by L.P. Nijnik, S.P. Belinsky, V.G. Romanov, T.P. Pukhnachova, D.K. Durdiev, Zh.D. Totieva and others. V.G. Romanov[9] considered the system of equations

[9] В.Г. Романов, К задаче определения правой част гиперболических систем, Диф. уравнения, 1977, 13:3, с. 509-515.

$$u_t + \sum_{j=1}^{n} A_j u_{x_j} + Bu_z + Du = F, \qquad (1)$$

for vector function $u = (u_1, u_2, \dots, u_m)$, $m \geq 2$ in the domain $\Omega = \{(x, z, t): x \in \mathbb{R}^n, 0 < z < H, t > 0\}$. $(A_j, B, D)(z)$ are $m \times m$ dimensional matrices, $F = (F_1, \dots, F_m)(x, z, t)$, $x = (x_1, \dots, x_n)$. Assuming that the system of equations (1) is t-hyperbolic and $F(x, z, t) = \Phi(t) f(x, z)$, the problem of finding the function $f(x, z)$ when $\Phi(t)$ is known on the basis of the conditions given at the boundary of the domain Ω. When the solution of the equations system (1) is known at the boundary of the domain Ω, S.P. Belinsky va V.G. Romanov[10] investigated the problem of determining matrices $(A_j, B, D)(z)$. The inverse problem of determining the convolution kernels of integral terms from a system of viscoelasticity of general form with two independent variables was studied in D.K. Durdiev and Zh.D. Totieva's works. The theorem of local existence, global uniqueness and stability estimates is obtained. In the work of D.K. Durdiev and Kh.Kh. Turdiev was applied to the investigating of the inverse problem of determining the diagonal relaxation matrix from the system of Maxwell's integro-differential equations. The theorem of global existence and uniqueness is obtained.

In this dissertation, the problem of finding the matrix $R(t)$ when the solution of the equation (1) is given on the boundary of the domain Ω in the case of

$$F = F(x, z, t; u) = \int_0^t R(t - \tau) u(x, z, \tau) d\tau + f(x, z, t), \qquad (2)$$

is studied. The Bolsano-Volterra integral relation between stress tensor and deformations of viscoelasticity theory is adopted as the physical model of equation (1). Isotropic, anisotropic media are considered. Various problems for different cases of the kernel matrix $R(t)$ are investigated, which represents the viscosity of the media. The results about the existence and uniqueness of the solution of the problems are obtained.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out.

The dissertation research is done in accordance with planned of scientific research Bukhara branch Institute of Mathematics named after Romanovskiy on the topic "Inverse problems of mathematical physics" and Southern Institute of Mathematics of the Vladikavkaz Scientific Center of the Russian Academy of Sciences of the project No. 075-02-2022-896 .

The aim of research work. The purpose of this dissertation is to construct methods for determining the kernels of integral terms in two- and three-dimensional systems of viscoelasticity and to prove the uniqueness and existence of solutions to these inverse problems.

[10] С.П. Белинский, В.Г. Романов, К задаче определения коэффициентов t-гиперболической системы, Матем. заметки, 1980, 28:4, с. 525-532.

Tasks of the research:

to prove theorems of existence and uniqueness of the solution to the inverse problems for a two-dimensional system of viscoelasticity equations;

to obtain theorems on the existence and uniqueness of the solution of the initial-boundary problems for the viscoelastic system in a vertically layered medium;

to prove the theorems on the global unique solvability of the inverse problems of determining memory from the system of viscoelasticity in isotropic media;

to obtain the theorems of existence and uniqueness of the solution to the problems of determining the kernel from the system of viscoelasticity in the anisotropic media with tetragonal form of elasticity module.

The research object is one-dimensional inverse problems for the system of integro-differential equations of first order hyperbolic type.

The research subject is inverse problems for a two- and three-dimensional system of viscoelasticity equations.

Research methods: The research used methods of functional analysis, partial differential equations and theory of integral equations. The unique solvability of inverse problems are proven by replacing these problems with a system of nonlinear integral equations of the second kind of Volterra type and the principle of contraction mappings.

The scientific novelty of the research is defined by the following points:

theorems of the existence and uniqueness of the solution of the inverse problems for a two- dimensional system of viscoelasticity equations are proved;

theorem on the existence and uniqueness of the solution of the initial-boundary problems for the viscoelastic system in a vertically layered medium are proved;

theorem on the global unique solvability of the inverse problem of determining memory from the system of viscoelasticity in isotropic media is proved;

the existence of a unique solution to the inverse problem of determining a diagonal matrix kernel from the system of viscoelasticity in the anisotropic media with tetragonal form of elasticity module is proved.

Practical results of the research:

The research results can be used in seismology, in the development of oil and gas fields and in delivering special courses of lectures on the subject of mathematical physics for senior undergraduate and graduate courses.

The reliability of the results of the study. Results have been obtained using the theory of differential equations, the theory of inverse problems, and the application of mathematic and functional analysis methods. The obtained results are mathematically strongly proved.

Scientific and practical significance of research results. The scientific significance of the research results is explained by further developing the theory of inverse problems for the system of viscoelasticity integro-differential equations of mathematical physics and constructing methods for determining one-dimensional kernels.

The practical significance of the study's results is explained by its study of the propagation of elastic and electromagnetic waves in media with memory.

Implementation of the research results. The results of solving inverse problems for a system of hyperbolic equations with first-order memory obtained in the dissertation were used in the following research projects:

using the method of determining kernels for a system of integro-differential equations of viscoelasticity in the fundamental project AAAA-A19-119032590069-3 "Mathematical modeling in sociology, geophysics and engineering sciences" (Certificate No. 81, Southern Mathematical Institute branch of the Federal State Budgetary Institution Federal Scientific Center Vladikavkaz Scientific Center of the Russian Academy of Sciences dates dated October 31, 2023). The application of the scientific result made it possible to determine the core in the form of a convolution from a system of viscoelastic equations in vertically layered isotropic media and prove its global existence and uniqueness;

determination of the displacement vector for a system of viscoelasticity equations in an isotropic medium were used in the grant OT-F4-01 "Natural disasters of Kamchatka - earthquakes and volcanic eruptions" (Certificate No. AAAA-A19-119072290002-9, Kamchatka State University named after Vitus Bering, RAS dated November 13, 2023). The application of scientific results made it possible to prove the existence and uniqueness of the problem of determining the kernel.

Approbation of the research results. The main results of the research have been discussed at 3 international and 2 national scientific conferences.

Publications of the research results. On the topic of the dissertation, 13 research papers have been published in scientific journals, 8 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the Doctor of Philosophy thesis, in addition 4 of them were published in international journals of mathematics and physics and 4 paper published in a national mathematical journal.

The structure and volume of the dissertation. The dissertation includes an introduction, three chapters, conclusion and references. The general volume of the thesis is 95 pages.

THE MAIN CONTENT OF THE DISSERTATION

In the introduction the motivation of the research theme and correspondence to the priority research areas of science and technology of the Republic is given, we present a review of international research on the theme of the dissertation and degree of scrutiny of the problem, formulate our goals and objectives, identify the object and subject of study, and state the scientific novelty and practical results of the research. Furthermore, we give the theoretical and practical importance of the obtained results, and also give information on the implementation of the research results, the published works and the structure of the dissertation.

Chapter I of the PhD thesis entitled **"The problem of determining kernels in a 2D system of viscoelasticity equations"** is mainly of an introductory nature, it presents the main concepts and definitions used in the dissertation. Furthermore,

the scientific research and the obtained results are described within the scope of this scientific research theme. In the second section, studied initial-boundary problem for a two dimensional system of the viscoelastic equations.

Let $\overline{x} = (x_1, x_2) \in \mathbb{R}^2$. Denote by $\sigma_{ij}, i,j = 1,2$ the projection onto the x_i, $i = 1,2$ axis of the stress acting on the area with the normal parallel to the x_j, $j = 1,2$ axis, and $\overline{u}_i, i = 1,2$ is the projection onto the x_i axis of the particle displacement vector. According to Hooke's law for viscoelastic media, stresses and strains are related by the formulas[11]:

$$\sigma_{ij}(\overline{x},t) = \mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) + \delta_{ij}\lambda \mathrm{div}\overline{u} +$$

$$+ \int_0^t K_{ij}(t-\tau)\left[\mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) + \delta_{ij}\lambda \mathrm{div}\overline{u}\right](\overline{x}, t-\tau)d\tau, \qquad i,j = 1,2, \quad (3)$$

where $\bar{u}(\bar{x}) = (\bar{u}_1, \bar{u}_2)$, $\mu = \mu(x_2) > 0$, $\lambda = \lambda(x_2) > 0$ are the Lame coefficients, δ_{ij} is the Kronecker symbol, $K_{ij}(t)$ are functions responsible for the viscosity of the medium and $K_{ij} = K_{ji}$, $i,j = 1,2$.

The equations of motion of a viscoelastic body practicles in the absence of external forces have the form

$$\rho \frac{\partial^2 \overline{u}_i}{\partial t^2} = \sum_{j=1}^{2} \frac{\partial \sigma_{ij}}{\partial x_j}, \quad i = 1,2, \qquad (4)$$

here $\rho = \rho(x_2) > 0$ is the density of the medium.

Let us pay attention to the fact that (3) can be considered Volterra integral equations of the second kind concerning the expression $\mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) + \delta_{ij}\lambda \mathrm{div}\overline{u}$. For each fixed pair (i,j), solving these equations, we obtain

$$\sigma_{ij}(\overline{x},t) = \mu\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) + \delta_{ij}\lambda \mathrm{div}\overline{u} + \int_0^t r_{ij}(t-\tau)\sigma_{ij}(\overline{x},\tau)d\tau, \quad i,j = 1,2, \quad (5)$$

where r_{ij} are the resolvents of the kernels K_{ij}, and integral relations interconnect them:

$$r_{ij}(t) = -K_{ij}(t) - \int_0^t K_{ij}(t-\tau)r_{ij}(\tau)d\tau, \quad i,j = 1,2. \qquad (6)$$

From the condition $K_{ij} = K_{ji}$ implies $r_{ij} = r_{ji}$.

Differentiating (5) with respect to t and introducing the notation $u_i = \frac{\partial}{\partial t}\overline{u}_i$, $(i = 1,2)$, we obtain

$$\frac{\partial}{\partial t}\sigma_{ij}(\overline{x},t) = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \delta_{ij}\lambda \mathrm{div}u +$$

[11] Л.А. Галин, Контактные задачи теории упругости и вязкоупругости. М. : Наука, 1980. 242 с.

$$+r_{ij}(0)\sigma_{ij}(\overline{x},t)+\int_0^t r'_{ij}(t-\tau)\sigma_{ij}(\overline{x},\tau)d\tau, \qquad i,j=1,2. \tag{7}$$

In this case, $r_{11}(0), r_{12}(0), r_{22}(0)$ are considered to be given.

With this in mind, the equations (4) and (7) for the speed u_i and stress σ_{ij} can be written as system of first-order integro-differential equations. For convenience, denoting $x_1 := x,\ x_2 := y$, we have

$$A\frac{\partial \boldsymbol{U}}{\partial t}-B\frac{\partial \boldsymbol{U}}{\partial x}-C\frac{\partial \boldsymbol{U}}{\partial y}-D\boldsymbol{U}=\int_0^t R(t-\tau)\boldsymbol{U}(x,y,\tau)d\tau, \tag{8}$$

here $\boldsymbol{U}=(u_1,u_2,\sigma_{11},\sigma_{22},\sigma_{12})^*$, $*$ is transposition sign,

$$A=\operatorname{diag}(\rho,\rho,1,1,1)$$

$$B=\begin{pmatrix}0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1\\ \lambda+2\mu & 0 & 0 & 0 & 0\\ \lambda & 0 & 0 & 0 & 0\\ 0 & \mu & 0 & 0 & 0\end{pmatrix}, C=\begin{pmatrix}0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1 & 0\\ 0 & \lambda & 0 & 0 & 0\\ 0 & \lambda+2\mu & 0 & 0 & 0\\ \mu & 0 & 0 & 0 & 0\end{pmatrix},$$

$$D=\operatorname{diag}(0,0,r_{11}(0),r_{22}(0),r_{12}(0)),\ R(t)=\operatorname{diag}(0,0,r'_{11},r'_{22},r'_{12}).$$

System (8) can be reduced to normal system with respect to the variables t and y. To do this, multiply (8) on the left by A^{-1} and compose equation

$$|A^{-1}C-\nu I|=0. \tag{9}$$

Equation (9) has roots

$$\nu_1=-\nu_2=\sqrt{\frac{\mu}{\rho}}, \qquad \nu_3=-\nu_4=\sqrt{\frac{\lambda+2\mu}{\rho}}, \qquad \nu_5=0. \tag{10}$$

Now we choose a nonsingular matrix $\Upsilon(y,t)$ so that the equality

$$\Upsilon^{-1}A^{-1}C\Upsilon=\Lambda, \tag{11}$$

where Λ is a diagonal matrix whose diagonal contains eigenvalues (for each fixed y) (10) of the matrix $A^{-1}C$.

We get following the equality by using formula (11) $A^{-1}C\Upsilon=\Upsilon\Lambda$, which means that the i-th column of the matrix Υ is an eigenvector of the matrix $A^{-1}C\Upsilon$ corresponding to the eigenvalue ν_i[12]. Direct calculations show that a matrix Υ satisfying the above conditions can be chosen as (not uniquely)

$$\Upsilon=\begin{pmatrix}1 & 1 & 0 & 0 & 0\\ 0 & 0 & -\dfrac{1}{\sqrt{\rho(\lambda+2\mu)}} & \dfrac{1}{\sqrt{\rho(\lambda+2\mu)}} & 0\\ 0 & 0 & \dfrac{\lambda}{\lambda+2\mu} & \dfrac{\lambda}{\lambda+2\mu} & 1\\ 0 & 0 & 1 & 1 & 0\\ -\sqrt{\mu\rho} & \sqrt{\mu\rho} & 0 & 0 & 0\end{pmatrix}.$$

[12] Ф.Р. Гантмахер, *Теория матриц*, Наука, Гл. ред. физ.-мат. лит., М, 1988.

Let us introduce the vector function $\boldsymbol{U}$ by the equality

$$\boldsymbol{U} = \Upsilon\boldsymbol{\vartheta}.$$

After making this replacement in equation (8) and then multiplying it from the left by $\Upsilon^{-1}A^{-1}$, we obtain

$$\frac{\partial\boldsymbol{\vartheta}}{\partial t} + \Lambda\frac{\partial\boldsymbol{\vartheta}}{\partial y} + B_1\frac{\partial\boldsymbol{\vartheta}}{\partial x} + C_1\boldsymbol{\vartheta} = \int_0^t R_1(y, t-\tau)\boldsymbol{\vartheta}(x, y, \tau)d\tau, \qquad (12)$$

where $\boldsymbol{\vartheta} = (\vartheta_1, \dots, \vartheta_5)^*$,

$$B_1 = \Upsilon^{-1}A^{-1}B\Upsilon = \left(b_{ij}\right)_{5\times5},\ C_1(y,t) = \Upsilon^{-1}A^{-1}C\frac{\partial\Upsilon}{\partial y} + \Upsilon^{-1}D\Upsilon = \left(c_{ij}\right)_{5\times5},$$

$$R_1(y,t) = \Upsilon^{-1}A^{-1}R\Upsilon = \left(\tilde{r}_{ij}\right)_{5\times5}.$$

We consider the system of equations (12) in the domain

$$\Omega = \{(x, y, t): x \in \mathbb{R}, 0 < y < H, t > 0\}, \quad H = const,$$

with initial

$$\vartheta_i|_{\Gamma_0} = \varphi_i(x, y), \qquad i = \overline{1,5}, \qquad (13)$$

and boundary conditions

$$\vartheta_i|_{\Gamma_1} = \psi_i(x, t), \qquad i = 1,3, \quad \vartheta_i|_{\Gamma_2} = \psi_i(x, t), \qquad i = 2,4. \qquad (14)$$

Here $\Gamma = \Gamma_0 \cup \Gamma_1 \cup \Gamma_2$:

$$\Gamma_0 = \{(x, y, t): x \in \mathbb{R}, 0 \le y \le H, t = 0\},$$

$$\Gamma_1 = \{(x, y, t): x \in \mathbb{R}, y = 0, t > 0\}, \quad \Gamma_2 = \{(x, y, t): x \in \mathbb{R}, y = H, t > 0\}.$$

It is known that problem (12)-(14) is well posed[13]. Assume that the functions $\varphi_i(x, y)$, $\psi_i(x, y)$ are finite in x for each fixed y, t and are smooth to some extent.

Since the given in problems (12)-(14) are finite with respect to variable x, the solution of this problem is also finite. So, we can use the Fourier transform on variable x. As a result, we study the inverse problem using the Fourier transforms of the solution of the initial-boundary problem.

The *inverse problem* is to determine the nonzero components of the matrix kernel R_1 in (12) if the following conditions are known:

$$\tilde{\vartheta}_1|_{\tilde{\Gamma}_2,\xi=0} = \tilde{h}_1(t), \quad \tilde{\vartheta}_3|_{\tilde{\Gamma}_2,\xi=0} = \tilde{h}_3(t), \quad \tilde{\vartheta}_5|_{\tilde{\Gamma}_1,\xi=0} = \tilde{h}_5(t), \qquad (15)$$

where $\widetilde{\boldsymbol{\vartheta}}(\xi, y, t) = \int_{\mathbb{R}} e^{i\xi x}\boldsymbol{\vartheta}(x, y, t)dx$, are Fourier transforms of functions ϑ, ξ is conversion parameter, $\tilde{h}_i(t)$ are given smooth vector functions and $\tilde{\Gamma}$ is the projection of Γ onto the plane y, t.

Denoting $\boldsymbol{V}(y, t) := \widetilde{\boldsymbol{\vartheta}}(\xi, y, t)|_{\xi=0}$. Direct calculations show that $\boldsymbol{V}(y, t) = (V_1, \dots, V_5)^*$ satisfies the equation

$$\frac{\partial\boldsymbol{V}}{\partial t} + \Lambda\frac{\partial\boldsymbol{V}}{\partial y} + C_1\boldsymbol{V} = \int_0^t R_1(y, \tau)\boldsymbol{V}(y, t-\tau)d\tau, \qquad (16)$$

and conditions (13), (14) correspond to the conditions

$$V_i|_{\tilde{\Gamma}_0} = \tilde{\varphi}_i(y), \qquad i = \overline{1,5}, \qquad (17)$$

$$V_i|_{\tilde{\Gamma}_1} = \tilde{\psi}_i(t), \qquad i = 1,3, \quad V_i|_{\tilde{\Gamma}_2} = \tilde{\psi}_i(t), \qquad i = 2,4, \qquad (18)$$

[13] В. Г. Романов, Обратные задачи математической физики. М.: Наука. 1984. с. 264.

where $\tilde{\varphi}_i(y),\ i=\overline{1,5}$, $\tilde{\psi}_i(t),\ i=\overline{1,4}$ are the Fourier images of the corresponding functions from (13), (14) for $\xi=0$.

The given functions $\tilde{\varphi}_i$ and $\tilde{\psi}_i$ must satisfy the matching conditions:

$$\tilde{\varphi}_i(0)=\tilde{\psi}_i(0),\quad i=1,3,\qquad \tilde{\varphi}_i(0)=\tilde{\psi}_i(H),\quad i=2,4. \tag{19}$$

Such a problem has a unique solution in a bounded subdomain $\Omega_{HT}=\{(y,t):0<y<H,\ 0<t<T\},\ T>0$ is some fixed number, the domain Ω.

Lemma 1. *Let* $\{\rho(y),\mu(y),\lambda(y)\}\in C^1[0,H],\ \tilde{\varphi}_i(y)\in C[0,H], i=\overline{1,5}$, $\tilde{\psi}_i(t)\in C[0,T],\ i=\overline{1,4},\ r_{ij}(t)\in C^1[0,T],\ i,j=1,2$, *and conditions (19) are satisfied. Then there is a unique solution to problem (16)-(18) in the domain* Ω_{HT} *belong to class* $C^1(\Omega_{HT})\cap C(\overline{\Omega}_{HT})$.

Let us introduce the vector function $\boldsymbol{\omega}(y,t)=\frac{\partial \boldsymbol{V}}{\partial t}(y,t)$. To obtain a problem for a function $\boldsymbol{\omega}(y,t)$ similar to (16)-(18), we differentiate equation (16) and boundary conditions (18) with respect to the variable t, and we find the condition at $t=0$ using equations (16) and initial conditions (17). In doing so, we get

$$\frac{\partial \omega_i}{\partial t}+\nu_i\frac{\partial \omega_i}{\partial y}+\sum_{j=1}^{4}c_{ij}(y)\omega_j(y,t)=\int_0^t\sum_{j=1}^{4}\tilde{r}_{ij}(y,\tau)\omega_j(y,t-\tau)d\tau+$$

$$+\sum_{j=1}^{4}\tilde{r}_{ij}(y,t)\tilde{\psi}_j(y),\quad i=\overline{1,5}, \tag{20}$$

$$\omega_i(y,t)|_{t=0}=-\nu_i\frac{d\tilde{\varphi}_i(y)}{dy}-\sum_{j=1}^{5}c_{ij}(y)\tilde{\varphi}_i(y)=\Phi_i(y),\ \ i=\overline{1,5}, \tag{21}$$

$$\omega_i(y,t)|_{y=0}=\frac{d\tilde{\psi}_i(t)}{dt},\ \ i=1,3,\quad \omega_i|_{y=H}=\frac{d\tilde{\psi}_i(t)}{dt},\ \ i=2,4, \tag{22}$$

For function ω_i additional conditions (15) following view

$$\omega_1|_{y=H}=\frac{d\tilde{h}_1(t)}{dt},\quad \omega_3|_{y=H}=\frac{d\tilde{h}_3(t)}{dt},\quad \omega_5|_{y=0}=\frac{d\tilde{h}_5(t)}{dt}. \tag{23}$$

Let the conditions

$$-\nu_i\frac{d\tilde{\varphi}_i(y)}{dy}\bigg|_{y=0}-\sum_{j=1}^{5}c_{ij}(0)\tilde{\varphi}_i(0)=\frac{d\tilde{\psi}_i(t)}{dt}\bigg|_{t=0},\quad i=1,3, \tag{24}$$

$$-\nu_i\frac{d\tilde{\varphi}_i(y)}{dy}\bigg|_{y=H}-\sum_{j=1}^{5}c_{ij}(H)\tilde{\varphi}_i(H)=\frac{d\tilde{\psi}_i(t)}{dt}\bigg|_{t=0},\quad i=2,4. \tag{25}$$

Theorem 1. *Let the conditions of Lemma 1 be satisfied. Besides,* $\tilde{\varphi}_i(y)\in C^1[0,H]$, $i=\overline{1,5}$, $\tilde{\psi}_i(t)\in C^1[0,T]$, $i=\overline{1,4}$ *and conditions (24),(25) are satisfied. Then there is a unique solution to problem (20)-(22) in the domain* Ω_{HT} *from class* $C^1(\Omega_{HT})\cap C(\overline{\Omega}_{HT})$.

In the third paragraph of the first chapter of the dissertation, studied the inverse problem the system for hyperbolic equations of the first order.

In what follows, we will assume that

$$\tilde{\varphi}_1(0) \neq \tilde{\varphi}_2(0), \quad \tilde{\varphi}_3(0) \neq -\tilde{\varphi}_4(0),$$
$$\lambda[\tilde{\varphi}_3(0) + \tilde{\varphi}_4(0) + \tilde{\varphi}_5(0)] + 2\mu(0)\tilde{\varphi}_5(0) \neq 0. \tag{26}$$

We require matching conditions of agreement

$$\frac{d}{dt}\tilde{h}_i(t)\Big|_{t=0} = -\nu_i \frac{d}{dy}\tilde{\varphi}_i(y)\Big|_{y=0} - \sum_{j=1}^{5} c_{ij}(0)\tilde{\varphi}_i(0), \quad i = 1,3, \tag{27}$$

$$\frac{d}{dt}\tilde{h}_5(t)\Big|_{t=0} = -\sum_{j=1}^{5} c_{5j}(0)\tilde{\varphi}_j(0). \tag{28}$$

Theorem 2. *Let the conditions of Theorem 1 for given functions be satisfied. Besides,* $\tilde{\varphi}_i(y) \in C^2[0,H]$, $i = \overline{1,5}$, $\tilde{\psi}_i(t) \in C^2[0,\mu_0]$, $i = \overline{1,4}$, $\tilde{h}_i(t) \in C^2[0,\mu_0]$, $i = 1,3,5$ *and conditions (26)-(28) be hold. Then for any* $H > 0$ *on the interval* $[0,\mu_0]$ *there is a unique solution to the inverse problem (20)-(23) from the class* $C^1[0,\mu_0]$.

Here $\mu_i(H) = \int_0^H \frac{1}{\nu_i(s)} ds$, $\mu_0 = \max\{\mu_1(H), \mu_3(H)\}$.

Based on the found functions $r'_{11}(t)$, $r'_{22}(t)$, $r'_{12}(t)$, the functions $r_{11}(t)$, $r_{22}(t)$, $r_{12}(t)$ are found by the formulas

$$r_{ij}(t) = r_{ij}(0) + \int_0^t r'_{ij}(\tau)d\tau, \quad i,j = 1,2.$$

Note that the functions $r_{11}(t)$, $r_{22}(t)$, $r_{12}(t)$ functions $K_{11}(t)$, $K_{22}(t)$, $K_{12}(t)$ are determined as solutions of integral equations (6).

In the second chapter is devoted "**Inverse problem for 3D viscoelastic system in a vertically layered medium**" and this section the problem of determining the one-dimensional kernel of the integral term is studied. The first paragraph formulates the formulations of the initial-boundary and inverse problems. In this section, at the beginning, viscoelastic three-dimensional system of integro-differential equations is reduced to canonical form. In addition, the existence and uniqueness theorems of initial-boundary problem solutions are presented. The second paragraph presents the derivation of integral equations equivalent to the inverse problem. The third paragraph presents the existence and uniqueness theorem of the solution of the inverse problem and its proof.

Vibrations of the space without external forces can be described by the system of equations

$$\begin{cases} \rho \dfrac{\partial u_i}{\partial t} = \displaystyle\sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j}, \quad i = 1,2,3, \\ \dfrac{\partial \sigma}{\partial t} = \mathrm{K}\varepsilon + Q\sigma + \displaystyle\int_0^t P(t-\tau)\,\sigma(\tau)\,d\tau, \end{cases} \quad (x_1,x_2) \in \mathbb{R}^2, \ x_3 \in [0,H], \ t > 0, \tag{29}$$

where $u = (u_1,u_2,u_3)^*$ is the velocity vector, σ $(\sigma_{ij} = \sigma_{ji}, i,j = 1,2,3)$ is stress tensor and $\sigma = (\sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{22}, \sigma_{23}, \sigma_{33})^*$, $\varepsilon = (\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4,\varepsilon_5,\varepsilon_6)^*$:

$$\varepsilon = \begin{cases} \varepsilon_1 = \dfrac{\partial u_1}{\partial x_1}, \quad \varepsilon_4 = \dfrac{\partial u_2}{\partial x_2}, \quad \varepsilon_6 = \dfrac{\partial u_3}{\partial x_3}, \\ \varepsilon_2 = \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_2} + \dfrac{\partial u_2}{\partial x_1}\right), \\ \varepsilon_3 = \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_3} + \dfrac{\partial u_3}{\partial x_1}\right), \\ \varepsilon_5 = \dfrac{1}{2}\left(\dfrac{\partial u_2}{\partial x_3} + \dfrac{\partial u_3}{\partial x_2}\right), \end{cases}$$

$$K = \begin{pmatrix} K_{11} & O_{3\times 3} \\ O_{3\times 3} & K_{22} \end{pmatrix}, \quad K_{11} = (k_{ij}) = \begin{cases} \lambda + 2\mu, & i = j, \\ \lambda, & i \neq j, \end{cases} \quad K_{22} = \operatorname{diag}(\mu, \mu, \mu),$$

$$Q = \operatorname{diag}\,(r_{11}(0),0,0,r_{22}(0),0,r_{33}(0)),\ P = \operatorname{diag}\,(r'_{11},0,0,r'_{22},0,r'_{33}),$$

and $\lambda = \lambda(x_3) > 0$, $\mu = \mu(x_3) > 0$, are Lame coefficients, $\rho = \rho(x_3) > 0$ is the density of the medium, $r_{11}(t), r_{22}(t), r_{33}(t)$ are functions responsible for the viscosity of the medium.

We obtain the system of hyperbolic integro-differential equations (29) to the canonical form:

$$\frac{\partial \boldsymbol{\vartheta}}{\partial t} + \Lambda \frac{\partial \boldsymbol{\vartheta}}{\partial x_3} + B_1 \frac{\partial \boldsymbol{\vartheta}}{\partial x_1} + C_1 \frac{\partial \boldsymbol{\vartheta}}{\partial x_2} + F_1 \boldsymbol{\vartheta} = \int_0^t R_1(t-\tau, x_3)\, \boldsymbol{\vartheta}(x_1, x_2, x_3, \tau) d\tau, \tag{30}$$

where $\boldsymbol{\vartheta} = (\vartheta_1, \dots, \vartheta_9)^*$,

$$\Lambda = \operatorname{diag}(\nu_i) = \operatorname{diag}\left(-\sqrt{\frac{\lambda+2\mu}{\rho}}, -\sqrt{\frac{\mu}{\rho}}, -\sqrt{\frac{\mu}{\rho}}, 0,0,0, \sqrt{\frac{\mu}{\rho}}, \sqrt{\frac{\mu}{\rho}}, \sqrt{\frac{\lambda+2\mu}{\rho}}\right),$$

$$B_1(x_3) = \Upsilon^{-1} A^{-1} B \Upsilon = \left(b_{ij}\right)_{9\times 9},$$

$$C_1(x_3) = \Upsilon^{-1} A^{-1} C \Upsilon = \left(c_{ij}\right)_{9\times 9},$$

$$F_1(x_3) = \Upsilon^{-1} A^{-1} D \frac{\partial \Upsilon}{\partial x_3} + \Upsilon^{-1} A^{-1} F \Upsilon = \left(p_{ij}\right)_{9\times 9},$$

$$R_1(x_3, t) = \Upsilon^{-1} A^{-1} R \Upsilon = \left(\tilde{r}_{ij}\right)_{9\times 9},$$

$$\Upsilon(x_3) = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \dfrac{\lambda}{\nu_9} & 0 & 0 & 1 & 0 & 1 & 0 & 0 & -\dfrac{\lambda}{\nu_9} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho\nu_8 & 0 & 0 & 0 & 0 & -\rho\nu_8 & 0 \\ \dfrac{\lambda}{\nu_9} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\dfrac{\lambda}{\nu_9} \\ 0 & \rho\nu_8 & 0 & 0 & 0 & 0 & -\rho\nu_8 & 0 & 0 \\ \rho\nu_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho\nu_9 \end{pmatrix}.$$

Consider the system of equations (30) in the domain

$\Omega = \{(x_1,\ x_2,\ x_3,\ t)\colon (x_1,\ x_2) \in \mathbb{R}^2,\ \ x_3 \in (0,H), t > 0\},\ H = \text{const},$

with following initial and boundary conditions:

$$\vartheta_i|_{t=0} = \varphi_i(x_1,x_2,x_3), \qquad i = \overline{1,9}, \tag{31}$$

$$\vartheta_i\rfloor_{x_3=H} = g_i(x_1,x_2,t), i = 1,2,3, \quad \vartheta_i|_{x_3=0} = g_i(x_1,x_2,t),\ \ i = 7,8,9. \tag{32}$$

Here $\varphi_i(x_1,x_2,x_3),\ g_i(x_1,x_2,t)$ are given functions.

The *inverse problem* is to determine the nonzero components of the matrix kernel R_1, if the following conditions are known:

$$\vartheta_i\rfloor_{x_3=0} = h_i(x_1,x_2,t), i = 1,4,6, \tag{33}$$

where $h_i(x_1,x_2,t), i = 1,4,6,$ are the given functions.

Let functions $\varphi_i(\bar{x}),\ i = \overline{1,9}$ and $g_i(x_1,x_2,t), i = 1,2,3,7,8,9$ have compact support in $x_1,\ x_2$ for each fixed x_3, t. From the existence of the system (30) of a compact support domain of dependence and compact support concerning x_1, x_2 of the right-hand side (30) and data (31), (32) implies the compact support in x_1, x_2 solutions to the problem (30)-(32).

Let us study the property of a solution to this problem. More precisely, we restrict ourselves to studying the Fourier transform in the variables x_1, x_2 of the solution. In what follows, for convenience, we set $x_3 = z$ and introduce

$$\widetilde{\boldsymbol{\vartheta}}(\eta_1,\eta_2,z,t) = \int_{\mathbb{R}^2} \boldsymbol{\vartheta}(x_1,x_2,z,t) e^{i[\eta_1 x_1 + \eta_2 x_2]} dx_1 dx_2, \tag{34}$$

where η_1 and η_2 are transformation parameters. We obtain $\eta_1 = 0, \eta_2 = 0$ and for convenience, we introduce the notation $\widetilde{\boldsymbol{\vartheta}}(\eta_1,\eta_2,z,t) = \widehat{\boldsymbol{\vartheta}}(z,t)$.

In terms of the function $\widehat{\boldsymbol{\vartheta}}(z,t)$ we write the inverse problem (30)-(33) as

$$\left(\frac{\partial}{\partial t} + \nu_j \frac{\partial}{\partial z}\right)\hat{\vartheta}_j(z,t) =$$

$$= \sum_{k=1}^{9} p_{jk}(z)\hat{\vartheta}_k(z,t) + \int_0^t \sum_{k=1}^{9} \hat{r}_{jk}(z,\tau)\hat{\vartheta}_k(z,t-\tau)\, d\tau,\ j = \overline{1,9}, \tag{35}$$

$$\hat{\vartheta}_i\big|_{t=0} = \hat{\varphi}_i(z), \qquad i = \overline{1,9}, \tag{36}$$

$$\hat{\vartheta}_i\big|_{z=H} = \hat{g}_i(t),\ \ i = 12,3, \qquad \hat{\vartheta}_i\big|_{z=0} = \hat{g}_i(t),\ \ i = 7,8,9, \tag{37}$$

$$\hat{\vartheta}_i\big|_{z=0} = \hat{h}_i(t), \quad i = 1,4,6. \tag{38}$$

For further research let us introduce the vector function $\boldsymbol{\omega}(z,t) = \frac{\partial \widehat{\boldsymbol{\vartheta}}}{\partial t}(z,t)$. To obtain a problem for a function $\boldsymbol{\omega}(z,t)$ similar to (35)-(38) differentiate the equations (35) and the conditions (37), (38) for the variable t, and the condition for $t = 0$ is found using the equations (35) and the initial conditions (36). In this case, we get

$$\left(\frac{\partial}{\partial t} + \nu_j \frac{\partial}{\partial z}\right)\omega_j(z,t) = \sum_{k=1}^{9} \hat{p}_{jk}(z)\omega_k(z,t) + \sum_{k=1}^{9} \tilde{r}_{jk}(z,\tau)\hat{\varphi}_k(z) +$$

$$+ \int_0^t \sum_{k=1}^{9} \tilde{r}_{jk}(z,\tau)\omega_k(z,t-\tau)\, d\tau, \quad j = \overline{1,9}, \tag{39}$$

$$\omega_i|_{t=0} = -\nu_j \frac{d\hat{\varphi}_i(z)}{dz} + \sum_{j=1}^{9} \hat{p}_{ij}(z)\hat{\varphi}_j(z) =: \Phi_i(z), \quad i = \overline{1,9}, \tag{40}$$

$$\omega_i|_{z=H} = \frac{d}{dt}\hat{g}_i(t), \ \ i = 1,2,3, \qquad \omega_i|_{z=0} = \frac{d}{dt}\hat{g}_i(t), \ \ i = 7,8,9, \tag{41}$$

$$\omega_i|_{z=0} = \frac{d}{dt}\hat{h}_i(t), \quad i = 1,4,6. \tag{42}$$

Let the following conditions hold

$$\hat{\varphi}_i(H) = \hat{g}_i(0), i = 1,2,3, \qquad \hat{\varphi}_i(0) = \hat{g}_i(0), \ \ i = 7,8,9,$$

$$\left.\frac{d\hat{g}_i(t)}{dt}\right|_{t=0} = -\nu_i \left.\frac{d\hat{\varphi}_i(z)}{dz}\right|_{z=H} + \sum_{j=1}^{9} p_{ij}(H)\hat{\varphi}_j(H), \qquad i = 1,2,3,$$

$$\left.\frac{d\hat{g}_i(t)}{dt}\right|_{t=0} = -\nu_i \left.\frac{d\hat{\varphi}_i(z)}{dz}\right|_{z=0} + \sum_{j=1}^{9} p_{ij}(0)\hat{\varphi}_j(0), \qquad i = 7,8,9. \tag{43}$$

Theorem 3. Let be $\{\rho(z),\ \mu(z),\ \lambda(z),\ \hat{\varphi}(z)\} \in C^1[0,H]$, $\hat{g}(t) \in C^1[0,T]$, $r_{ii}(t) \in C^1[0,T]$, $i = 1,2,3$ and conditions (43) are satisfied. Then there is a unique solution to the problem (39)-(41) in the domain $\Omega_{HT} = \{(z,t): 0 < z < H, 0 < t < T\}$.

In what follows, we will assume that

$$\lambda[\hat{\varphi}_1(0) - \hat{\varphi}_9(0)] + \nu_p\hat{\varphi}_6(0) + \nu_p\hat{\varphi}_4(0) \neq 0, \quad \hat{\varphi}_5(0) \neq 0,$$
$$\hat{\varphi}_3(0) - \hat{\varphi}_8(0) \neq 0, \quad \lambda[\hat{\varphi}_1(0) - \hat{\varphi}_9(0)] + \nu_p\hat{\varphi}_6(0) \neq 0, \tag{44}$$
$$\hat{\varphi}_2(0) - \hat{\varphi}_7(0) \neq 0, \quad \hat{\varphi}_1(0) - \hat{\varphi}_9(0) \neq 0.$$

We require the fulfillment of the matching conditions

$$\nu_i \left.\frac{d\hat{\varphi}_i(z)}{dz}\right|_{z=0} + \sum_{j=1}^{9} p_{ij}(0)\omega_j(0) = \left.\frac{d}{dt}\hat{h}_i(t)\right|_{t=0}, \ \ i = 1,4,6. \tag{45}$$

Theorem 4. *Let the conditions of Theorem 3 are satisfied, besides* $\hat{\varphi}_i(z) \in C^2[0,H]$, $i = \overline{1,9}$, $\hat{g}_i(t) \in C^2[0,\mu_0]$, $i = 1,2,3,7,8,9$, $\hat{h}_i(t) \in C^2[0,\mu_0]$, $i = 1,4,6$, *inequalities (44) and matching conditions (45) hold. Then for any fixed* $H > 0$ *on the segment* $[0,\mu_0]$ *there is a unique solution to the inverse problems (39)-(42) from class* $C^1[0,\mu_0]$. *Here* $\mu_0 = \int_0^H \frac{1}{\nu_1(s)} ds$.

The third chapter is called "**An inverse problem for the system of viscoelasticity in the anisotropic media**" it considers the initial-boundary value problem for a three-dimensional integro-differential system of viscoelasticity equations, besides the inverse problem of determining a one-dimensional memory kernel.

In the anisotropic medium, the stress tensor σ and the velocity u can be expressed by the following system of integro-differential equations

$$\begin{cases} \rho\dfrac{\partial u_i}{\partial t} = \sum\limits_{j=1}^{3}\dfrac{\partial \sigma_{ij}}{\partial x_j}, \qquad i,j = 1,2,3, \\ \dfrac{\partial}{\partial t}\sigma_{ij} = c_{ijkl}\left(\dfrac{\partial u_k}{\partial x_l} + \dfrac{\partial u_l}{\partial x_k}\right) + r_{ij}(0)\sigma_{ij}(x,t) + \int\limits_0^t r'_{ij}(t-\tau)\sigma_{ij}(x,\tau)d\tau, \end{cases} \tag{46}$$

where $x = (x_1, x_2, x_3)$, $c_{ijkl} = c_{ijkl}(x_3)$ is module of elasticity, $\rho = \rho(x_3) > 0$ is the density of the medium, $r_{ij}(t)$ are functions responsible for the viscosity of the medium and $r_{ij} = r_{ji}$, and $r_{11}(t) = r_{12}(t)$.

Let's $c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klji}$. The symmetry of the stress tensor reduces the number of independent elastic module from 81 to 21. If we assume that $c_{\alpha\beta} = c_{ijkl}$ where $\alpha = (ij)$ and $\beta = (kl)$, in accordance with the notation $(11) \to 1,\ (22) \to 2,\ (33) \to 3,\ (23) = (32) \to 4,\ (13) = (31) \to 5,\ (12) = (21) \to 6$, then the matrix of the independent elastic module can be given the form of a 6×6 symmetrical matrix. We will consider anisotropic media with a matrix of independent elastic module of the following form[14]:

$$c_{\alpha\beta} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{11} & c_{13} & 0 & 0 & -c_{16} \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ c_{16} & -c_{16} & 0 & 0 & 0 & c_{66} \end{pmatrix}.$$

The system of equations (46) can be written with respect to the vector function $\boldsymbol{U} = (u, \sigma)^*$ as follows:

$$\left(A\frac{\partial}{\partial t} + B\frac{\partial}{\partial x_1} + C\frac{\partial}{\partial x_2} + D\frac{\partial}{\partial x_3} + F\right)\boldsymbol{U}(x,t) = \int\limits_0^t R(t-\tau)\boldsymbol{U}(x,\tau)d\tau, \tag{47}$$

where

$$A = \mathrm{diag}(\rho, \rho, \rho, 1,1,1,1,1,1),$$
$$F = \mathrm{diag}\left(0,0,0, r_{11}(0), r_{22}(0), r_{33}(0), r_{11}(0), r_{13}(0), r_{23}(0)\right),$$
$$R = R(t) = \mathrm{diag}\,(0,0,0, r'_{11}, r'_{22}, r'_{33}, r'_{11}, r'_{13}, r'_{23}),$$
$$B = \begin{pmatrix} & & & -1 & 0 & 0 & 0 & 0 & 0 \\ & \mathbf{O}_{3\times 3} & & 0 & 0 & 0 & -1 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & -1 & 0 \\ -c_{11} & 0 & 0 & & & & & & \\ -c_{12} & 0 & 0 & & & & & & \\ -c_{13} & 0 & 0 & & & & & & \\ 0 & -c_{44} & 0 & & & \mathbf{O}_{6\times 6} & & & \\ 0 & 0 & -c_{44} & & & & & & \\ -c_{16} & 0 & 0 & & & & & & \end{pmatrix},$$

[14] Э. Дьелесан, Д. Руайе, Упругие волны в твердых телах, М.:Наука, 1982.

$$C = \begin{pmatrix} & \mathbf{O}_{3\times3} & & \begin{matrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{matrix} \\ \begin{matrix} 0 \\ 0 \\ 0 \\ -c_{44} \\ 0 \\ 0 \end{matrix} & \begin{matrix} -c_{12} \\ -c_{11} \\ -c_{13} \\ 0 \\ 0 \\ c_{16} \end{matrix} & \begin{matrix} -c_{16} \\ -c_{16} \\ 0 \\ 0 \\ 0 \\ -c_{66} \end{matrix} & \mathbf{O}_{6\times6} \end{pmatrix},$$

$$D = \begin{pmatrix} & \mathbf{O}_{3\times3} & & \begin{matrix} 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{matrix} \\ \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ -c_{44} \\ 0 \end{matrix} & \begin{matrix} -c_{16} \\ c_{16} \\ 0 \\ 0 \\ 0 \\ -c_{66} \end{matrix} & \begin{matrix} -c_{13} \\ -c_{13} \\ -c_{33} \\ 0 \\ 0 \\ 0 \end{matrix} & \mathbf{O}_{6\times6} \end{pmatrix}.$$

Let us reduce system (47) to the canonical form with respect to the variables t and x_3. As is known from linear algebra, in the case under consideration, there exists a nonsingular matrix Υ. Such conditions can be satisfied, for example, by a matrix:

$$\Upsilon(x_3) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -c_{13}\sqrt{\frac{\rho}{c_{33}}} & 0 & -c_{16}\sqrt{\frac{\rho}{c_{66}}} & 1 & 0 & 1 & c_{16}\sqrt{\frac{\rho}{c_{66}}} & 0 & c_{13}\sqrt{\frac{\rho}{c_{33}}} \\ -c_{13}\sqrt{\frac{\rho}{c_{33}}} & 0 & c_{16}\sqrt{\frac{\rho}{c_{66}}} & 0 & 1 & 0 & -c_{16}\sqrt{\frac{\rho}{c_{66}}} & 0 & c_{13}\sqrt{\frac{\rho}{c_{33}}} \\ -\sqrt{c_{33}\rho} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{c_{33}\rho} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\sqrt{c_{44}\rho} & 0 & 0 & 0 & 0 & 0 & \sqrt{c_{44}\rho} & 0 \\ 0 & 0 & -\sqrt{c_{66}\rho} & 0 & 0 & 0 & \sqrt{c_{66}\rho} & 0 & 0 \end{pmatrix}.$$

We introduce a new function in equation (47) using the equality $\boldsymbol{U} = \Upsilon\boldsymbol{\vartheta}$ and multiply this equation on the left by the matrix $\Upsilon^{-1}A^{-1}$. Then, for the function $\boldsymbol{\vartheta}$, after obvious transformations, we obtain the equation

$$\left(I\frac{\partial}{\partial t} + \Lambda\frac{\partial}{\partial x_3} + B_1\frac{\partial}{\partial x_1} + C_1\frac{\partial}{\partial x_2} + F_1\right)\boldsymbol{\vartheta}(x,t) = \int_0^t R_1(t-\tau, x_3)\boldsymbol{\vartheta}(x,\tau)d\tau\,, (48)$$

Where $\boldsymbol{\vartheta} = (\vartheta_1, \dots, \vartheta_9)^*$,

$$\Lambda = \mathrm{diag}(\nu_i) = \mathrm{diag}\left(-\sqrt{\frac{\rho}{c_{33}}}, -\sqrt{\frac{\rho}{c_{44}}}, -\sqrt{\frac{\rho}{c_{66}}}, 0,0,0, \sqrt{\frac{\rho}{c_{66}}}, \sqrt{\frac{\rho}{c_{44}}}, \sqrt{\frac{\rho}{c_{33}}}\right),$$

$$B_1(x_3) = \Upsilon^{-1}A^{-1}B\Upsilon = \left(b_{ij}\right)_{9\times 9},$$
$$C_1(x_3) = \Upsilon^{-1}A^{-1}C\Upsilon = \left(c_{ij}\right)_{9\times 9},$$
$$F_1(x_3) = \Upsilon^{-1}A^{-1}D\frac{\partial \Upsilon}{\partial x_3} + \Upsilon^{-1}A^{-1}F\Upsilon = \left(p_{\mathrm{ij}}\right)_{9\times 9},$$
$$R_1(x_3,t) = \Upsilon^{-1}A^{-1}R\Upsilon = \left(\tilde{r}_{ij}\right)_{9\times 9}.$$

We consider the system of equations (48) in the domain

$\Omega = \{(x,t): (x_1,x_2) \in \mathbb{R}^2, x_3 \in (0,H), t > 0\}, \ \ H = const,$

with initial

$$\vartheta|_{t=0} = \varphi_i(x), \qquad i = \overline{1,9}, \tag{49}$$

and boundary conditions

$$\vartheta_i|_{x_3=H} = g_i(x_1,x_2,t), \ \ i = \overline{1,3}, \ \ \vartheta_i|_{x_3=0} = g_i(x_1,x_2,t), \ \ i = \overline{7,9}. \tag{50}$$

Here $\varphi_i(x), i = \overline{1,9}, \ g_i(x_1,x_2,t), \ i = 1,2,3,7,8,9$ are given functions.

The *inverse problem* is to determine the nonzero components of the matrix kernel R_1, that is $r_{ij}(t)$, if the following conditions are known:

$$\vartheta_i|_{x_3=0} = h_i(x_1,x_2,t), i = \overline{1,5}, \tag{51}$$

where $h_i(x_1,x_2,t), i = \overline{1,5}$ are the given functions.

Let functions $\varphi_i(x), g_i(x_1,x_2,t)$ and $h_i(x_1,x_2,t)$ have compact support in x_1,x_2 for each fixed x_3,t. Then, by (34), we write the problem (48)-(51) as

$$\left(\frac{\partial}{\partial t} + \nu_j\frac{\partial}{\partial z}\right)\hat{\vartheta}_j(z,t) =$$
$$= \sum_{k=1}^{9} p_{jk}(z)\,\hat{\vartheta}_k(z,t) + \int_0^t \sum_{k=1}^{9} \tilde{r}_{jk}(z,\tau)\hat{\vartheta}_j(z,t-\tau)d\tau\,, \ \ j = \overline{1,9}, \tag{52}$$
$$\hat{\vartheta}_i\big|_{t=0} = \hat{\varphi}_i(z), \qquad i = \overline{1,9}, \tag{53}$$
$$\hat{\vartheta}_i\big|_{z=H} = g_i(t), i = \overline{1,3}, \quad \hat{\vartheta}_i\big|_{z=0} = g_i(z), i = \overline{7,9}, \tag{54}$$
$$\hat{\vartheta}_i\big|_{z=0} = \hat{h}_i(t), \ \ i = \overline{1,5}, \tag{55}$$

where $\hat{\varphi}_i(z)$, $\hat{g}_i(t)$, and $\hat{h}_i(t)$ are the Fourier images of the corresponding functions from $(49)-(51)$ for $\eta_1 = \eta_2 = 0$.

The given functions $\hat{\varphi}_i$ and $\hat{g}_i$ must satisfy the matching conditions at the corner points of the domain Ω, that is

$$\hat{\varphi}_i(0) = \hat{g}_i(H), i = 1,2,3, \qquad \hat{\varphi}_i(0) = \hat{g}_i(0), i = 7,8,9. \tag{56}$$

Let us introduce the vector function $\boldsymbol{\omega}(z,t) = \frac{\partial \hat{\boldsymbol{\vartheta}}}{\partial t}(z,t)$ and we get

$$\left(\frac{\partial}{\partial t} + \nu_i\frac{\partial}{\partial z}\right)\omega_i(z,t) = \sum_{k=1}^{9} p_{ik}(z)\omega_k(z,t) +$$
$$+ \sum_{k=1}^{9} \tilde{r}_{ik}(z,t)\hat{\varphi}_i(z) + \int_0^t \sum_{k=1}^{9} \tilde{r}_{ik}(z,t)\omega_k(z,t-\tau)d\tau, \ \ i = \overline{1,9}, \tag{57}$$

$$\omega_i|_{t=0} = \Phi_i(z) = -\nu_i \frac{d\hat{\varphi}_i(z)}{dz} + \sum_{j=1}^{9} p_{ji}(z)\hat{\varphi}_i(z), \qquad i = \overline{1,9}, \quad (58)$$

$$\omega_i|_{z=H} = \frac{d}{dt}\hat{g}_i(t), i = \overline{1,3}, \quad \omega_i|_{z=0} = \frac{d}{dt}\hat{g}_i(t), i = \overline{7,9}, \qquad (59)$$

$$\omega_i|_{z=0} = \frac{d}{dt}\hat{h}_i(t), \quad i = \overline{1,5}. \qquad (60)$$

Let the following conditions hold

$$\frac{d\hat{g}_i(t)}{dt}\Big|_{t=0} = -\nu_i \frac{d\hat{\varphi}_i(z)}{dz}\Big|_{z=H} + \sum_{j=1}^{9} p_{ij}(H)\hat{\varphi}_j(H), \quad i = \overline{1,3}, \qquad (61)$$

$$\frac{d\hat{g}_i(t)}{dt}\Big|_{t=0} = -\nu_i \frac{d\hat{\varphi}_i(z)}{dz}\Big|_{z=0} + \sum_{j=1}^{9} p_{ij}(0)\hat{\varphi}_j(0), \quad i = \overline{7,9}. \qquad (62)$$

Theorem 5. *Let be* $\{\rho(z), c_{33}(z), c_{44}(z), c_{66}(z), \hat{\varphi}(z)\} \in C^1[0,H]$, $g(t) \in C^1[0,T], r_{ij}(t) \in C^1[0,T]$, $i,j = 1,2,3$ *and conditions (56), (61), (62) are satisfied. Then there is a unique solution to the problem (57)-(59) in the domain* $\Omega_{HT} = \{(z,t): 0 < z < H, 0 < t < T\}$.

In what follows, we will assume that

$$\frac{c_{13}}{\nu_1}\big(\hat{\varphi}_1(0) - \hat{\varphi}_9(0)\big) + \frac{c_{16}}{\nu_3}\big(\hat{\varphi}_3(0) - \hat{\varphi}_7(0)\big) + \hat{\varphi}_4(0) + \hat{\varphi}_6(0) \neq 0,$$
$$\frac{c_{13}}{\nu_1}\big(\hat{\varphi}_9(0) - \hat{\varphi}_1(0)\big) + \frac{c_{16}}{\nu_3}\big(\hat{\varphi}_7(0) - \hat{\varphi}_3(0)\big) + \hat{\varphi}_5(0) \neq 0, \qquad (63)$$
$$\hat{\varphi}_2(0) - \hat{\varphi}_8(0) \neq 0, \qquad \hat{\varphi}_3(0) - \hat{\varphi}_7(0) \neq 0, \qquad \hat{\varphi}_1(0) - \hat{\varphi}_9(0) \neq 0.$$

We require the fulfillment of the matching conditions

$$-\nu_i \frac{d\hat{\varphi}_i(z)}{dz}\Big|_{z=0} + \sum_{j=1}^{9} \hat{p}_{ij}(0)\,\hat{\varphi}_j(0) = \frac{d\hat{h}_i(t)}{dt}\Big|_{t=0}, \qquad i = \overline{1,5}. \qquad (64)$$

Theorem 6. *Let the conditions of Theorem 5 for given functions are satisfaed, besides* $\hat{\varphi}_i(z) \in C^2[0,H]$, $i = \overline{1,9}$, $\hat{g}_i(t) \in C^2[0,\mu_0]$, $i = 1,2,3,7,8,9$, $\hat{h}_i(t) \in C^2[0,\mu_0]$, $i = \overline{1,5}$, *equality (63) and matching condition (64) hold. Then for any* $H > 0$ *on the segment* $[0,\mu_0]$ *there is a unique solution to the inverse problems (57)-(60) from class* $C^1[0,\mu_0]$. *Here* $\mu_i(H) = \int_0^H \frac{1}{\nu_i(s)} ds$, $\mu_0 = \max\{\mu_1(H), \mu_2(H), \mu_3(H)\}$.

CONCLUSION

The thesis examines initial-boundary problems for systems of hyperbolic viscoelasticity integro-differential equations and inverse problems for determining the kernel of the integral term. In order to solve the inverse problems, they are reduced into the system of integral equations of the Volterra second type. Inverse problems are considered for the hyperbolic system of integro-differential equations with the integral operator on the right side of the winding view.

During the research presented in the dissertation, the following results were obtained:

a method has been developed for determining the one-dimensional kernel of the equation of viscoelasticity in the space $\mathbb{R}^2$ for concentrated sources of disturbances in a vertically inhomogeneous medium. Conditions for global unique solvability have been obtained;

theorems for the global unique solvability for the initial-boundary problem of the viscoelasticity equation in the space $\mathbb{R}^3$ were proved;

based on the principle of contraction mappings in the class of continuous functions with exponential weight, a theorem on the global unique solvability of the inverse problem of determining memory from a system of three-dimensional integro-differential equations of anisotropic viscoelasticity in normal form is proved.

The conditions for unique solvability of the one-dimensional problem of determining elastic moduli in the system of equations of anisotropic viscoelasticity with a known matrix memory kernel are determined.

These results made it possible to solve the problem of global solvability of one-dimensional problems of determining the kernels of integral convolution operators in equations of hyperbolic type, when the nonlinearity in inverse problems is of a convolutional nature.

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БОЛТАЕВ АСЛИДДИН АСКАР УГЛИ

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования настоящей диссертационной работы является построение методов решений обратных задач по определению ядер интегральных членов в двумерной и трехмерной системах интегро-дифференциальных уравнений вязкоупругости, а также исследование единственности и существования решений этих обратных задач.

Объектом исследования является одномерные обратные задачи для системы интегро-дифференциальных уравнений гиперболического типа первого порядка.

Научная новизна исследования состоит из следующих:

доказаны теоремы существования и единственности решении обратной задач для двумерной системы уравнений вязкоупругости;

доказаны теоремы существования и единственности решения начально-граничная задачи для вязкоупругой системы в вертикально-слоистой среде;

доказана теорема о глобальной однозначной разрешимости обратной задачи определения памяти по системе вязкоупругости в изотропных средах;

доказано существование единственного глобального решения обратной задачи определения диагонального матричного ядра из системы вязкоупругости в анизотропных средах с тетрагональной формой модуля упругости.

Внедрение результатов исследования. Полученные в диссертации результаты решения обратных задач для системы гиперболических уравнений с памятью первого порядка были использованы в следующих научно-исследовательских проектах:

исследование задач определения ядер для системы интегро-дифференциальных уравнений вязкоупругости проекте «Математическое моделирование в социологии, геофизике и инженерных науках» регистрационный номер AAAA-A19-119032590069-3 (справка № 81 от «31» октября 2023 года, Южного Математического Института филиала ФГНБУ ФНЦ Владикавказский научный центр РАН). Применение научного результата позволило определить ядро свёртки из системы вязкоупругих уравнений в вертикально-слоистых изотропных средах и доказать его глобальное существование и единственность;

определение вектора смещения для системы уравнений вязкоупругости в изотропной среде, были использованы в проекте “Природные катастрофы Камчатки-землетрясения и извержение вулканов”, регистрационный номер ОТ-Ф4-01 (справка № AAAA-A19-119072290002-9 от «13» ноября 2023 года, Камчатский государственный университет имени Витуса Беринга, РАН). Применение научных результатов позволило доказать существование и единственность задачи определения ядра;

Структура и объем диссертации. Диссертационная работа состоит из введения, трех глав, заключения, списка использованной литературы. Общий объем диссертации составляет 95 страницы.

E’LON QILINGAN ISHLAR RO‘YXATI
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I bo‘lim (Part I; Часть I)

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