V.I.ROMONOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

O‘ZBEKISTON MILLIY UNIVERSITETI

DEXQONOV FARRUXJON NURIDDIN O‘G‘LI

ISSIQLIK ALMASHINISH JARAYONINI CHEGARAVIY BOSHQARISH

01.01.02 – Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI bo‘yicha falsafa doktori (PhD) dissertatsiyasi AVTOREFERATI

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Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

V.I.ROMONOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

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Fizika-matematika fanlari bo‘yicha falsafa doktori (Doctor of Philosophy) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Vazirlar Mahkamasi huzuridagi Oliy attestatsiya komissiyasida № B2022.1.PhD/FM669 raqam bilan ro‘yxatga olingan.

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Dissertatsiya himoyasi V.I.Romanovskiy nomidagi Matematika instituti huzuridagi DSc.02/30.12.2019.FM.86.01 raqamli Ilmiy kengashning 2022 yil « 28 » iyun soat 16:00 dagi majlisida bo‘lib o‘tadi. (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko‘chasi, 9- uy. Tel.: (+99871) 207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zaruriyati. Jahon miqyosida olib borilayotgan ko‘plab ilmiy-amaliy tadqiqotlar ba’zi hollarda matematik fizika tenglamalari bilan tavsiflanadigan boshqarish masalalariga keltiriladi. Fizika va texnikadagi ayrim masalalarning matematik modellari parametrning ma’lum qiymatlarini olish uchun zarur bo‘lgan boshqaruv jarayonlarni tavsiflaydi. Masalan, issiqlik almashinish jarayonini boshqarish yoki to‘lqin tarqalish jarayonlarini boshqarish masalasi. To‘lqin tarqalish jarayonlarida, berilgan sohaning qaysidir qismida ma'lum bir to‘lqin shakliga erishish uchun soha chegarasining bir qismida to‘lqin parametrini nazorat qilish kerak. Issiqlik almashinish jarayonida sohaning ayrim qismida berilgan o‘rtacha haroratga erishish uchun harorat parametrini soha chegarasining bir qismida nazorat qilish kerak. Shuning uchun chegaraviy boshqaruv masalalari ham nazariy, ham amaliy jihatdan dolzarb bo‘lib, zamonaviy matematikaning dolzarb yo‘nalishlaridan biri hisoblanadi.

Hozirgi kunda matematik fizikaning dolzarb muammolaridan biri chegaraviy boshqaruv masalasi hisoblanadi. Chegaraviy masalalarni o‘rganish matematik fizikaning teskari va nokorrekt qo‘yilgan muammolari bilan chambarchas bog‘liq, shuning uchun, ular Fredgolm yoki Volterra integral tenglamalarini o‘rganishga olib kelinadi. So‘nggi paytlarda issiqlik tarqalish jarayonlarini chegaraviy boshqarish masalalari jadal rivojlanmoqda, chunki ular amaliyotga tatbiqi bilan chambarchas bog‘liq. Bunday masalalar ixtiyoriy chegaralgan sohada parabolik tipdagi tenglamalarga qo‘yilgan boshqaruv masalalarini yechishga keltiriladi. Bu borada: issiqlik almashinish jarayonlarini boshqarishning chegaraviy masalalar va qo‘shimcha integral shartli boshlang‘ich-chegaraviy masalalarni yechish ilmiy tadqiqotlardan hisoblanadi.

Mamlakatimizdagi fundamental fanlarning ilmiy va amaliy tatbiqiga ega bo‘lgan matematika, fizika va optimal boshqruv yo‘nalishlariga e’tibor kuchaytirildi. Jumladan, o‘yinlar nazaryasi va optimal boshqaruv masalalarini tadqiq qilishga alohida e’tibor qaratildi. Oddiy differensial tenglamalar, xususiy hosilali differensial tenglamalar va ularning sistemalari, oddiy differensial tenglamalar va ularning sistemalari orqali tavsiflanadigan jarayonlarni boshqarish nazariyasiga doir ajoyib natijalarga erishildi. “Funksional analiz, differensial tenglamalar va matematik fizika fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish asosiy vazifalar va faoliyat yo‘nalishlari” etib belgilandi[1]. Qaror ijrosini ta’minlashda differensial tenglamalar va matematik fizika masalalarini hal qilishda chegaraviy boshqaruv masalalarini rivojlantirish muhim ahamiyatga ega.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi PF-4947-son Farmoni, 2019 yil 9 iyuldagi “Matematika ta’limi va fanlarni yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston

[1] O‘zbekiston Respublikasi Vazirlar Maxkamasining 2017-yil 18-maydagi “O‘zbekiston Respublikasi Fanlar Akademiyasining yangidan tashkil etilgan ilmiy tadqiqot muassasalari faoliyatini tashkil etish to‘g‘risida”gi 292-sonli qarori.

Respublikasi Fanlar akademiyasining V.I. Romonovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi PQ-4387-son Qarori va 2020-yil 7-maydagi “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora tadbirlari to‘g‘risida”gi PQ-4708-sonli Qarori hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnalogiyalar rivojlanishining ustuvor yo‘nalishlariga bog‘liqligi. Ushbu tadqiqot O‘zbekiston Respublikasi fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarildi.

Muammoning o‘rganilganlik darajasi. Boshqaruv masalalarining matematik nazariyasi o‘tgan asrning o‘rtalarida jadal rivojlana boshlagan. Oddiy differensial tenglamalar va ularning sistemalari uchun boshqaruv masalalarining matematik nazariyasiga L.S. Pontryagin, V.G. Boltyanskiy, R.V. Gamkrelidze, E.F. Mishchenko R. Bellmanlarning ilmiy ishlarida asos solingan va M.I. Zelikin, V.M. Tixomirovlar tomonidan rivojlantirilgan. Taxminan shu davrda kosmonavtika ehtiyoji bilan AQShda R. Bellman, La-Caol, Dj. Leytman kabi matematiklar tomonidan optimal boshqaruv masalalariga variatsion hisob, xususan, Gamilton-Yakobi usulini qo‘llashga asoslangan yondashuv rivojlantirildi. Oddiy differensial tenglamalar va ularning sistemalarini boshqarish masalalarining matematik nazariyasi bo‘yicha N.N. Krasovskiy asos solgan Yekaterinburg ilmiy maktabi vakillari A.B. Kurjanskiy, Yu.S.Osipov, A.I. Subbotin, A.V. Kryajimskiy, N.Yu. Lukoyanov, N.N. Subbotina, V.N. Ushakov va boshqalar tomonidan keng ko‘lamda tadqiqotlar olib borildi. 1970-yilda N.Yu. Satimov Toshkentda optimal boshqaruv va differensial o‘yinlar nazaryasi sohasida ilmiy maktabga asos soldi. Dastlab bu maktab vakillari asosan quvish va qochish masalalari bilan shug‘ullangan bo‘lsa, XX asr oxirlaridan, xususiy hosilali differensial tenglamalarda boshqaruv masalalari bilan N.Yu. Satimov, A. Azamov, M. To‘xtasinov, M. Mamatov va boshqalar shug‘ullandilar. Xususan, A. Azamov, M. Ruziboyev va boshqalar tomonidan boshqariluvchi evalyutsion sistemalarda F.L. Cherniusko taklif etgan boshqaruv funksiyasini qurish usuli rivojlantirilmoqda.

Parabolik tipdagi xususiy hosilali differensial tenglamalar uchun vaqtni optimal boshqaruv masalasi birinchi bo‘lib H.O. Fattorini va A. Friedmanlar tomonidan tadqiq qilingan. Optimal boshqarish muammolari haqida ko‘plab ma’lumotlar A.V. Fursikov va J.L. Lions monografiyalarida batafsil berilgan. Shuningdek xususiy hosilali differensial tenglamalar bilan tavsiflanadigan jarayonlar bilan bog‘liq boshqaruv masalalari K.A. Lur’e, V.A. Il’in va E.I. Moiseyev, A.G. Butkovskiy, A.I. Yegorov, E.S. Efimova, V.I. Maksimov, N. Chen, Y. Wang, D. Yang, E. Fernandez-Cara, E. Zuazua va boshqalar tomonidan o‘rganilgan. Mazkur dissertatsiyada o‘rganilgan parabolik tipdagi tenglamalar uchun boshqaruv masalalari bilan bog‘liq so‘nggi ilmiy natijalar Sh.A. Alimov, S. Albeverio, Z.K. Fayazova va boshqalar tomonidan tadqiq qilingan.

Giperbolik tipdagi tenglamalar va ularning sistemalari orqali tavsiflanadigan jarayonlarni chegaraviy boshqarish masalalari V.A.Il'in va E.I. Moiseyevlarning ishlarida keltirilgan. Parabolik va giperbolik tipdagi tenglamalar uchun turli xil cheklovlar ostida boshqaruv masalalari Y. Luo, I. Lasieska, R. Triggiani, V. Barbu, A. Rascanu, G. Tessitore, M. Gueye, W. Littman , Sule S. Sener, Murat Sulasi, P. Cannarsa, P. Martinez, J. Vancostenoble va boshqalar tomonidan tadqiq qilingan. Optimal boshqaruv masalalarini tatbiqlari A. Altmyuller va L. Grünelarning ishlarida uchraydi. Issiqlik tarqalish tenglamasini optimal boshqaruvining amaliy masalalari S. Dubljevic va P.D. Christofideslarning ishlarida keltirilgan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta'lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog'liqligi. Dissertatsiya tadqiqoti O'zbekiston Milliy universiteti ilmiy tadqiqot ishlari va V.I. Romanovskiy nomidagi matematika instituti F-FA-2021-424–sonli "Butun va kasr tartibli xususiy hosilali differensial tenglamalar uchun chegaraviy masalalarning yechimi" mavzusidagi loyihasi doirasida bajarilgan.

Tadqiqotning maqsadi tekislik va fazoda berilgan sohada jismni o'rtacha haroratgacha isitish uchun ketadigan minimal vaqtga baho topish va asosiy integral tenglamani yechimi mavjudligini ko'rsatishdan iborat.

Tadqiqotning vazifalari:

sterjenni tez isitish uchun zarur bo'lgan minimal vaqtga baho olish hamda asosiy integral tenglamaning yechimi mavjudligini isbotlash;

berilgan sohaning qismidagi o'rtacha haroratga erishish uchun ketadigan minimal vaqtga baho topish;

n-o'lchamli chegaralangan sohada jismni isitish uchun ketadigan minimal vaqtga baho topish va mos integral tenglamaning yechimi mavjudligi isbotlash;

uch o'lchamli sohada o'rtacha temperatura kritik qiymatga yaqin bo'lganda, minimal vaqt harorat jarayoni parametrlariga bog'liq ekanligini isbotlash.

Tadqiqotning ob'ekti. Parabolik tipdagi tenglamalar, birinchi va ikkinchi turdagi Volterra integral tenglamalari.

Tadqiqotning predmeti. Parabolik tipdagi tenglamalar uchun qo'shimcha integral shartlar bilan berilgan chegaraviy boshqaruv masalalari.

Tadqiqotning usullari. Dissertatsiya ishida matematik fizikaning zamonaviy usullari, integral tenglamalar, Laplas almashtirishi va Furye qatorlari nazariyalaridan keng foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

intervalda parabolik tenglama uchun boshlang'ich-chegaraviy masalada sterjenni tez isitish uchun zarur bo'lgan minimal vaqtga baho olingan hamda asosiy integral tenglamaning yechimi mavjudligi isbotlangan;

berilgan sohaning qismidagi o'rtacha haroratga erishish uchun ketadigan minimal vaqtni yuqori chegarasi topilib, asosiy integral tenglamaning yechimi mavjudligi Laplas almashtirishi usuli orqali isbotlangan;

uch o'lchamli sohada o'rtacha temperatura kritik qiymatga yaqin bo'lganda, minimal vaqt harorat jarayoni parametrlariga bog'liq ekanligi isbotlangan;

n-o‘lchamli chegaralangan sohada issiqlik almashinish tenglamasi uchun boshlang‘ich-chegaraviy masalada berilgan jismni isitish uchun ketadigan minimal vaqtga baho topilgan.

Tadqiqotning amaliy natijasi dissertatsiya ishida olingan natijalar va qo‘llanilgan usullar oliy o‘quv yurtlari magistrantlari va doktorantlari uchun magistratura bosqichi sifatida o‘qitilish va parabolik tipdagi tenglama uchun issiqlik almashinish jarayonlarini chegaraviy boshqarish bo‘yicha olingan natijalar boshqa turdagi tenglamalar uchun qo‘llash mumkinligidan iborat.

Tadqiqot natijalarining ishonchliligi. Natijalar matematik fizikaning zamonaviy usullari, matematik tahlil usullaridan foydalanilgan holda olingan. Olingan natijalarning isbotlari matematik jihatdan to‘g‘ri.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarning ilmiy ahamiyati shundan iboratki, masalani yechishda bir, ikki va uch o‘lchamli sohada Dirixle yoki Neyman shartlari bilan berilgan parabolik tenglamalar uchun chegaraviy boshqaruv masalalari yechimini topish usulini va berilgan sohada o‘rtacha haroratga erishish uchun ketadigan minimal vaqtga baholar olishda foydalanilganligi bilan izohlanadi.

Tadqiqotning amaliy ahamiyati esa magistrlarni tayyorlashda hamda berilgan jismni o‘rtacha temperaturagacha isitish uchun ketadigan minimal vaqtni topish masalasini matematik modellashtirishda qo‘llash imkoni bilan belgilaniladi.

Tadqiqot natijalarining joriy qilinishi. Issiqlik almashinish jarayonini chegaraviy boshqarish masalasi bo‘yicha olingan natijalar asosida:

Volterra birinchi tur integral tenglamsini yadrosiga va bu yadroning Laplas almashtirishiga olingan baholardan OT-F-4-(36+32) raqamli “Matematik fizika va optimal boshqaruv masalalarini yechishning yangi usullarini ishlab chiqish+toq tartibli xususiy hosilali tenglamalar uchun noklassik boshlang‘ich, spektral masalalar va ularning tatbiqlari” mavzusidagi fundamental loyihada Volterra birinchi tur integral tenglamasini yechimi mavjudligini ko‘rsatishda foydalanilgan (O‘zbekiston Milliy universitetining 2022-yil 20-apreldagi №04/11-2261-sonli ma’lumotnomasi). Ilmiy natijalarning qo‘llanishi bir va ikki o‘lchamli sohada parabolik tenglama uchun qo‘yilgan boshlang‘ich-chegaraviy masalaning umumlashgan yechimini topish imkonini bergan;

chegarasi bo‘lakli silliq bo‘lgan chegaralangan ikki va *n*-o‘lchamli sohalarda parabolik tipdagi tenglamalar uchun boshlang‘ich-chegaraviy masalalarning umumlashgan yechimidan OT-F4-02 raqamli “Matematik fizikaning holatlar to‘plami cheksiz bo‘lgan modellari termodinamikasi” mavzusidagi fundamental loyihada Grin funksiyasi xossalari isbotlanishida foydalanilgan (Buxoro davlat universiteti 2022-yil 13-maydagi №04-04/01-0963-sonli ma’lumotnomasi). Ilmiy natijalarning qo‘llanishi fundamental loyiha ishida matematik fizika tenglamalari uchun qo‘yilgan aralash tipdagi masalalarning umumlashgan yechimni topish imkonini bergan.

Tadqiqot natijalarini aprobatsiyasi. Mazkur tadqiqot natijalari 4 ta xalqaro va 3 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e'lon qilinganligi. Dissirtatsiya mavzusi bo'yicha jami 13 ta ilmiy ish chop etilgan, shulardan, O'zbekiston Respublikasi Oliy attestatsiya komissiyasining falsafa doktori dissertatsiyalarini asosiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 6 ta maqola, jumladan, 1 tasi xorijiy va 5 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzulishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan tashkil topgan. Dissertatsiyaning hajmi 86 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirishda dissertatsiya mavzusining dolzarbligi va zarurligi asoslangan, tadqiqotning respublika fan va texnologiyasini rivijlantirishning ustuvor yo'nalishlariga mosligi ko'rsatilgan, xorijiy va mahalliy ilmiy tadqiqotlar haqida umumiy ma'lumot berilgan, muammoning o'rganilganlik darajasi, maqsad va vazifalari belgilangan, tadqiqotning ob'ekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari ochib berilgan. Olingan natijalarning nazariy va amaliy ahamiyati bayon etilgan, ilmiy-tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzulishi bo'yicha ma'lumotlar keltirilgan.

Dissertatsiyaning **"Sterjenni tez isitish bilan bog'liq boshqaruv masalasi"** deb nomlangan birinchi bobida parabolik tipdagi tenglama uchun intervalda boshlang'ich-chegaraviy masala qaralgan. Birinchi paragrafda issiqlik tarqalish tenglamasi uchun birinchi tur boshlang'ich-chegaraviy masalani yechimi Furye usuli bilan topilgan.

Ikkinchi paragrafda parabolik tenglama bilan bog'liq chegaraviy nazorat masalasini ko'rib chiqiladi. Sterjenni berilgan o'rtacha temperaturaga erishish uchun ketadigan minimal vaqtga baho olinadi va asosiy integral tenglamani yechimi mavjudligi ko'rsatiladi.

Biz $\Omega = \{x \in R:\ 0 < x < l\}$ sohada

$$u_t(x,t) = u_{xx}(x,t), \quad x \in (0,l), \quad t > 0, \tag{1}$$

bir jinsli issiqlik tarqalish tenglamasini

$$u(0,t) = \mu(t), \quad u(l,t) = 0, \quad t > 0, \tag{2}$$

chegaraviy shartlar va

$$u(x,0) = 0, \quad 0 \le x \le l, \tag{3}$$

boshlang'ich shart bilan ko'rib chiqamiz.

Aytaylik, $M > 0$ o'zgarmas son berilgan bo'lsin. Agar $\mu(t)$ funksiya $t \ge 0$ da differensiallanuvchi va

$$\mu(0) = 0, \quad |\mu(t)| \le M$$

shartlarni qanoatlantirsa, bu funksiya *joiz boshqaruv* deyiladi.

Biz $[0,l]$ segmentda differensiallanuvchi va

$$\rho(x) \ge 0, \quad \rho'(x) \le 0, \quad \int_0^l \rho(x)dx = 1,$$

shartlarni qanoatlantiradigan $\rho(x)$ vazn funksiyasini kiritamiz.

1-masala. Aytaylik, $\theta > 0$ o‘zgarmas son berilgan bo‘lsin. $T > 0$ ning shunday minimal qiymatini topingki, $t > 0$ uchun (1)-(3) boshlang‘ich-chegaraviy masalani $u(x,t)$ yechimi hamda $\mu(t)$ boshqaruv funksiyasi mavjud va quyidagi

$$\int_0^l \rho(x)u(x,t)dx = \theta,\ t \ge T,$$

tenglamani qanoatlantirsin.

1-teorema. Berilgan θ uchun

$$0 < \theta < \frac{M \cdot l \cdot \rho_1}{\pi}$$

shart o‘rinli va

$$T_0 = -\left(\frac{l}{\pi}\right)^2 \ln\left(1 - \frac{\theta \cdot \pi}{\rho_1 \cdot l \cdot M}\right)$$

berilgan bo‘lsin. U holda 1-masalaning $T_{\min}$ yechimi mavjud va $T_{\min} \le T_0$ baho o‘rinli bo‘ladi.

1-ta’rif. Berilgan (1) - (3) masalaning yechimi deganda

$$u(x,t) = \frac{l-x}{l}\mu(t) + \vartheta(x,t)$$

ko‘rinishda ifodalangan $u(x,t)$ funksiyani tushunamiz, bu yerda $\vartheta(x,t)$ funksiya $C\left([0,T] \to \tilde{W}_2^1\right)$ sinfdan olingan quyidagi aralash masalaning umumlashgan yechimi:

$$\vartheta_t(x,t) - \Delta\vartheta(x,y,t) = -\frac{l-x}{l}\mu'(t),$$

$$\vartheta(x,t)\big|_{\partial\Omega} = 0, \qquad \vartheta(x,0) = 0, \quad 0 \le x \le l.$$

1-lemma. Aytaylik, $g(y)$ funksiya $y \in [0,\infty)$ da nomanfiy kamayuvchi funksiya bo‘lsin. U holda quyidagi tengsizlik o‘rinli:

$$\int_0^{n\pi} g(y)\sin y dy \ge 0,\ n = 1,2,3,....$$

Quyidagi belgilashlarni kiritamiz:

$$Q(t) = \int_0^t K(s)ds, \qquad Q^* = \lim_{t\to\infty} Q(t) = \int_0^\infty K(s)ds.$$

1-tasdiq. Aytaylik, $0 < \theta < MQ^*$ shart o‘rinli bo‘lsin. U holda $T > 0$ va haqiqiy qiymatli $\mu(t)$ o‘lchamli funksiya mavjud va quyidagi

$$\int_0^T K(T-s)\mu(s)ds = \theta \tag{4}$$

tenglik o‘rinli bo‘ladi.

2-tasdiq. Aytaylik,

$$0 < \theta < \frac{M \cdot l \cdot \rho_1}{\pi}$$

shart o‘rinli bo‘lsin. U holda shunday $T > 0$ mavjudki, quyidagi

$$T < -\left(\frac{l}{\pi}\right)^2 \ln\left(1 - \frac{\theta \cdot \pi}{\rho_1 \cdot l \cdot M}\right)$$

baho va (4) tenglik o‘rinli bo‘ladi.

Dissertatsiyaning **“Ikki o‘lchamli sohada isitish jarayoni bilan bog‘liq boshqaruv masalasi”** deb nomlangan ikkinchi bobida to‘g‘ri to‘rtburchak sohada issiqlik tarqalish tenglamasi uchun boshlang‘ich-chegaraviy masala tadqiq qilingan. To‘rtburchakning qismi bo‘lgan sohadagi o‘rtacha temperaturaga erishish uchun ketadigan minimal vaqtga baho topilgan.

Birinchi paragrafda berilgan chekli sohada boshlang‘ich-chegaraviy masala ko‘rib chiqilgan va masalaning yechimi Furye usulida topilgan. To‘g‘ri to‘rtburchakli sohada o‘rtacha temperaturagacha erishish uchun ketadigan minimal vaqt topish masalasi qo‘yilgan.

Berilgan chekli $\Omega = \{(x,y) \in R^2 : 0 < x < a,\ 0 < y < b\}$ sohada

$$u_t(x,y,t) = u_{xx}(x,y,t) + u_{yy}(x,y,t),\ \ (x,y) \in \Omega,\ \ t > 0, \tag{5}$$

bir jinsli issiqlik tarqalish tenglamasini

$$u(0,y,t) = \varphi(y)\mu(t),\ \ u(a,y,t) = \psi(y)\mu(t), \tag{6}$$

$$u(x,0,t) = u(x,b,t) = 0, \tag{7}$$

chegaraviy shartlar va

$$u(x,y,0) = 0,\ \ 0 \le x \le a,\ \ 0 \le y \le b, \tag{8}$$

boshlang‘ich shart bilan qaraymiz, bu yerda

$$\mu(0) = 0,\ \ \varphi(0) = \varphi(b) = 0,\ \ \ \psi(0) = \psi(b) = 0.$$

2-masala. Aytaylik, $\alpha, \beta \ge 1$ va $\theta > 0$ o‘zgarmas sonlar berilgan bo‘lsin. $T > 0$ning shunday minimal qiymatini topingki, $t > 0$ uchun (5)-(8) boshlang‘ich-chegaraviy masalani $u(x,y,t)$ yechimi hamda $\mu(t)$ joiz boshqaruv mavjud va ba’zi $T_1 > T$ uchun quyidagi tenglikni qanoatlantirsin:

$$\int_0^{b/\alpha} \int_0^{a/\beta} u(x,y,t)\,dxdy = \theta,\ \ \ T \le t \le T_1.$$

Quyidagi belgilashlarni kiritamiz:

$$\rho_{nm} = \frac{8b}{a\pi m}(\varphi_m - (-1)^n \psi_m)\sin^2\frac{n\pi}{2\beta}\sin^2\frac{m\pi}{2\alpha},\ \ \ n,m = 1,2,3,... \tag{9}$$

va $\gamma = \dfrac{\pi^2(a^2+b^2)}{a^2b^2}$.

2-teorema. Berilgan θ uchun

$$0 < \theta < \frac{\rho_{11} M}{\gamma}$$

shart o‘rinli va

$$T_0 = -\frac{1}{\gamma}\ln\left(1-\frac{\theta\gamma}{\rho_{11}M}\right)$$

berilgan bo‘lsin. U holda 2-masalaning T_{min} yechimi mavjud va $T_{min} \le T_0$ baho o‘rinli bo‘ladi.

Ikkinchi paragrafda birinchi paragrafda keltirilgan minimal vaqt masalasini yechish uchun yordamchi masalani yechish usuli qurilgan.

Uchinchi paragrafda biz birinchi paragrafda keltirilgan asosiy masalaning yechimini topish bilan shug‘ullanilgan. Bunda asosan ikkinchi paragrafda keltirilgan yordamchi masalani yechimini topishda qo‘llanilgan usullardan foydalaniladi. Ya’ni to‘rtburchakning ikki yon chetidan issiqlik oqimi berilganda, minimal vaqtga baho olingan va mos integral tenglamani yechimi mavjudligi isbotlangan.

2-ta’rif. Berilgan (5) - (8) masalaning yechimi deganda biz

$$u(x,y,t) = \mu(t)(\psi(y) + \frac{a-x}{a}(\varphi(y)-\psi(y))) - \vartheta(x,y,t)$$

ko‘rinishda ifodalangan $u(x,y,t)$ funksiyani tushunamiz, bu yerda $\vartheta(x,y,t)$ funksiya $C\left([0,T]\to\tilde{W}_2^1\right)$ sinfdan olingan quyidagi aralash masalaning umumlashgan yechimi:

$$\vartheta_t(x,y,t) - \Delta\vartheta(x,y,t) =$$

$$= \mu'(t)(\psi(y) + \frac{a-x}{a}(\varphi(y)-\psi(y))) - \mu(t)\cdot[\psi''(y) + \frac{a-x}{a}(\varphi''(y)-\psi''(y))]$$

$$\vartheta(x,y,t)|_{\partial\Omega} = 0, \quad \vartheta(x,y,0) = 0, \quad 0 \le x \le a, \quad 0 \le y \le b.$$

Ushbu

$$B(t) = \sum_{n,m=1}^{\infty} \rho_{nm} e^{-\lambda_{nm}t} \tag{10}$$

belgilashni kiritamiz, bu yerda ρ_{nm} (9) tenglik bilan ifodalaniladi.

Quyidagi asosiy integral tenglamani qaraymiz:

$$\int_0^t B(t-s)\mu(s)ds = f(t), \ t > 0. \tag{11}$$

Biz

$$\| f \|^2_{W_2^2(R_+)} \le M_0$$

shartni qanoatlantiradigan va $t \le 0$ uchun $f(t) = 0$ bo‘ladigan $f \in W_2^2(-\infty,+\infty)$ funksiyalar to‘plamini $W\left(M_0\right)$ orqali belgilaymiz.

2-lemma. Shunday $M_0 > 0$ mavjudki, har qanday $f \in W\left(M_0\right)$ funksiya uchun (11) tenglamaning $\mu(t)$ yechimi mavjud va

$$\left|\mu(t)\right| \le M$$

baho o‘rinli bo‘ladi.

Quyidagi belgilashlarni kiritamiz:

$$Q(t)=\int_0^t B(s)ds,\quad Q^*=\lim_{t\to\infty}Q(t)=\int_0^\infty B(s)ds.$$

3-tasdiq. Aytaylik, $0<\theta<MQ^*$ shart o‘rinli bo‘lsin. U holda $T>0$ va haqiqiy qiymatli $\mu(t)$ o‘lchovli funksiya mavjud va quyidagi tenglik o‘rinli:

$$\int_0^T B(T-s)\mu(s)ds=\theta. \tag{12}$$

4-tasdiq. Aytaylik,

$$0<\theta<\frac{\rho_{11}M}{\gamma}$$

shart o‘rinli bo‘lsin. U holda shunday $T>0$ mavjudki, ushbu

$$T<-\frac{1}{\gamma}\ln\left(1-\frac{\theta\gamma}{\rho_{11}M}\right)$$

baho va (12) tenglik o‘rinli bo‘ladi.

Dissertatsiyaning **“Chegaralangan sohada issiqlik almashinish jarayonini chegaraviy boshqarish masalasi”** deb nomlangan uchinchi bobida biz chegaralangan sohada issiqlik o‘tkazuvchanlik tenglamasi uchun boshlang‘ich-chegaraviy masalani tadqiq qilingan. Boshqaruv parametri issiq yoki sovuq havoning chiqishi kattaligiga teng va chegaraning ma’lum bir qismida aniqlangan.

Birinchi paragrafda *n*-o‘lchamli chegaralangan sohada issiqlik almashinish tenglamasi uchun boshlang‘ich-chegaraviy masala ko‘rib chiqildi. Berilgan jismni isitish uchun ketadigan minimal vaqtga baho topiladi va mos integral tenglamaning yechimi mavjudligi isbotlangan.

Biz $\partial\Omega$ chegarasi bo‘lakli silliq bo‘lgan chegaralangan $\Omega\subset\mathbb{R}^n$ sohada

$$u_t(x,t)=\Delta u(x,t),\quad x\in\Omega,\quad t>0, \tag{13}$$

issiqlik tarqalish tenglamasini ushbu

$$\frac{\partial u}{\partial n}+h(x)u(x,t)=0,\quad x\in\partial\Omega\setminus\Gamma,\quad t>0, \tag{14}$$

$$\frac{\partial u}{\partial n}=a(x)\mu(t),\quad x\in\Gamma,\quad t>0, \tag{15}$$

chegaraviy shartlar va

$$u(x,0)=0, \tag{16}$$

boshlang‘ich shart bilan qaraymiz.

Biz $x\in\Gamma$ uchun $h(x)=0$ va $x\notin\Gamma$ uchun $a(x)=0$ larni kiritish orqali ikkala $h(x)$ va $a(x)$ funksiyalarni butun chegara $\partial\Omega$ ga kengaytira olamiz. Bu holda (14) va (15) shartlarni

$$\frac{\partial u(x,t)}{\partial n}+h(x)u(x,t)=a(x)\mu(t),\quad x\in\partial\Omega,\quad t>0, \tag{17}$$

ko‘rinishda yozib olamiz.

Aytaylik, $M>0$ o‘zgarmas son berilgan bo‘lsin. Agar $\mu(t)$ funksiya $t\ge 0$ da o‘lchovli va ushbu

$$|\mu(t)|\le M$$

shartni qanoatlantirsa bu funksiya *joiz boshqaruv* deyiladi.

Aytaylik $\rho:\overline{\Omega}\to R$ uzluksiz funksiya quyidagi shartlarni qanoatlantirsin:

$$\int_\Omega \rho(x)dx=1,\quad \rho(x)>0.$$

3-masala. Aytaylik, $\theta>0$ o‘zgarmas son berilgan bo‘lsin. $T>0$ ning shunday minimal qiymatini topingki, $t>0$ uchun (13)-(16) boshlang‘ich-chegaraviy masalani $u(x,t)$ yechimi hamda $\mu(t)$ joiz boshqaruv mavjud va quyidagi

$$\int_\Omega \rho(x)u(x,t)dx=\theta,\ t\ge T, \tag{18}$$

tenglikni qanoatlantirsin.

Quyidagi belgilashlarni kiritamiz:

$$d_0=\min_{x\in\Omega\cup\partial\Omega}\rho(x) \tag{19}$$

va

$$B_k=\frac{1}{\Lambda_k}\int_{\partial\Omega}\vartheta_k(x)a(x)d\sigma(x),\ \text{bu yerda}\ \ \Lambda_k=\|\vartheta_k(x)\|_{C(\bar\Omega)}. \tag{20}$$

3-teorema. Aytaylik, θ uchun

$$0<\theta<\frac{d_0B_1M}{\lambda_1}$$

shart o‘rinli va

$$T_0=-\frac{1}{\lambda_1}\ln\left(1-\frac{\theta\lambda_1}{d_0B_1M}\right)$$

berilgan bo‘lsin. U holda 3-masalaning T_{min} yechimi mavjud va $T_{min}\le T_0$ baho o‘rinli bo‘ladi.

Quyidagi Grin funksiyasini kiritamiz:

$$G(x,y,t)=\sum_{k=1}^{\infty}e^{-\lambda_k t}\vartheta_k(x)\vartheta_k(y),\ x\in\Omega,\ y\in\Omega,\ t>0.$$

5-tasdiq. Aytaylik barcha $x\in\partial\Omega$ da $h(x)\ge 0$ bo‘lsin. U holda (13)-(16) boshlang‘ich-chegaraviy masalaning yechimi bo‘lgan Grin funksiyasi nomanfiy bo‘ladi.

Quyidagi belgilashni kiritamiz:

$$H(x,t)=\int_\Omega\rho(y)G(x,y,t)dy,\ x\in\Omega,\ t>0. \tag{21}$$

6-tasdiq. Aytaylik, Λ_1 mavjud bo‘lsin. U holda quyidagi baho o‘rinli:

$$H(x,t)\ge\frac{d_0}{\Lambda_1}e^{-\lambda_1 t}\vartheta_1(x).$$

Quyidagi belgilashni kiritamiz:

$$R(t)=\int\limits_{\partial\Omega}H(y,t)a(y)d\sigma(y). \tag{22}$$

7-tasdiq. Ushbu

$$R(t)\ge d_0B_1e^{-\lambda_1 t}$$

baho o‘rinli, bu yerda B_1 koeffitsient (21) orqali aniqlanadi.

Ma’lumki (13) + (15) + (17) boshlang‘ich-chegaraviy masalaning yechimi Grin funksiyasi orqali quyidagicha topialdi:

$$u(x,t)=\int\limits_0^t\mu(s)ds\int\limits_{\partial\Omega}G(x,y,t-s)a(y)d\sigma(y).$$

(13)-(16) aralash masalaning yechimi hamda (18) shart va (22) tenglikdan biz

$$\int\limits_0^t R(t-s)\mu(s)ds=\theta,\quad t\ge T, \tag{23}$$

asosiy integral tenglamani olamiz.

Quyidagi belgilashlarni kiritamiz:

$$Q(t)=\int\limits_0^t R(s)ds,\qquad Q^*=\lim_{t\to\infty}Q(t)=\int\limits_0^\infty R(s)ds.$$

8-tasdiq. Aytaylik, $0<\theta<MQ^*$ shart o‘rinli bo‘lsin. U holda $T>0$ va haqiqiy qiymatli $\mu(t)$ o‘lchovli funksiya mavjud va quyidagi

$$\int\limits_0^T R(T-s)\mu(s)ds=\theta \tag{24}$$

tenglik o‘rinli bo‘ladi.

9-tasdiq. Aytaylik,

$$0<\theta<\frac{d_0B_1M}{\lambda_1}$$

shart o‘rinli bo‘lsin. U holda shunday $T>0$ mavjudki, ushbu

$$T<-\frac{1}{\lambda_1}\ln\left(1-\frac{\theta\lambda_1}{d_0B_1M}\right) \tag{25}$$

baho va (24) tenglik o‘rinli bo‘ladi.

10-tasdiq. Aytaylik, $T>0$ (24) tenglamani va (25) shartni qanoatlantirsin. U holda shunday $T_1>T$ va haqiqiy qiymatli $\mu(t)$ funksiya mavjud, shuning uchun $|\mu(t)|\le M$ va ushbu

$$\int\limits_\Omega\rho(x)u(x,t)dx=\theta,\ T\le t\le T_1,$$

tenglik o‘rinli bo‘ladi.

Ikkinchi paragrafda uch o‘lchamli sohada o‘rtacha temperatura kritik qiymatga yaqin bo‘lganda, minimal vaqt harorat jarayonining parametrlariga bog‘liqlik ekanligi isbotlangan.

Biz $\partial\Omega$ chegarasi bo‘lakli silliq bo‘lgan chegaralangan $\Omega \subset \mathbb{R}^3$ sohada (13)-(16) aralash masalani ko‘rib chiqamiz.

Aytaylik, $\rho : \overline{\Omega} \to R$ funksiya quyidagi shartlarni qanoatlantirsin:

$$\int_{\Omega} \rho(x)dx = 1, \quad \rho(x) \geq 0.$$

Har qanday $\theta > 0$ uchun quyidagi belgilashni kiritamiz:

$$\int_{\Omega} u(x,t)\rho(x)dx = \theta. \tag{26}$$

Biz haroratning o‘rtacha qiymati θ ga erishish uchun ketadigan minimal vaqtni $T(\theta)$ orqali belgilaymiz. Buning ma’nosi (26) tenglik $t = T(\theta)$ uchun bajariladi va $t < T(\theta)$ uchun bajarilmaydi.

Biz quyidagi funksiyani ko‘rib chiqamiz:

$$U(t) = \int_{\Omega} u(x,t)\rho(x)dx. \tag{27}$$

Quyidagi belgilashlarni kiritamiz:

$$\theta^* = M \int_{\Gamma} [(-\Delta)^{-1}\rho(x)]a(x)d\sigma(x) \tag{28}$$

va

$$b = \frac{M}{\lambda_1} \cdot (\rho, \vartheta_1) \int_{\Gamma} \vartheta_1(y)a(y)d\sigma(y). \tag{29}$$

4-teorema. Aytaylik, $\theta^* > 0$ (28) tenglik bilan aniqlangan bo‘lsin. U holda quyidagilar o‘rinli:

1) $0 < \theta < \theta^*$ intervaldan olingan θ uchun $T(\theta)$ mavjud va quyidagilar o‘rinli:

$$U(t) < \theta, \quad 0 < t < T(\theta), \quad U\big(T(\theta)\big) = \theta.$$

2) $\theta \to \theta^*$ uchun quyidagi

$$T(\theta) = \frac{1}{\lambda_1} \ln \frac{1}{\varepsilon(\theta)} + \frac{1}{\lambda_1} \ln b + O(\varepsilon^{(\lambda_2 - \lambda_1)/\lambda_1})$$

baho o‘rinli, bu yerda $\varepsilon = |\theta^* - \theta|$.

3) $\theta \geq \theta^*$ uchun $T(\theta)$ mavjud emas.

Quyidagi belgilashni kiritamiz:

$$H(x,t) = \int_{\Omega} \rho(y)G(x,y,t)dy, \ x \in \Omega, \ t > 0. \tag{30}$$

Bunda biz $L_2(\Omega, dx)$ dagi spektral teoremadan foydalanib

$$H(x,t) = \int_0^{\infty} e^{-\lambda t} dE_{\lambda}\rho(x)$$

tenglikni yozishimiz mumkin.

Biz (30) tenglikda aniqlangan funksiyani

$$H(x,t) = (\rho, \vartheta_1)e^{-\lambda_1 t}\vartheta_1(x) + H_1(x,t), \quad t \geq 0, \tag{31}$$

ko‘rinishda yozib olamiz, bu yerda

$$H_1(x,t)=\int_{\lambda_2}^{\infty}e^{-\lambda t}dE_\lambda\rho(x).$$

Quyidagi belgilashni kiritamiz:

$$G_2(x,y)=\sum_{k=2}^{\infty}\frac{\vartheta_k(x)\vartheta_k(y)}{\lambda_k^2}.$$

11-tasdiq. $H_1(x,t)$ funksiya uchun

$$|H_1(x,t)|\le\|\Delta\rho\|\cdot\sqrt{G_2(x,x)}e^{-\lambda_2 t},\quad t\ge 0,$$

baho o‘rinli.

Ushbu

$$K(t)=\int_{\Gamma}H(y,t)a(y)d\sigma(y) \tag{32}$$

funksiyani kiritamiz.

(13)-(16) aralash masalaning yechimi hamda (27) shart va (32) tenglikdan

$$\int_0^t K(t-s)\mu(s)ds=U(t)$$

asosiy integral tenglamani olamiz.

12-tasdiq. Aytaylik $f(r)$ funksiya $(0,1]$ intervalda o‘suvchi va $\beta>0$ uchun

$$f(r)=br+O(r^{1+\beta})$$

ko‘rinishda aniqlangan bo‘lsin. U holda $r=f^{-1}(s)$ teskari funksiya uchun ushbu

$$\ln\frac{1}{r}=\ln\frac{1}{s}+\ln b+O(s^{\beta})$$

baho o‘rinli, bu yerda $b>0$.

XULOSA

Parabolik tipdagi tenglamalar bilan ifodalanadigan jarayonlarni chegaraviy boshqarishning asosiy muammosi optimal vaqtni topish va ruxsat etilgan boshqaruv funksiyasini mavjudligini isbotlashdir.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1. Bir o‘lchamli va ikki o‘lchamli sohalarda parabolik tipdagi tenglama uchun boshlang‘ich-chegaraviy masalaning umumlashgan yechimlari topildi.
2. Berilgan intervalda parabolik tenglama uchun boshlang‘ich-chegaraviy masalada sterjenni tez isitish uchun zarur bo‘lgan minimal vaqtga baho olindi.
3. To‘g‘ri to‘rtburchakli sohada issiqlik almashinish tenglamasi qo‘shimcha integral shart ostida tadqiq qilindi. Berilgan sohaning qismidagi o‘rtacha haroratga erishish uchun ketadigan minimal vaqtga baho topildi va asosiy integral tenglamaning yechimi mavjudligi Laplas almashtirishi usuli orqali isbotlandi.
4. Uch o‘lchamli sohada o‘rtacha temperatura kritik qiymatga yaqin bo‘lganda, minimal vaqt harorat jarayoni parametrlariga bog‘liq ekanligi isbotlandi.
5. $n-$o‘lchamli chegaralangan sohada issiqlik almashinish tenglamasi uchun boshlang‘ich-chegaraviy masala tadqiq qilindi. Qo‘shimcha integral shart ostiga musbat aniqlangan vazn funksiyasi qo‘yilganda, berilgan jismni o‘rtacha temperaturagacha isitish uchun ketadigan minimal vaqtga baho topildi va mos integral tenglamaning yechimi mavjudligi isbotlandi.

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NATIONAL UNIVERSITY OF UZBEKISTAN

DEXQONOV FARRUXJON NURIDDIN O‘G‘LI

THE BOUNDARY CONTROL OF THE HEAT EXCHANGE PROCESS

01.01.02-Differential equations and mathematical physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

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INTRODUCTION

Actuality and demand of the theme of the thesis. Many scientific and practical studies at the world level, in many cases, are reduced to the study of the theory of control of systems described by the equations of mathematical physics. Scientific applied research can be reduced to the problems of managing various systems. Mathematical models of some problems in physics and technology describe processes in which it is necessary to control the process in such a way as to obtain certain parameter values. An example is the problem of heat transfer control or wave processes. In the case of wave processes, it is necessary to control the wave parameter on a part of the region boundary in order to achieve a certain waveform in some part of the given region. In the case of heat transfer, it is necessary to control the temperature parameter on a part of the region boundary in order to achieve a given average temperature in some part of the region. Therefore, boundary control problems are both theoretically and practically relevant and are considered one of the topical areas of modern mathematics.

At present, one of the topical problems of mathematical physics is boundary control. The study of boundary problems is closely related to the inverse and ill-posed problems of mathematical physics, so they also reduce to the study of Fredholm or Volterra integral equations (more often to the Volterra equation of the first kind). Recently, the control of heat transfer processes from various points of view has been intensively developed, since they are closely related to the tasks of the application. Especially in the bounded domain, the solution of boundary control problems of parabolic type equations is a topical issue. Control problems for mathematical physics equations often lead to the first type of Volterra integral equation, and the Laplace transform method is used to prove that there exists is a solution to this equation. Therefore, the study of boundary problems of control of heat transfer processes is relevant, both from the point of view of theory and application.

In our country, special attention was paid to topical areas of mathematical physics and optimal control, which have scientific and practical applications in the fundamental sciences. In particular, special attention was paid to the study of optimal control and game theory. Significant results have been obtained in the control theory of processes described by ordinary differential equations and their systems, partial differential equations and their systems. “Conducting scientific research at the level of international standards in the priority areas of mathematical sciences in the field of functional analysis, differential equations and mathematical physics are the main tasks and directions”[1]. In the implementation of this decision, the development of the solution of boundary value problems for differential equations and equations of mathematical physics is of great importance.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees and Resolutions of the President of the Republic of

[1] Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May « On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan» № 292 dated May 17, 2017.

Uzbekistan of February 7, 2017, PF-4947 , «On the strategy of action for the further development of the Republic of Uzbekistan», PQ-4387 dated July 9, 2019 «On state support for the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I. Romanovskiy of the Academy of Sciences of the Republic of Uzbekistan», PQ-4708 of May 7, 2020 «On measures to improve the quality of education and research in the field of mathematics» as well as in other regulations related to basic sciences.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of the Republic of Uzbekistan IV, «Mathematics, Mechanics and Computer Science».

The degree of scrutiny of the problem. The mathematical theory of control problems began to develop rapidly in the middle of the last century. Mathematical Theory of control problems for simple differential equations and their systems were founded in the scientific work of R. Bellman, L.S. Pontryagin, V.G. Boltyanskiy, R.V. Gamkrelidze, E.F. Mishchenko and developed by M.I. Zelikin, V.M. Tikhomirovs. Around this time, with the need for cosmonautics, mathematicians such as R. Bellman, La-Caol, Dj. Leitman developed an approach to optimal control problems based on the application of variational calculus, particularly the Hamilton-Jacobi method. Based on the mathematical theory of simple differential equations and the control of their systems, extensive research has been conducted by researches of Yekaterinburg Scientific School, founded by N.N. Krasovsky, such as A.B. Kurjanskiy, Yu.S.Osipov, A.I. Subbotin, A.V. Kryajimskiy, N.Yu. Lukoyanov, N.N. Subbotina, V.N. Ushakov et al. In 1970, N.Yu. Satimov founded a scientific school in Tashkent in the field of optimal control and differential game theory. Initially, the members of this school were mainly concerned with the issues of chase and escape, but since the end of the twentieth century, control problems of partial differential equations have been studied by N.Yu. Satimov, A. Azamov, M. Tukhtasinov, M. Mamatov and others. In particular, in controllable evolutionary systems the method of constructing the control function proposed by F.L. Cherniusko is being developed by A. Azamov, M. Ruziboyev, and others.

We recall that the time-optimal control problem for partial differential equations of parabolic type was first concerned H.O. Fattorini and A. Friedman. Detailed information on the problems of optimal control for distributed parameter systems is given in the monographs A.V. Fursikov and J.L. Lions. Also, control problems related to processes represented by partial differential equations are studied by K.A. Lur'e, V.A. Il'in and E.I. Moisiev, A.G. Butkovskiy, A.I. Yegorov, E.S. Efimova, V.I. Maximov, N. Chen, Y. Wang, D. Yang, E. Fernandez-Cara, E. Zuazua and other mathematicians. Recent scientific findings on control problems for parabolic-type equations studied in this dissertation have been studied by Sh.A. Alimov, S. Albeverio, Z.K.Fayazova and et al.

The problems of boundary control of processes described by equations of hyperbolic type and their systems were considered in the works of V.A.Il'in and E.I.

Moisiev. Control under various constraints for parabolic and hyperbolic type equations has been studied by Y. Luo, I. Lasieska, R. Triggiani, V. Barbu, A. Rascanu, G. Tessitore, M. Gueye, W. Littman, Sule S. Sener, Murat Sulasi, P. Cannarsa, P. Martinez, J. Vancostenoble, and others. General numerical optimization an optimal boundary control have been studied a great number of publications such as A. Altmyuller and L. Grüne. The practical approaches to optimal control of the heat equation are described in publications like S. Dubljevic and P.D. Christofides.

The connection of the theme of the thesis with the research plans of the higher education institute, where the research on the thesis is carried out. The dissertation research is done in accordance with the planned theme of scientific research project F-FA-2021-424 "Solution of boundary value problems for differential equations with integer and fractional orders" at the Institute of Mathematics named after V.I. Romanovskiy and in the researches at the National University of Uzbekistan.

The aim of the research work to estimate the minimum time required to heat an object to an average temperature in a given domain and to show the existence of a solution to the corresponding integral equation.

Research problems:

to estimate the minimum time required for fast heating of the rod and prove the existence of a solution of the main integral equation;

to estimate the minimum time to reach the average temperature in a part of a given domain;

to estimate the minimum time required to heat a given object in an $n-$ dimensional bounded domain and prove the existence of a solution of the corresponding integral equation.

to prove that the minimum time to be depend on the parameters of the temperature process when the average temperature in the three-dimensional domain is near to the critical value.

The research object. Equations of parabolic type, Volterra integral equations of the first and second types.

The research subject. Boundary control problems with additional integral conditions for parabolic type equations.

Research methods. The dissertation work uses modern methods of mathematical physics, integral equations and including the methods of Fourier series.

Scientific novelty of the research work consists of the following:

the initial-boundary value problem for the parabolic equation was considered in the interval as well as the minimum time required to heat the rod rapidly was estimated and the existence of a solution of the main integral equation was proved;

an estimate of the minimum time required to reach the average temperature in a given part of the domain was found and the existence of a solution of the main integral equation was proved by the Laplace transform method;

the minimum time to be depend on the parameters of the temperature process when the average temperature in a three-dimensional domain is close to the critical value was proved;

an initial-boundary value problem for the heat transfer equation in an $n-$ dimensional bounded domain was considered as well as the minimum time required to heat a given body is estimated.

Practical results of the research. The taken results and used methods in the dissertation can be taught as a graduate course for masters and doctoral students of higher education institutions. In addition, the results of the dissertation concerning the boundary control of the heat exchange processes for the parabolic type equation allow to verify hypothesis about other type equations.

The reliability of the results of the study. The results have been obtained by using the methods of modern methods of mathematical physics, mathematical anaysis. The proof of obtained results are mathematically correct.

Scientific and practical significance of the research results. The scientific significance of the results of the research is that in solving the problem, it is proved that there is a solution of boundary control problems for parabolic equations given by Dirichlet or Neumann conditions in one, two and three dimensional domains. In particular, the minimum time required to reach the average temperature in a given domain is estimated.

The practical significance of the research is determined by the possibility of using it in the preparation of masters and in mathematical modeling to find the minimum time required to heat a given object to an average temperature.

Implementation of the research results. Based on the results obtained on the problem of the boundary control of the heat exchange process:

the estimates for the kernel of Volterra's first-order integral equation and the Laplace substitution of this kernel were used to show that there is a solution for the main Volterra first-order integral equation by using Laplace substitution in the fundamental scientific project No. OT-F-4-(36+32) "Development of new methods for solving problems of mathematical physics and optimal control+nonclassical initial and spectral problems and their applications for partial differential equations of odd order" (Reference from National University of Uzbekistan named after Mirzo Ulugbek, No. 04/11-2261, 20.04.2022). The application of scientific results allowed to find a generalized solution of the initial-boundary value problem for the parabolic equation in one and two-dimensional domains;

a generalized solution of the initial-boundary problems for parabolic-type equations is found in the bounded two and $n-$dimensional fields with smooth boundary segments was used to study the properties of the Green function in the research project No. OT-F4-02 "Thermodynamics of models of mathematical physics with an infinite set of states" (Reference from Bukhara State University reference, No. 04-04/01-0963, 13.05.2022). The application of scientific results allowed to find a generalized solution of mixed problems for the equations of mathematical physics in the fundamental project work.

Approbation of the research results. The main results of the research have been discussed at 4 international and 3 national scientific conferences.

Publications of the research results. On the topic of the dissertation 13 research papers have been published in the scientific journals, 6 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis, in addition 1 of them were published in international journals and 5 papers published in national mathematical journals.

The structure and volume of the thesis. The dissertation consists of an introduction, four chapters, conclusion and bibliography. The general volume of the thesis is 86 pages.

THE MAIN CONTENT OF THE THESIS

The **introduction** part of the thesis includes the actuality and the demand of the research, the relevance of the research to the prioritity areas of science and technology, the review of foreign research on the topic, the degree of scrutiny of the problem. Also the introductory part contains the aim of the research work, research problems, the object and the subject of the research, scientific novelty and practical results, theoretical, practical significance and the statement of the research results, published works and the information on the structure of the thesis.

The first chapter of the thesis, titled **"Control problem associated with fast heating of a rod"**, we consider the initial-boundary problem in the interval. In the first section, the solution of the first-order initial-boundary value problem for the heat dissipation equation is found by the Fourier method.

The second section, deals with the boundary control problem associated with the parabolic equation. The minimum time required to heat the rod to a given average temperature is estimated, and the solution of the basic Volterra integral equation is shown.

We consider the following heat conduction equation along the domain $\Omega = \{x \in R:\ 0 < x < l\}$:

$$u_t(x,t) = u_{xx}(x,t), \quad x \in (0,l), \quad t > 0, \tag{1}$$

with boundary condition

$$u(0,t) = \mu(t), \quad u(l,t) = 0, \quad t > 0, \tag{2}$$

and initial condition

$$u(x,0) = 0, \quad 0 \le x \le l. \tag{3}$$

Let $M > 0$ be some given constant. We say that the function $\mu(t)$ is an admissible control if this function is differentiable on the half-line $t \ge 0$ and satsfies the following constrains

$$\mu(0) = 0, \quad |\mu(t)| \le M$$

We consider also the weight function $\rho(x)$, which is smooth on the interval $[0,l]$ and satisfies the following conditions

$$\rho(x) \geq 0, \quad \rho'(x) \leq 0, \quad \int_0^l \rho(x)dx = 1.$$

Problem 1. Given constant $\theta > 0$. Find the minimum value of $T > 0$ so that for $t > 0$ the solution $u(x,t)$ of the initial-boundary value problem (1) - (3) with admissible control $\mu(t)$ exists and for some $T_1 > T$ satisfies the following equation

$$\int_0^l \rho(x)u(x,t)dx = \theta, \ T \leq t \leq T_1.$$

Theorem 1. Let

$$0 < \theta < \frac{M \cdot l \cdot \rho_1}{\pi}.$$

Set

$$T_0 = -\left(\frac{l}{\pi}\right)^2 \ln\left(1 - \frac{\theta \cdot \pi}{\rho_1 \cdot l \cdot M}\right).$$

Then a solution $T_{\min}$ of the Problem 1 exists and the estimate $T_{\min} \leq T_0$ is valid.

Definition 1. By the solution of the problem (1) - (3) we mean function $u(x,t)$, represented in the form

$$u(x,t) = \frac{l-x}{l}\mu(t) + \vartheta(x,t),$$

where the function $\vartheta(x,t)$ is a generalized solution from $C\left([0,T] \to \tilde{W}_2^1\right)$ of the problem

$$\vartheta_t(x,t) - \Delta\vartheta(x,y,t) = -\frac{l-x}{l}\mu'(t), \quad t > 0,$$
$$\vartheta(x,t)\big|_{\partial\Omega} = 0, \qquad \vartheta(x,0) = 0, \quad 0 \leq x \leq l.$$

Lemma 1. Let $g(y)$ function is decreasing and non-negative on $y \in [0,\infty)$. Then the following inequality is valid:

$$\int_0^{n\pi} g(y)\sin y dy \geq 0, \ n = 1,2,3,....$$

Set

$$Q(t) = \int_0^t K(s)ds, \qquad Q^* = \lim_{t\to\infty} Q(t) = \int_0^\infty K(s)ds.$$

Proposition 1. Let $0 < \theta < MQ^*$. Then there exists $T > 0$ and a real-valued measurable function $\mu(t)$ and the following equality is valid:

$$\int_0^T K(T-s)\mu(s)ds = \theta. \tag{4}$$

Proposition 2. Let

$$0 < \theta < \frac{M \cdot l \cdot \rho_1}{\pi}.$$

Then there exists $T > 0$ so that

$$T < -\left(\frac{l}{\pi}\right)^2 \ln\left(1 - \frac{\theta \cdot \pi}{\rho_1 \cdot l \cdot M}\right),$$

and the equality (4) is fulfilled.

The second chapter of the thesis, titled **"Control problem associated with the heating process in a two-dimensional domain"**, we consider the initial-boundary value problem for the heat conduction equation in a rectangular domain. We estimate the minimum time it takes to reach the average temperature in an domain that is part of a right rectangle.

In the first section, we consider the initial-boundary value problem in the given domain, and the solution of the problem is found by the Fourier method. The problem is to find the minimum time required to reach the average temperature in a rectangular domain.

We consider the following heat conduction equation in the domain $\Omega = \{(x, y) \in R^2 : 0 < x < a,\ 0 < y < b\}$

$$u_t(x,y,t) = u_{xx}(x,y,t) + u_{yy}(x,y,t),\ \ (x,y) \in \Omega,\ \ t > 0, \tag{5}$$

with boundary conditions

$$u(0,y,t) = \varphi(y)\mu(t),\ \ u(a,y,t) = \psi(y)\mu(t), \tag{6}$$

$$u(x,0,t) = u(x,b,t) = 0, \tag{7}$$

and initial condition

$$u(x,y,0) = 0,\ \ 0 \le x \le a,\ \ 0 \le y \le b, \tag{8}$$

where $\mu(0) = 0,\ \ \varphi(0) = \varphi(b) = 0,\ \ \psi(0) = \psi(b) = 0.$

Problem 2. Given constants $\alpha, \beta \ge 1$ and $\theta > 0$. Find the minimum value of $T > 0$ so that for $t > 0$ the solution $u(x,y,t)$ of the initial-boundary value problem (5) - (8) with admissible control $\mu(t)$ exists and for some $T_1 > T$ satisfies the following equation

$$\int_0^{b/\alpha} \int_0^{a/\beta} u(x,y,t)\,dx\,dy = \theta, \quad T \le t \le T_1.$$

Set

$$\rho_{nm} = \frac{8b}{a\pi m}(\varphi_m - (-1)^n \psi_m)\sin^2\frac{n\pi}{2\beta}\sin^2\frac{m\pi}{2\alpha}, \quad n,m = 1,2,3,... \tag{9}$$

and $\gamma = \dfrac{\pi^2(a^2 + b^2)}{a^2 b^2}$.

Theorem 2. Let

$$0 < \theta < \frac{\rho_{11} M}{\gamma}.$$

Set

$$T_0 = -\frac{1}{\gamma}\ln\left(1 - \frac{\theta\gamma}{\rho_{11}M}\right).$$

Then a solution $T_{\min}$ of the Problem 2 exists and the estimate $T_{\min} \le T_0$ is valid.

In the second section, an auxiliary problem is considered to solve the problem given in the first section.

In the third section, we find the solution to the main problem presented in the first section. In doing so, the methods used in finding the solution to the auxiliary problem given in the second section are used. That is, when a heat air is given from both sides of a right rectangle, the minimum time is estimated and the solution of the main integral equation is proved.

Definition 2. By the solution of the problem (5) - (8) we mean function $u(x,y,t)$, represented in the form

$$u(x,y,t) = \mu(t)(\psi(y) + \frac{a-x}{a}(\varphi(y) - \psi(y))) - \vartheta(x,y,t),$$

where the function $\vartheta(x,y,t)$ is a generalized solution from $C\left([0,T] \to \tilde{W}_2^1\right)$ of the problem

$$\vartheta_t(x,y,t) - \Delta\vartheta(x,y,t) =$$

$$= \mu'(t)(\psi(y) + \frac{a-x}{a}(\varphi(y) - \psi(y))) - \mu(t)\cdot[\psi''(y) + \frac{a-x}{a}(\varphi''(y) - \psi''(y))]$$

$$\vartheta(x,y,t)|_{\partial\Omega} = 0, \quad \vartheta(x,y,0) = 0, \quad 0 \le x \le a, \quad 0 \le y \le b.$$

Set

$$B(t) = \sum_{n,m=1}^{\infty} \rho_{nm} e^{-\lambda_{nm} t}, \tag{10}$$

where ρ_{nm} defined by (9) equality.

We consider the following integral equation

$$\int_0^t B(t-s)\mu(s)ds = f(t), \ t > 0. \tag{11}$$

Denote by $W\left(M_0\right)$ the set of function $f \in W_2^2(-\infty,+\infty)$, $f(t) = 0$ for $t \le 0$ which satisfies the condition

$$\| f \|^2_{W_2^2(R_+)} \le M_0.$$

Lemma 2. There exists $M_0 > 0$ such that for any function $f \in W\left(M_0\right)$ the solution $\mu(t)$ of the equation (11) exists and satifies the following estimate

$$\left|\mu(t)\right| \le M.$$

Set

$$Q(t) = \int_0^t B(s)ds, \ Q^* = \lim_{t\to\infty} Q(t) = \int_0^{\infty} B(s)ds.$$

Proposition 3. Let $0 < \theta < MQ^*$. Then there exists $T > 0$ and a real-valued measurable function $\mu(t)$ and the following equality is valid:

$$\int_0^T B(T-s)\mu(s)ds = \theta. \tag{12}$$

Proposition 4. Let

$$0 < \theta < \frac{\rho_{11} M}{\gamma}.$$

Then there exists $T > 0$ such that

$$T < -\frac{1}{\gamma}\ln\left(1 - \frac{\theta\gamma}{\rho_{11} M}\right),$$

and the equality (12) is fulfilled.

In the third chapter of the thesis, titled **"Boundary control problem of the heat exchange process in a bounded domain"**, we consider the initial-boundary value problem for the equation of thermal conductivity in a bounded domain.. The control parameter is equal to the magnitude of output of hot or cold air and is defined on a given part of the boundary.

In the first section, the initial-boundary value problem for the heat transfer equation in an $n-$dimensional bounded domain is considered. An estimate of the minimal time for achieving the given average temperature is found

Consider in the bounded domain $\Omega \subset \mathbb{R}^n$ with piecewise smooth boundary $\partial\Omega$ the heat conduction equation

$$u_t(x,t) = \Delta u(x,t), \quad x \in \Omega, \quad t > 0, \tag{13}$$

with boundary conditions

$$\frac{\partial u}{\partial n} + h(x)u(x,t) = 0, \quad x \in \partial\Omega \setminus \Gamma, \quad t > 0, \tag{14}$$

$$\frac{\partial u}{\partial n} = a(x)\mu(t), \quad x \in \Gamma, \quad t > 0, \tag{15}$$

and initial condition

$$u(x,0) = 0. \tag{16}$$

We may extend both functions $h(x)$ and $a(x)$ to the whole boundary $\partial\Omega$ by setting $h(x) = 0$ for $x \in \Gamma$, and $a(x) = 0$ for $x \notin \Gamma$. In this case we may write the conditions (14) and (15) in the following form

$$\frac{\partial u(x,t)}{\partial n} + h(x)u(x,t) = a(x)\mu(t), \quad x \in \partial\Omega, \quad t > 0. \tag{17}$$

Let $M > 0$ be some given constant. We say that the function $\mu(t)$ is an admissible control if the function is measurable on $t \geq 0$ and satisfies the following constraint

$$|\mu(t)| \leq M.$$

Let the continuous function $\rho : \overline{\Omega} \to R$ satisfies conditions

$$\int_{\Omega} \rho(x)dx = 1, \quad \rho(x) > 0.$$

Problem 3. Given constant $\theta > 0$. Find the minimum value of $T > 0$ so that for $t > 0$ the solution $u(x,t)$ of the initial-boundary value problem (13) - (16) with admissible control $\mu(t)$ exists and satisfies the following equality

$$\int_{\Omega} \rho(x)u(x,t)dx = \theta, \ t \geq T. \tag{18}$$

Set

$$d_0 = \min_{x \in \Omega \cup \partial\Omega} \rho(x), \tag{19}$$

$$B_k = \frac{1}{\Lambda_k} \int_{\partial\Omega} \vartheta_k(x)a(x)d\sigma(x), \text{ where } \Lambda_k = \| \vartheta_k(x) \|_{C(\bar{\Omega})}. \tag{20}$$

Theorem 3. Let

$$0 < \theta < \frac{d_0 B_1 M}{\lambda_1}.$$

Set

$$T_0 = -\frac{1}{\lambda_1} \ln\left(1 - \frac{\theta\lambda_1}{d_0 B_1 M}\right).$$

Then a solution $T_{\min}$ of the Problem 3 exists and the estimate $T_{\min} \leq T_0$ is valid.

Set

$$G(x,y,t) = \sum_{k=1}^{\infty} e^{-\lambda_k t} \vartheta_k(x)\vartheta_k(y), \ x \in \Omega, \ y \in \Omega, \ t > 0.$$

Proposition 5. Let $h(x) \geq 0$ for all $x \in \partial\Omega$. Then the Green function of the initial-boundary value problem (13)-(16) is non-negative.

Set

$$H(x,t) = \int_{\Omega} \rho(y)G(x,y,t)dy, \ x \in \Omega, \ t > 0. \tag{21}$$

Proposition 6. There exsists Λ_1, so that inequality

$$H(x,t) \geq \frac{d_0}{\Lambda_1} e^{-\lambda_1 t} \vartheta_1(x)$$

is valid.

Set

$$R(t) = \int_{\partial\Omega} H(y,t)a(y)d\sigma(y). \tag{22}$$

Proposition 7. The following estimate

$$R(t) \geq d_0 B_1 e^{-\lambda_1 t}$$

is valid, where the coeffitsient B_1 is taken from the condition (21).

It is well-know that the solution of the initial-boundary value problem (13) + (15) + (17) may be represented by the Green function:

$$u(x,t)=\int_0^t \mu(s)ds \int_{\partial\Omega} G(x,y,t-s)a(y)d\sigma(y).$$

Using solutions of mixed problem (13) - (16) and (18), (22) we obtain the following integral equation

$$\int_0^t R(t-s)\mu(s)ds=\theta, \quad t\geq T. \tag{23}$$

Set

$$Q(t)=\int_0^t R(s)ds, \qquad Q^*=\lim_{t\to\infty} Q(t)=\int_0^\infty R(s)ds.$$

Proposition 8. Let $0<\theta<MQ^*$. Then there exsist $T>0$ and real valued measurable function $\mu(t)$ and the following equality is valid:

$$\int_0^T R(T-s)\mu(s)ds=\theta. \tag{24}$$

Proposition 9. Let

$$0<\theta<\frac{d_0B_1M}{\lambda_1}.$$

Then there exists $T>0$ so that

$$T<-\frac{1}{\lambda_1}\ln\left(1-\frac{\theta\lambda_1}{d_0B_1M}\right), \tag{25}$$

and the equality (24) is fulfilled.

Proposition 10. Let $T>0$ satisfies the equality (24) and condition (25). Then there exists $T_1>T$ and measurable real valued function $\mu(t)$ so that $|\mu(t)|\leq M$ and the following equality is valid:

$$\int_\Omega \rho(x)u(x,t)dx=\theta, \ T\leq t\leq T_1.$$

In the second section, it was proved that the minimum time depends on the parameters of the temperature process when the average temperature is close to the critical value.

We consider in the bounded domain $\Omega\subset\mathbb{R}^3$ with piecewise smooth boundary $\partial\Omega$ the mixed problem (13)-(16).

Let the function $\rho:\overline{\Omega}\to R$ satisfies conditions

$$\int_\Omega \rho(x)dx=1, \quad \rho(x)\geq 0.$$

For any $\theta>0$ consider the condition

$$\int_\Omega u(x,t)\rho(x)dx=\theta. \tag{26}$$

Denote by the symbol $T(\theta)$ the minimal time required to reach the given value θ by the average value of the temperature. This means that the equation (26) is fulfilled for $t = T(\theta)$ and is not valid for $t < T(\theta)$.

Recall that we consider the behavior of the function

$$U(t) = \int_{\Omega} u(x,t)\rho(x)dx. \tag{27}$$

Set

$$\theta^* = M\int_{\Gamma}[(-\Delta)^{-1}\rho(x)]a(x)d\sigma(x), \tag{28}$$

$$b = \frac{M}{\lambda_1}\cdot(\rho,\vartheta_1)\int_{\Gamma}\vartheta_1(y)a(y)d\sigma(y). \tag{29}$$

Theorem 4. Let $\theta^* > 0$ be defined by equation (28). Then

1) for every θ from the interval $0 < \theta < \theta^*$ there exists $T(\theta)$ such that

$$U(t) < \theta, \quad 0 < t < T(\theta),$$

and $U\big(T(\theta)\big) = \theta.$

2) for every $\theta \to \theta^*$ the following estimate is valid:

$$T(\theta) = \frac{1}{\lambda_1}\ln\frac{1}{\varepsilon(\theta)} + \frac{1}{\lambda_1}\ln b + O(\varepsilon^{(\lambda_2-\lambda_1)/\lambda_1})$$

where $\varepsilon = |\theta^* - \theta|$.

3) for every $\theta \geq \theta^*$ the $T(\theta)$ does not exists.

Set

$$H(x,t) = \int_{\Omega}\rho(y)G(x,y,t)dy, \ x \in \Omega, \ t > 0. \tag{30}$$

In this using the spectral theorem in $L_2(\Omega, dx)$ we may write

$$H(x,t) = \int_0^{\infty} e^{-\lambda t}dE_{\lambda}\rho(x).$$

We write function $H(x,t)$ in the following form

$$H(x,t) = (\rho,\vartheta_1)e^{-\lambda_1 t}\vartheta_1(x) + H_1(x,t), \quad t \geq 0, \tag{31}$$

where

$$H_1(x,t) = \int_{\lambda_2}^{\infty} e^{-\lambda t}dE_{\lambda}\rho(x).$$

Set

$$G_2(x,y) = \sum_{k=2}^{\infty}\frac{\vartheta_k(x)\vartheta_k(y)}{\lambda_k^2}.$$

Proposition 11. For the $H_1(x,t)$ function the following estimate is valid:

$$|H_1(x,t)| \leq \|\Delta\rho\| \cdot \sqrt{G_2(x,x)}e^{-\lambda_2 t}, \quad t \geq 0.$$

Set

$$K(t) = \int_{\Gamma} H(y,t)a(y)d\sigma(y). \tag{32}$$

Using solutions of mixed problem (13) - (16) and (27), (32) we obtain the following integral equation

$$\int_0^t K(t-s)\mu(s)ds = U(t).$$

Proposition 12. Let $f(r)$ be increasing on the interval $(0,1]$ and for some $\beta > 0$

$$f(r) = br + O(r^{1+\beta}).$$

Then for inverse function $r = f^{-1}(s)$ the following estimate is valid:

$$\ln\frac{1}{r} = \ln\frac{1}{s} + \ln b + O(s^{\beta}), \text{ where } b > 0.$$

CONCLUSION

The main problem in the boundary control process associated with parabolic equation is to find the optimal time and prove the existence of an admissible control function.

Basic results of the research are as follows:

1. Generalized solutions of the initial-boundary value problem have been found for parabolic-type equations in one and two-dimensional domains;
2. The initial-boundary value problem for the parabolic equation was considered in the interval as well as the minimum time required to heat the rod rapidly was estimated;
3. An estimate of the minimum time required to reach the average temperature in a given part of the domain was found and the existence of a solution of the main integral equation was proved by the Laplace transform method;
4. The minimum time to be depend on the parameters of the temperature process when the average temperature in a three-dimensional domain is close to the critical value was proved;
5. An initial-boundary value problem for the heat transfer equation in an $n-$ dimensional bounded domain was considered as well as the minimum time required to heat a given body is estimated when a positively defined weight function is placed under the additional integral condition and the solution of the corresponding integral equation is proved.

НАУЧНЫЙ СОВЕТ DSc.02/30.12.2019.FM.86.01
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ
ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО

НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ УЗБЕКИСТАНА

ДЕХКОНОВ ФАРРУХЖОН НУРИДДИН УГЛИ

ГРАНИЧНОЕ УПРАВЛЕНИЕ ПРОЦЕССОМ ТЕПЛООБМЕНА

01.01.02 – Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)
ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

ТАШКЕНТ-2022

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Кабинете Министров Республики Узбекистан за B2022.1.PhD/FM669.

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Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице по адресу http://kengash.mathinst.uz и на Информационно-образовательном портале «ZiyoNet» по адресу http://www.ziyonet.uz.

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С диссертацией можно ознакомиться в Информационно-ресурсном центре Института Математики имени В.И.Романовского (зарегистрирована за № 138). (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9.Тел.: (+99871) 207-91-40).

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ВВЕДЕНИЕ (аннотация диссертации доктора философии(PhD))

Целью исследования оценить минимальное время, необходимое для нагрева объекта до средней температуры в заданной области, и показать существование решения соответствующего интегрального уравнения.

Объектом исследования являются краевые задачи для параболических уравнений с граничными условиями типа Дирихле, интегральные уравнения Вольтерра первого и второго рода.

Научная новизна исследования состоит в следующем:

на интервале рассмотрена начально-краевая задача для параболического уравнения, а также оценено минимальное время, необходимое для быстрого нагрева стержня;

найдена оценка минимального времени достижения средней температуры в заданной части области и доказано существование решения основного интегрального уравнения методом преобразования Лапласа;

определено минимальное время зависимости от параметров температурного процесса, когда средняя температура в трехмерной области близка к критическому значению;

рассмотрена начально-краевая задача для уравнения теплопереноса в $n-$ мерной ограниченной области, а также оценено минимальное время, необходимое для нагрева заданного тела при помещении положительно определенной весовой функции в дополнительное интегральное условие и решено соответствующее интегральное уравнение.

Внедрение результатов исследования. Научные результаты, полученные в ходе работы над диссертацией, были использованы в следующих исследовательских проектах:

на основе оценок ядра интегрального уравнения Вольтерра первого рода и преобразования Лапласа этого ядра показано существование решения основного интегрального уравнения Вольтерра первого порядка с помощью преобразования Лапласа в фундаментальном научном проекте № ОТ- F-4-(36+32) «Разработка новых методов решения задач математической физики и оптимального управления. Неклассические начальные и спектральные задачи и их приложения для уравнений в частных производных нечетного порядка» (справка из Национального университета от 20 апреля 2022 г., № 04/11-2261). Применение научных результатов позволило найти обобщенное решение начально-краевой задачи для параболического уравнения в одномерной и двумерной областях.

найденные обобщенные решения начально-краевых задач для уравнений параболического типа в ограниченных двумерных и многомерных областях с кусочно-гладкой границей были использованы для исследования свойств функции Грина в научно-исследовательском проекте № ОТ-Ф4-02 «Термодинамика моделей математической физики с бесконечным множеством состояний» (справка Бухарского государственного университета от 13 мая 2022 г., № 04-04/01-0963). Применение научных результатов позволило найти

обобщенное решение смешанных задач для уравнений математической физики в фундаментальном проекте.

Структура и объем диссертации. Диссертация состоит из введения, четырёх глав, разбитых на одиннадцать параграфов, заключения и списка использованной литературы. Объем диссертации составляет 86 страниц.

E’LON QILINGAN ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo‘lim (part I, I часть)

1. Alimov Sh.A., Dekhkonov F.N. On a control problem associated with fast heating of a thin rod. // Bulletin of National University of Uzbekistan: Mathematics and Natural Sciences. –2019. –Vol. 2(1). –P. 1-14. (01.00.00, № 8).
2. Alimov Sh.A., Dekhkonov F.N. On the time-optimal control of the heat exchange process. // Uzbek Mathematical Journal. –2019. No. 2. –P. 4-17. (01.00.00, № 6).
3. Dekhkonov F.N. On the control problem for heat transfer equation. // Scientific Bulletin of Namangan State University: –2020. –Vol. 2(6). –P. 3-10. (01.00.00, №14).
4. Dekhkonov F.N. On time-optimal control problem associated with parabolic equation. // Bulletin of National University of Uzbekistan: Mathematics and Natural Sciences: –2021. –Vol. 4(1). –P. 54-63. (01.00.00, № 8).
5. Dekhkonov F.N. On a time-optimal control of thermal processes in a boundary value problem. // Lobachevskii journal of mathematics, –2022. –Vol. 43(1). –P. 192-198. (3.Scopus. IF=0.53).
6. Дехқонов Ф.Н. Иссиқлик алмашиниш жараёни билан боғлик бошқарув масаласи. // Ўз Р Фанлар академиясининг марузалари, –2022. –No. 1. –P. 3-6. (01.00.00, № 7).

II bo‘lim (part II, II часть)

7. Dexqonov F.N. Issiqlik almashinuvini boshqarishda minimal vaqt masalasi. // “Yosh olimlarning yangi teoremalari-2018” ilmiy konferensiya. Namangan 18-19 oktabr 2018, –P. 147-148.
8. Dekhkonov F.N. The control problem for the heat exchange process. // “Современные проблемы теории вероятностей и математической статистики”, Ташкент, 30 апреля - 1 мая, 2019. –С. 235-236.
9. Dekhkonov F.N. The boundary control problem of the heat exchange process. // Сборник материалов международной конфренция КРОМШ-2019, Симферополь, Полипринт, 17-25 сентабря, 2019. –С. 203-204.
10. Dekhkonov F.N. The time optimal control problem for parabolic equation. // «Современные методы математической физики и их приложения», Ташкент, 17-18 ноября, 2020. –С. 173-174.
11. Dekhkonov F.N. Proceedings of scientific conference “Actual problems of stochastic analysis”. February 20-21, 2021. –P. 201-204.
12. Dekhkonov F.N. On the time-optimal control problem for heat transfer equation. // Современные проблемы математики и физики.

Международной научной конференции, 12-15 сентября, Стерлитамак, 2021. –С. 145-149.
13. Dekhkonov F.N. On the boundary control problem for the heat conduction equation in the space. // Annual International April Mathematical Conference-2022, Kazakstan, 6-7 April, 2022. –С. 178-179.

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