V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

MATEMATIKA INSTITUTI

BOXONOV ZAFAR SAYDIMAXMUDOVICH

BIOLOGIYADA RATSIONAL AKSLANTIRISHLARNING DINAMIK SISTEMALARI

01.01.01 – Matematik analiz

FIZIKA-MATEMATIKA FANLARI BO'YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

TOSHKENT - 2023 yil

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Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, Fan va Innovatsiyalar Vazirligi huzuridagi Oliy attestasiya komissiyasida B2023.3.PhD/FM907 raqam bilan ro'yxatga olingan.

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida olib borilayotgan ko'plab ilmiy va amaliy tadqiqotlar aksariyat hollarda ratsional dinamik sistemalarni tadqiq qilish kabi masalalariga keltiriladi. Dinamik sistemalar nazariyasining ahamiyati sistemaning hozirgi holatini bilgan holda kelajakda qanday holatlar kelib chiqishini baholashdan iborat bo'lib, mexanika, fizika, biologiya, tibbiyot, iqtisodiyot va boshqa masalalarni hal etishda muhim rol o'ynaydi. Bu esa insonlarga murakkab sistemalarni tushunish va optimal qarorlar qabul qilishda yordam beradi. Ta'kidlash joizki, dinamik sistemalar odatda ayirmali tenglamalar, differensial tenglamalar hamda differensial-ayirmali tenglamalar orqali ifodalanadi. Biologiya va ekologiya sohalaridagi tadqiqotlarning asosiy obyektlaridan biri ratsional evolyutsion operatorlarning dinamik sistemalari hisoblanadi. Shuningdek, matematik biologiya masalalarida populyatsiyaning evolyutsiyasini tadqiq qilishga ratsional evolyutsion operatorlaridan hosil qilingan dinamik sistemalar bilan bog'liq natijalar asos sifatida xizmat qiladi. Shu sababli, ratsional akslantirishlarning dinamik sistemalarini tadqiq etish dinamik sistemalar nazariyasidagi muhim va dolzarb vazifalardan biri bo'lib qolmoqda.

Hozirgi vaqtda ratsional dinamik sistemalar nazariyasi ko'plab amaliy masalalarning xarakterini tushunishda, tahlil qilishda hamda optimal yechimini topishda asosiy vosita sifatida qo'llanilmoqda. Jumladan, dinamik sistemalar nazariyasi uchun trayektoriyaning limit nuqtalar to'plamini tavsiflash, qo'zg'almas va davriy nuqtalarning mavjudligini hamda ularning turini aniqlash, invariant to'plamlarni topish, bifurkatsiya sodir bo'lishini tekshirish, sistemaning turg'unligi yoki xaotik xarakterga egaligini aniqlash kabi muammolar ko'plab amaliy masalalarni tahlil qilishda keng qo'llanilmoqda. Bu borada chuqur biologik tadqiqotlarga asoslangan optimal matematik modellarni yaratish hamda ularni matematik biologiyaning populyatsiya masalalariga qo'llash maqsadli ilmiy tadqiqotlardan hisoblanadi.

Mamlakatimizda so'nggi yillarda fundamental fanlarning ilmiy va amaliy tatbiqiga ega bo'lgan tibbiyot, biologiya, matematika va fizika fanlariga e'tibor kuchaytirildi. Jumladan, oxirgi yillarda iqtisodiy, fizik va biologik sistemalarda keng tatbiqiga ega bo'lgan ratsional dinamik sistemalar nazariyasini rivojlantirishga alohida ahamiyat berildi. Diskret vaqtli uzluksiz ratsional operatorlar dinamikasi bo'yicha salmoqli natijalarga erishildi. "Funksional analiz, matematik fizika va statistik fizika" fanlarining ustivor yo'nalishlari bo'yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalari va faoliyat yo'nalishlari etib belgilandi[1]. Shunday qilib ilmiy natijalarni fanning turdosh sohalarida qo'llash, qaror ijrosini ta'minlash maqsadida biologiyada ratsional aklantirishlarning dinamik sistemalarini tahlil qilish muhim ahamiyatga ega.

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[1] O'zbekiston Respublikasi Vazirlar mahkamasining 2017-yil 18-maydagi "O'zbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy tadqiqotlar muassasalari faoliyatini tashkil etish to'g'risida"gi 292-sonli qarori.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi va 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga mo‘ljallangan Yangi O‘zbekistonning Taraqqiyot strategiyasi to‘g‘risida”gi Farmonlari, 2019-yil 9-iyuldagi PQ-4387-son “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi va 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishi ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustivor yo’nalishi doirasida bajarilgan.

Muammoning o’rganilganlik darajasi. So‘ngi yillarda tabiiy fanlardagi ko‘pgina muammolarning matematik modellarini tushunish diskret vaqtli dinamik sistemalarni tadqiq etishga bo‘lgan qiziqishni uyg‘otdi. XIX asr oxirida olimlar tirik organizmlar populyatsiyalarining o‘sish va kamayish dinamikasini tadqiq qilish uchun populyatsiyani modellashtirish usullarini ishlab chiqa boshladilar. Ingliz olimi Tomas Maltus insoniyat taqdiri haqida fikr yuritar ekan, birinchilardan bo‘lib populyatsiyalar geometrik o‘sib borishini eng sodda model tuzish orqali asoslab berdi. Belgiyalik matematik Per Fransua Verxulst 1838 yilda populyatsiya o‘sishining muhim modellaridan biri bo‘lgan logistik modelni ishlab chiqdi. Sayyoramizda ozuqa resurslarining cheklanganligi sababli XX asrlarga kelib populyatsiyani modellashtirishga bo‘lgan qiziqish yanada ortdi. Shuningdek, populyatsiya modellari parazitlar, viruslar va kasalliklarning tarqalishini nazorat qilish uchun ham qo‘llanila boshlandi.

Muayyan populyatsiya uchun asosiy matematik muammo populyatsiya evolyutsiyasini, ya’ni holatlarning vaqtga bog‘liq dinamikasini tavsiflashdir. Bu kabi muammolarni hal qilishda foydalaniladigan matematik usullar funksional tahlillar, differensial tenglamalar, ehtimollar nazariyasi, stoxastik jarayonlar hamda dinamik sistemalar nazariyasiga asoslanadi. Populyatsiya dinamikasini tavsiflash uchun nochiziqli (xususan, ratsional va kvadratik) evolyutsion operatorlari O. Reiersol, Y.I. Lyubich, X. Kesten va boshqalar tomonidan kiritilgan. Ko‘pgina jarayonlarni matematik modellashtirish ratsional diskret vaqtli dinamik sistemalarga olib kelinadi. Ratsional diskret vaqtli dinamik sistemalar dinamik sistemalarning muhim sinfi hisoblanib, so‘ngi yillarda M. Kulenovich, O. Merino, E. Pilav, D. Clark, C.D. Lynd, E. Camouzis, A. Gilbert, M. Heissan, G. Ladas, E. Drymonis, E.A. Grove, Y. Kostrov, M. A. Radin, S. Schultz kabi matematiklar tomonidan olib borilgan izlanishlar ratsional dinamik sistema nazariyasini ishlab chiqishda muhim o‘rin egallaydi.

2020 yilda O‘.A. Roziqov tomonidan yozilgan “Populyatsiya dinamikasi: algebraik va ehtimolli yondoshuv” nomli kitobda populyatsiya modellari keltirilgan. Shuningdek, biologik modellarning bir nechta dinamik sistemalari (J.M.Kasas, M.Ladra, O‘.Roziqov, B.Mamurov, S.Xudayarovlar tomonidan tadqiq qilingan), jinsga bog‘liq populyatsiyaning dinamikasi (Yu.I.Lyubich, U.Jamilov, O‘.Roziqov tomonidan tekshirilgan), chivin populyatsiyasining dinamik sistemasi va evolyutsion algebrasi (M.Velasko, R.Varro, O‘.Roziqov) hamda okean ekosistemalari (S. Shoyimardonov, O‘.Roziqov) berilgan. Ratsional diskret vaqtli dinamik sistemalar juda oddiy ko‘rinishga ega bo‘lsa-da, umumiy nazariyaning mavjud emasligi tufayli operatorlar dinamikasini to‘liq tahlil qilishda qiyinchilik tug‘iladi va har bir operatorni o‘rganish tadqiqotchidan alohida ilmiy yondashuv talab etadi. Hozirgi kunga kelib, ratsional operatorlar dinamik sistemalari bo‘yicha yetarlicha ilmiy izlanishlar olib borilganligiga qaramay, ko‘plab ratsional operatorlar orqali hosil qilingan dinamik sistemalar uchun trayektoriyaning limit nuqtalari to‘plamini to‘la tavsifi olinmaganligini ta’kidlash joiz.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan ilmiy tekshirish instituti ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti V.I. Romanovskiy nomidagi Matematika institutining OT-F4-87 raqamli “Evklid va psevdo-Evklid fazolaridagi egri chiziqlar va sirtlarning global invariantlari nazariyasi va uning mexanikaga tatbiqlari” (2017-2020 yy), OT-F4-82 raqamli “Operatorlar va noassotsiativ algebralarda lokal differensiallash va avtomorfizmlar, nochiziqli dinamik sistemalarda faza almashishlar va xaos” (2017-2020 yy), YoFA-Ftex-2018-78 “Amenabel bo‘lmagan graflarda dinamik va termodinamik sistemalar” (2018-2019 yy) mavzusidagi ilmiy tadqiqot loyihalari doirasida bajarilgan.

Tadqiqot maqsadi chivin populyatsiyasi va jinsga bog‘liq bo‘lgan biologik sistemalarning evolyutsiya operatori sifatida qaraladigan aniq ratsional operatorlardan hosil qilingan diskret vaqtli dinamik sistemalarda ixtiyoriy boshlang‘ich nuqta uchun trayektoriyaning limit nuqtalari to‘plamini to‘la tavsiflashdan iborat.

Tadqiqotning vazifalari:

chivin populyatsiyasi va jinsga bog‘liq bo‘lgan biologik sistemlarning turg‘un holatini tavsiflovchi evolyutsion operatorlarning qo‘zg‘almas nuqtalari topish;

davriy nuqtalar to‘plamini tavsiflash va invariant to‘plamlarni aniqlash;

jinsga bog‘liq bo‘lgan populyatsiyaning evolyutsion operatorlaridan hosil qilingan trayektoriyalarning limit nuqtalarini tadqiq qilish;

yovvoyi chivin populyatsiyasidan hosil qilingan operator uchun trayektoriyalarning limit nuqtalari to‘plamini tavsiflash;

sog‘lom, kasallangan va tuzalganlarni aks ettiruvchi epidemik modellarining diskret vaqtli dinamikasini tavsiflash va Covid-19 virusi tarqalishiga tatbiq etish.

Tadqiqot ob’ekti: jinsga bog‘liq bo‘lgan populyatsiyaning evolyutsion operatorlari, yovvoyi chivin populyatsiyasining operatori, sog‘lom, kasallangan va tuzalganlarni aks ettiruvchi epidemik modellari.

Tadqiqot predmeti. Matematik analiz, funksional analiz, nochiziqli diskret vaqtli dinamik sistemalar nazariyasi.

Tadqiqot usullari. Tadqiqot ishida matematik analiz, funksional analiz, chiziqli algebra va diskret vaqtli dinamik sistemalar nazariyasi usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

ikki oʻlchamli va parametrlarning baʼzi shartlarida yuqori oʻlchamli kvadratik operator yordamida boshqariladigan jinsga bogʻliq evolyutsion operator uchun trayektoriyaning limit nuqtalari toʻplami toʻla tavsiflangan;

yovvoyi chivin populyatsiyasi bilan bogʻliq boʻlgan modelning diskret vaqtli dinamik sistemasiga mos evolyutsion operatorning barcha qoʻzgʻalmas nuqtalarini topilgan va qoʻzgʻalmas nuqtalarning tiplari aniqlangan;

parametrlarning alohida holatlari uchun mos keladigan modelning diskret vaqtli dinamikasi regulyar ekanligi isbotlangan hamda natijalarga biologik talqinlar berilgan;

sogʻlom, kasallangan va tuzalganlarni aks ettiruvchi epidemik modellarining diskret vaqtli dinamikasi tavsiflangan va uning Covid-19 virusi tarqalishiga qoʻllanishi koʻrsatilgan.

Tadqiqotning amaliy natijalari. Olingan natijalar va dissertatsiyada qoʻllanilgan usullar oliy oʻquv yurtlarida magistratura talabalari, tayanch doktorantlar uchun oʻquv kurs sifatida oʻqitilishi mumkin. Shuningdek, matematik biologiyadagi yovvoyi chivin populyatsiyasini nazorat qilish masalalarida foydalanish mumkin.

Tadqiqot natijalarining ishonchliligi. Tadqiqot natijalarida matematik va funksional analiz usullari, diskret vaqtli dinamik sistemalar nazariyasidan foydalanilgan. Olingan natijalar qatʼiy matematik mulohazalarga asoslanib isbotlangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati ratsional akslantirishlar dinamikasida trayektoriya limit nuqtalari toʻplamining tavsifidan nochiziqli diskret vaqtli dinamik sistemalar nazariyasida musbat orbitalarning topologik xossalarini tekshrishda foydalanish mumkinligi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati yovvoyi chivin populyatsiyasi bilan bogʻliq boʻlgan modelning diskret vaqtli dinamik sistemasiga mos evolyutsion operatorning qoʻzgʻalmas nuqtalari toʻplamining tavsifi baʼzi nochiziqli modellar uchun trayektoriyaning limit nuqtalari toʻplamini tavsiflash imkonini bergani bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Biologiyada ratsional akslantirishlarning dinamik sistemalari boʻyicha olingan natija asosida:

yovvoyi chivin populyatsiyasi bilan bogʻliq boʻlgan modelning diskret vaqtli dinamik sistemasiga mos evolyutsion operatorning qoʻzgʻalmas nuqtalari toʻplamining tavsifidan G00003447 raqamli “Kvant genetik algebralari va ularning tatbiqlari” mavzusidagi xorijiy grant loyihasida standart simpleksda aniqlangan bir nechta kvadratik stohastik operatorlar tomonidan hosil qilingan dinamik sistemalarni tekshirish uchun foydalanilgan (Birlashgan Arab Amirliklari universitetining 2023 yil 11-oktyabrdagi ma'lumotnomasi, BAA). Ilmiy natijaning qo'llanilishi nochiziqli modellar uchun trayektoriyaning limit nuqtalari to'plamini tavsiflash imkonini bergan;

ikki o'lchamli va parametrlarning ba'zi shartlarida yuqori o'lchamli kvadratik operator yordamida boshqariladigan jinsga bog'liq evolyutsion operator uchun trayektoriyaning limit nuqtalari to'plarini tavsiflashda qo'llangan usuldan FRGS 17-027-0593 raqamli "Xaos faza almashishini keltirib chiqaradi: p-adik Gibbs o'lchovlarining yangi sinfini topishda p-adik dinamik sistema usuli" mavzusidagi xorijiy grant loyihasida nochiziqli dinamik sistemalarni tadqiq qilishda foydalanilgan (Malayziya Xalqaro Islom universitetining 2023 yil 16-oktyabrdagi ma'lumotnomasi, Malayziya). Ilmiy natijaning qo'llanilishi nochiziqli operatorlarning davriy nuqtalarini aniqlash hamda trayektoriyalarni tahlil qilish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 9 ta ilmiy-amaliy anjumanlarda, jumladan 4 ta xalqaro va 5 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o'tkazilgan.

Tadqiqot natijalarining e'lon qilinganligi. Dissertatsiya tadqiqoti mavzusi bo'yicha jami 14 ta ilmiy ish chop etilgan, shulardan O'zbekiston Respublikasi Oliy Attestatsiya komissiyasining falsafa doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 5 ta maqola, jumladan 4 tasi xorijiy va 1 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, o'n bitta paragraf, xulosa va foydalanilgan adabiyotlar ro'yxatidan tashkil topgan. Dissertatsiyaning umumiy hajmi 103 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustivor yo'nalishlariga mosligi ko'rsatilgan, muammoning o'rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, ob'ekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo'yicha ma'lumotlar keltirilgan.

Dissertatsiyaning "**Jinsga bo'g'liq populyatsiyaning diskret-vaqt dinamik sistemalari va trayektoriyalari**" deb nomlanuvchi birinchi bobida dissertatsiya mavzusini to'la yoritish uchun zarur bo'lgan asosiy ta'riflar va muhim tushunchalar keltirilgan. Shuningdek, jinsga bog'liq evolyutsion operatorining diskret vaqtli dinamikasi ikki o'lchamli holida to'liq tadqiq qilingan. Bundan tashqari parametrlarning ba'zi shartlari uchun yuqori o'lchamli evolyutsion operator trayektoriyalarining xarakteri tadqiq qilingan.

Diskret vaqtli dinamik sistemani aniqlash uchun $f: X \to X, \quad X \subseteq \mathbb{R}$ funksiyani qaraylik. f funksiyaning o'z-o'ziga n marta kompozitsiyasi $f^n(x)$ kabi belgilanadi, ya'ni

$$f^n(x) = \underbrace{f(f(f...(f}_{n\ times}(x))...).$$

1**-ta'rif.** *Berilgan $x_0 \in X$ va $f: X \to X$ diskret vaqtli dinamik sistema uchun*

$$x_0,\ x_1 = f(x_0),\ x_2 = f^2(x_0),\ x_3 = f^3(x_0),\ \dots \qquad (1)$$

nuqtalar ketma-ketligi x_0 nuqtaning musbat orbitasi yoki musbat trayektoriyasi deyiladi.

2**-ta'rif.** *Agar $x \in X$ nuqta $f(x) = x$ tenglamani qanoatlantirsa, u holda x nuqta $f: X \to X$ funksiyaning qo'zg'almas nuqtasi deyiladi. Agar x nuqta $f^p(x) = x$ tenglamani qanoatlantirsa, u holda x nuqta f funksiyaning p davrli davriy nuqtasi deyiladi. $f^p(x) = x$ tenglikni qanoatlantiruvchi eng kichik musbat p soniga x nuqtaning asosiy davri deyiladi.*

Barcha qo'zg'almas va p davrli davriy nuqtalar to'plamini mos ravishda $Fix(f)$ va $\mathrm{P}er_p(f)$ orqali belgilanadi.

Ushbu

$$x_k' = F_i(x_1, x_2, \dots, x_m),\ k = 1, 2, \dots, m, \qquad (2)$$

nochiziqli

$$F: x = (x_1, x_2, ..., x_m) \in \mathbb{R}^m \to x' = F(x) = (x_1', x_2', ..., x_m') \in \mathbb{R}^m$$

akslantirishni qaraylik, bu yerda $F_i: \mathbb{R}^m \to \mathbb{R},\ \ i = 1, \dots, m$ uzluksiz differensial-lanuvchi bir qiymatli fuksiyalardir.

F akslantirish mos J_F Yakobi matritsasining xos qiymatlari chiziqli bo'lmagan dinamik sistemaning lokal xarakterini aniqlaydi. Ushbu fikrni yanada aniqroq bayon qilamiz.

3**-ta'rif.** x^* *nuqta F akslantirishning qo'zg'almas nuqtasi bo'lsin.*

- *agar F akslantirishning x^* nuqtadagi Yakobi matritsasi J_F birlik aylanada xos qiymatlarga ega bo'lmasa x^* giperbolik nuqta deyiladi;*
- *agar $J_F(x^*)$ Yakobi matritsaning barcha xos qiymatlarining absolyut qiymati 1 dan kichik bo'lsa, x^* nuqtaga tortuvchi qo'zg'almas nuqta deyiladi;*
- *agar $J_F(x^*)$ Yakobi matritsaning barcha xos qiymatlarining absolyut qiymati 1 dan katta bo'lsa, x^* nuqtaga itaruvchi qo'zg'almas nuqta deyiladi;*
- *qolgan hollarda x^* egar nuqta deyiladi.*

Topologik qo'shma tushunchasi dinamik sistemalarni o'rganishda muhim ahamiyatga ega.

4**-ta'rif.** *$f: U \to U$ va $g: V \to V$ akslantirishlar berilgan bo'lsin. Agar f va g akslantirishlar uchun shunday $h: U \to V$ gomeomorfizm topilib, $h \circ f = g \circ h$ tenglik bajarilsa, u holda f va g akslantirishlar topologik qo'shma deyiladi.*

$E = \{1, ..., m\}$ to'plamda

$$S^{m-1}=\{x=(x_1,...,x_m)\in\mathbb{R}^m : x_i\geq 0, \sum_{i=1}^{m} x_i=1\}. \tag{3}$$

kabi berilgan barcha ehtimollik o‘lchovlari to‘plamiga $(m-1)$ - o‘lchamli simpleks deyiladi.

5-ta’rif. *(3) simpleksni o‘ziga akslantiruvchi hamda*

$$W:\ x'_k=\sum_{i=1}^{m}\sum_{j=1}^{m} P_{ij,k}x_i x_j, \qquad k=1,...,m, \tag{4}$$

ko‘rinishda aniqlangan $W: S^{m-1}\to S^{m-1}$ *akslantirish kvadratik stoxastik operator (KSO) deyiladi, bu yerda* $P_{ij,k}$ *koeffisiyentlar quyidagi shartlarni qanoatlantiradi*

$$P_{ij,k}\geq 0,\ P_{ij,k}=P_{ji,k},\ \sum_{k=1}^{m} P_{ij,k}=1, \quad (i,j,k=1,...,m). \tag{5}$$

Agar $z\in S^{m-1}$ populyatsiyaning hozirgi avlodni ifodalovchi har bir ehtimollik holati uchun $z'\in S^{m-1}$ kelgusi avlodni tasvirlab bersa, $z\to z'$, $W: S^{m-1}\to S^{m-1}$ evolyutsion operator deb ataladi.

Populyatsiyaning holatlari quyidagi diskret vaqtli dinamik sistema orqali tavsiflanadi

$$z,\ z^{(1)}=s'=W(z),\ z^{(2)}=W^2(z),\ z^{(3)}=W^3(z),... .$$

Berilgan W operatordan hosil qilingan dinamik sistema uchun asosiy masala ixtiyoriy $z^{(0)}\in S^{m-1}$ boshlang‘ich nuqtaning $\{z^{(n)}\}_{n=0}^{\infty}$ trayektoriyasi limit holatini tavsiflashdir.

Har qanday $a\in(0,1)$ uchun quyidagi belgilashni qilamiz:

$$S_a\equiv S_a^{2m-2}=\left\{z=(x_1,...,x_m,y_1,...,y_m)\in S: \sum_{i=1}^{m} x_i=a, \sum_{j=1}^{m} y_j=1-a\right\}.$$

Birinchi bobda ushbu

$$W_a: \begin{cases} x_k'=(1-a)^{-1}\sum_{i=1}^{m}\sum_{p=1}^{m}\theta_{ipk}x_i y_p, \\ y_k'=a^{-1}\sum_{i=1}^{m}\sum_{p=1}^{m}\theta_{ipk}x_i y_p. \end{cases} \tag{6}$$

evolyutsion operator $W_a: S_a\to S_a$ tomonidan hosil qilingan dinamik sistema tadqiq qilingan.

1-lemma. *Agar* $z=(x,y)\in S_a$ *nuqta* W_a *operatorning qo‘zg‘almas nuqtasi bo‘lsa, u holda* $y_k=\beta x_k$, $k=1,\dots,m$ *bo ‘ladi, bu yerda* $\beta=(1-a)/a$.

Ushbu lemma yordamida W_a operatorning qo‘zg‘almas nuqtalarini topish masalasi quyidagi sistemaning yechimiga keltiriladi

$$x_k=a^{-1}\sum_{i=1}^{m}\sum_{p=1}^{m}\theta_{ipk}x_i x_p,\ k=1,\dots,m. \tag{7}$$

2-lemma. *Agar* $z^{(0)}=(x_1^{(0)},\dots,x_m^{(0)},y_1^{(0)},\dots,y_m^{(0)})\in S_a$ *boshlang'ich nuqta va* $z^{(n)}=(x_1^{(n)},\dots,x_m^{(n)},y_1^{(n)},\dots,y_m^{(n)})=W_a^n(z^{(0)})$ *bo'lsa, u holda har qanday* $k=1,\dots,m,\ n\geq 1$ *uchun* $y_k^{(n)}=\beta x_k^{(n)}$ *bo'ladi.*

Umuman olganda, (7) sistemani yechish qiyin, shuning uchun $m=2$ holatni qarab chiqamiz. Bu holda $x_2=a-x_1$ tenglikdan foydalanib yechimlari

$$x_{1,1}=\frac{a}{2}\cdot\frac{2\theta_3-\theta_2+1+\sqrt{(\theta_2-1)^2+4\theta_3(1-\theta_1)}}{\theta_1+\theta_3-\theta},$$
$$x_{1,2}=\frac{a}{2}\cdot\frac{2\theta_3-\theta_2+1-\sqrt{(\theta_2-1)^2+4\theta_3(1-\theta_1)}}{\theta_1+\theta_3-\theta}, \tag{8}$$

bo'lgan to'rt parametrli kvadrat tenglamaga keltiriladi, bu yerda

$$\theta_1=\theta_{111},\ \theta_2=\theta_{121}+\theta_{211},\ \theta_3=\theta_{221}.$$

Shunday qilib parametrlar

$$(\theta_2-1)^2+4\theta_3(1-\theta_1)\geq 0 \tag{9}$$

shartni qanoatlantirsa, qo'zg'almas nuqtalar quyidagicha bo'ladi:

$$z_j=(x_{1,j},x_{2,j},y_{1,j},y_{2,j}),\ j=1,2,$$

bu yerda $x_{2,i}=a-x_{1,i}$ va $y_{i,j}=\beta x_{i,j}$.

S_a^2 to'plamdan olingan har qanday boshlang'ich $z^{(0)}=\left(x_1^{(0)},x_2^{(0)},y_1^{(0)},y_2^{(0)}\right)$ nuqta trayektoriyasining limit nuqtalarini tavsifi uchun quyidagi teorema o'rinli.

1-teorema. $z^{(0)}=\left(x_1^{(0)},x_2^{(0)},y_1^{(0)},y_2^{(0)}\right)\in S_a^2\setminus Fix(W_a)$ *ixtiyoriy boshlang'ich nuqta bo'lsin.*

1) *agar* $\theta_1+\theta_3=\theta_2$ *va* $\theta_1\neq 1,\theta_3\neq 0$ *yoki* $\theta_1\neq 0,\theta_3\neq 1$ *bo'lsa, u holda*

$$\lim_{n\to\infty}W_a^{(n)}\left(z^{(0)}\right)=(x_1^*,a-x_1^*,\beta x_1^*,\beta(a-x_1^*)).$$

2) *agar* $\theta_1\neq\theta_2,\theta_1\in[0;1),\theta_2\in[0;1],\theta_3=0$ *yoki* $\theta_1=1,\theta_2\in[0;1),\theta_3=0$ *bo'lsa, u holda*

$$\lim_{n\to\infty}W_a^{(n)}\left(z^{(0)}\right)=(0,a,0,1-a).$$

3) *agar* $\theta_1+\theta_3\neq\theta_2,\theta_1=1,\theta_2\in[1;2],\theta_3\in(0;1]$ *yoki* $\theta_1=1,\theta_2\in(1;2],\theta_3=0$ *bo'lsa, u holda*

$$\lim_{n\to\infty}W_a^{(n)}\left(z^{(0)}\right)=(a,0,1-a,0).$$

4) *agar* $\theta_1+\theta_3\neq\theta_2,(\theta_2-1)^2+4\theta_3(1-\theta_1)\leq 4$ *va* $\theta_1\in(0;1),\theta_2\in[0;2],\theta_3\in(0;1]$ *yoki* $\theta_1=0,\theta_2\in[0;2],\theta_3\in(0;1)$ *bo'lsa, u holda*

$$\lim_{n\to\infty}W_a^{(n)}\left(z^{(0)}\right)=(x_2^*,a-x_2^*,\beta x_2^*,\beta(a-x_2^*)).$$

5) *agar* $\theta_1=1,\theta_2\in[0;1),\theta_3\in(0;1)$ *bo'lsa, u holda*

$$\lim_{n\to\infty}W_a^{(n)}\left(z^{(0)}\right)=(x_3^*,a-x_3^*,\beta x_3^*,\beta(a-x_3^*)).$$

6) *agar* $\theta_1 \in [0;1), \theta_2 \in (1;2], \theta_3 = 0$ *bo'lsa, u holda*

$$\lim_{n\to\infty} W_a^{(n)}\left(z^{(0)}\right) = (x_4^*, a - x_4^*, \beta x_4^*, \beta(a - x_4^*)).$$

7) *agar* $\theta_1 = 0, \theta_2 = 1, \theta_3 = 1$ *bo'lsa, u holda*

$$\lim_{n\to\infty} W_a^{(n)}\left(z^{(0)}\right) = \begin{cases} (x_1^{(0)}, a - x_1^{(0)}, \beta x_1^{(0)}, \beta(a - x_1^{(0)})), & agar \;\; n = 2k, \; k = 0,1,2,... \\ (a - x_1^{(0)}, x_1^{(0)}, \beta(a - x_1^{(0)}), \beta x_1^{(0)}), & agar \;\; n = 2k+1. \end{cases}$$

8) *Agar* $\theta_1 + \theta_3 \neq \theta_2, (\theta_2 - 1)^2 + 4\theta_3(1-\theta_1) > 4$ *va* $\theta_1 \in (0;1), \theta_2 \in [0;2], \theta_3 \in (0;1]$ *yoki* $\theta_1 = 0, \theta_2 \in [0;2], \theta_3 \in (0;1)$ *yoki* $\theta_1 = 0, \theta_2 \in [0;1) \cup (1;2], \theta_3 = 1$ *bo'lsa, u holda*

$$\lim_{n\to\infty} W_a^{(n)}\left(z^{(0)}\right) \subset \{\left(p_1, a - p_1, \beta p_1, \beta(a - p_1)\right), \left(p_2, a - p_2, \beta p_2, \beta(a - p_2)\right)\},$$

bu yerda

$$x_1^* = \frac{\theta_3 a}{\theta_3 - \theta_1 + 1}, \qquad x_2^* = \frac{a}{2} \cdot \frac{2\theta_3 - \theta_2 + 1 - \sqrt{(\theta_2 - 1)^2 + 4\theta_3(1-\theta_1)}}{\theta_1 + \theta_3 - \theta}, \qquad x_3^* = \frac{\theta_3 a}{\theta_3 - \theta_2 + 1},$$

$$x_4^* = \frac{(1-\theta_2)a}{\theta_1 - \theta_2}, \; p_{1,2} = \frac{2\theta_3 - \theta_2 - 1 \pm \sqrt{(\theta_2 - 1)^2 + 4\theta_3(1-\theta_1) - 4}}{2(\theta_1 + \theta_3 - \theta_2)} a.$$

Irsiyat koeffitsiyentlari θ_{ipk} quyidagi shartlarni qanoatlantirsin

$$\theta_{ipk} = \begin{cases} c_{ik}, & agar \;\; p = j, \\ 1, & agar \;\; p \neq j, k = l, \qquad j, l = 1, \ldots, m. \\ 0, & agar \;\; p \neq j, k \neq l, \end{cases}$$

bu yerda

$$c_{ik} = \begin{cases} 1, & i = k, \\ 0, & i \neq k. \end{cases}$$

U holda S_a^{2m-2} to'plamdan olingan har qanday boshlang'ich $z^{(0)}$ nuqta trayektoriyasining limit nuqtalarini tavsifi uchun quyidagi teorema o'rinli bo'ladi.

2-teorema. $z^{(0)} = \left(x_1^{(0)}, ..., x_m^{(0)}, y_1^{(0)}, ..., y_m^{(0)}\right) \in S_a^{2m-2}$ *ixtiyoriy boshlang'ich nuqta bo'lsin.*

1) *agar* $j = l$ *bo'lsa, u holda*

$$\lim_{n\to\infty} W^{(n)}\left(z^{(0)}\right) = \lim_{n\to\infty} z^{(n)} = (\underbrace{0,...,0,a}_{j}, \underbrace{0,...0}_{m-j}, \underbrace{0,...,0,1-a}_{j}, \underbrace{0,...,0}_{m-j}).$$

2) *agar* $j \neq l$ *bo'lsa, u holda*

$$\lim_{n\to\infty} W^{(n)}\left(z^{(0)}\right) = \lim_{n\to\infty} z^{(n)} = \begin{cases} z_{1,l} = (\underbrace{0,...,0,a}_{l}, \underbrace{0,...0}_{m-l}, \underbrace{0,...,0,1-a}_{l}, \underbrace{0,...,0}_{m-l}), & agar \;\; x_j^{(0)} \neq a, \\ z_{2,j} = (\underbrace{0,...,0,a}_{j}, \underbrace{0,...0}_{m-j}, \underbrace{0,...,0,1-a}_{j}, \underbrace{0,...,0}_{m-j}), & agar \;\; x_j^{(0)} = a. \end{cases}$$

Dissertatsiyaning **"Chivin populyatsiyasining diskret vaqtli dinamik sistemasi"** deb nomlangan ikkinchi bobida ikki o'zgaruvchidan iborat beshta parametrli chivin populatsiyasi modeli tuzilgan hamda uning dinamikasi tavsiflangan.

Quyidagi $W:\mathbb{R}^2\to\mathbb{R}^2$ operatorni qaraylik:

$$\begin{cases} x'=\beta y-\dfrac{\alpha x}{1+x}-(d_0+d_1x)x+x, \\ y'=\dfrac{\alpha x}{1+x}-\mu y+y \end{cases} \tag{10}$$

bu yerda $\alpha>0,\beta>0,\mu>0,d_0\geq 0,d_1\geq 0.$

(10) orqali berilgan W operator $\mathbb{R}^2\setminus\{(x,y):x=-1\}$ to‘plamda korrekt aniqlangan, lekin uzluksiz operatorning dinamik sistemasini populyatsiya deb qaralgani uchun $x\geq 0$ va $y\geq 0$ deb olamiz. Shuning uchun W operatorning parametrlarini shunday tanlab olamizki, $W:\mathbb{R}^2_+\to\mathbb{R}^2_+$ o‘rinli bo‘lsin, bu yerda $\mathbb{R}^2_+=\{(x,y):x,y\in\mathbb{R},x\geq 0,y\geq 0\}$.

Faraz qilaylik $\beta=\mu,\ d_0=d_1=0$ va $S_1=\{(x,y):x,y\in\mathbb{R}_+,x+y=1\}$ bo‘lsin.

Ushbu $W\left(S_1\right)\subset S_1$ munosabatni qanoatlantiruvchi S_1 to‘plam W operatorga nisbatan invariant to‘plam deyiladi.

Har bir berilgan $c>0$ son uchun $S_c=\{(x,y)\in\mathbb{R}^2_+:\ x+y=c\}$ to‘plam (10) orqali berilgan W operatorga nisbatan invariant to‘plam bo‘ladi. Bundan tashqari barcha S_c to‘plamlar birlashmasi $\mathbb{R}^2_+$ to‘plamni hosil qiladi. Shuning uchun dinamik sistemani har bir berilgan $c>0$ son uchun S_c to‘plamda o‘rganish yetarli. W operatorning har bir berilgan $c>0$ son uchun S_c to‘plamdagi dinamikasi S_1 to‘plamdagi kabi bo‘ladi.

$\beta=\mu,\ d_0=d_1=0$ holat uchun S_1 to‘plamdan olingan har qanday boshlang‘ich $z^{(0)}$ nuqta trayektoriyasining limit nuqtalarini tavsifi uchun quyidagi teorema o‘rinli.

3-teorema. $z^{(0)}=\left(x^{(0)},1-x^{(0)}\right)\in S$ *ixtiyoriy boshlang‘ich nuqta bo‘lsin.*

1) *agar* $\left(\alpha,\beta\right)\in\left(A\cup B\right)\setminus\left\{\left(2;1\right)\right\}$ *bo‘lsa, u holda*

$$\lim_{n\to\infty}W_0^n\left(z^{(0)}\right)=\left(\frac{\sqrt{\alpha^2+4\beta^2}-\alpha}{2\beta},\frac{2\beta+\alpha-\sqrt{\alpha^2+4\beta^2}}{2\beta}\right).$$

2) *agar* $\alpha=2,\beta=1$ *bo‘lsa, u holda*

$$\lim_{n\to\infty}W_0^n\left(z^{(0)}\right)=\begin{cases}\left(x^{(0)},1-x^{(0)}\right), & agar\ \ n=2k,k=0,1,2,... \\ \left(\dfrac{1-x^{(0)}}{1+x^{(0)}},\dfrac{2x^{(0)}}{1+x^{(0)}}\right), & agar\ \ n=2k+1.\end{cases}$$

Faraz qilaylik $\beta\neq\mu,\ d_0=d_1=0$ bo‘lsin.

Agar

$$0<\alpha\leq 1,\ \beta>0,\ 0<\mu\leq 1 \tag{11}$$

bo‘lsa, u holda (10) orqali berilgan W operator $\mathbb{R}_{+}^{2}$ to‘plamni o‘zini o‘ziga o‘tkazadi.

4-teorema. *(10) orqali berilgan W operator ((11) shartlar ostida) va ixtiyoriy $(x^{(0)}, y^{(0)}) \in \mathbb{R}_{+}^{2} \setminus Fix(W)$ boshlang‘ich nuqta uchun quyidagilar o‘rinli:*

$$\lim_{n\to\infty} x^{(n)} = \begin{cases} 0, & agar \ \ \beta < \mu, \\ +\infty, & agar \ \ \beta > \mu, \end{cases}$$

$$\lim_{n\to\infty} y^{(n)} = \begin{cases} 0, & agar \ \ \beta < \mu, \\ \alpha / \mu, & agar \ \ \beta > \mu, \end{cases}$$

bu yerda $\left(x^{(n)}, y^{(n)}\right) = W^{n}\left(x^{(0)}, y^{(0)}\right)$.

Faraz qilaylik $d_0 \neq 0$ bo‘lsin.

1-tasdiq. *Agar*

$$\alpha > 0, \beta > 0, 0 < \mu \leq 1, d_0 > 0, \alpha + d_0 \leq 1, d_1 = 0 \tag{12}$$

bo‘lsa, u holda (10) orqali berilgan W operator $\mathbb{R}_{+}^{2}$ to‘plamni o‘zini o‘ziga o‘tkazadi.

2-tasdiq. *(10) orqali berilgan W operator qo‘zg‘almas nuqtalari va ularning tipi quyidagicha:*

1) *agar* $\beta \leq \mu\left(1 + \dfrac{d_0}{\alpha}\right)$ *bo‘lsa, u holda (10) operator ((12) shartlar ostida) yagona* $(0,0)$ *qo‘zg‘almas nuqtaga ega va*

$$(0,0) - \begin{cases} tortuvchi, & agar \ \ \beta < \mu\left(1 + \dfrac{d_0}{\alpha}\right), \\ nogiperbolik, & agar \ \ \beta = \mu\left(1 + \dfrac{d_0}{\alpha}\right). \end{cases}$$

2) *agar* $\beta > \mu\left(1 + \dfrac{d_0}{\alpha}\right)$ *bo‘lsa, u holda (10) operator ikkita* $(0,0)$ *va* (x^{*}, y^{*}) *qo‘zg‘almas nuqtalarga ega, shuningdek* (x^{*}, y^{*}) *qo‘zg‘almas nuqta toryuvchi va*

$$(0,0) - \begin{cases} itaruvchi, & agar \ \ \beta > \mu\left(1 + \dfrac{d_0}{\alpha}\right) + \alpha^{*}, \\ egar, & agar \ \ \mu\left(1 + \dfrac{d_0}{\alpha}\right) < \beta < \mu\left(1 + \dfrac{d_0}{\alpha}\right) + \alpha^{*}, \\ nogiperbolik, & agar \ \ \beta = \mu\left(1 + \dfrac{d_0}{\alpha}\right) + \alpha^{*}, \end{cases}$$

bu yerda $\alpha^{*} = \left(4 - 2(\alpha + \mu + d_0)\right) / \alpha$.

5-teorema. *$p \geq 2$ holatlar uchun (10) orqali berilgan W operator ((12) shartlar ostida) $\mathbb{R}_{+}^{2}$ to‘plamda p-davrli davriy nuqtalarga ega emas.*

Ushbu

$$\Omega = \left\{ (x, y) \in \mathbb{R}_+^2,\ 0 \le x \le \frac{\alpha\beta}{\mu d_0},\ 0 \le y \le \frac{\alpha}{\mu} \right\}$$

to‘plamni qaraylik. Ω to‘plamni quyidagicha to‘rtta qismga ajratamiz:

$$\Omega_1 = \left\{ (x, y) \in \Omega,\ 0 \le x \le x^*,\ 0 \le y \le y^* \right\},$$

$$\Omega_2 = \left\{ (x, y) \in \Omega,\ x^* \le x \le \frac{\alpha\beta}{\mu d_0},\ y^* \le y \le \frac{\alpha}{\mu} \right\},$$

$$\Omega_3 = \left\{ (x, y) \in \Omega,\ x^* < x \le \frac{\alpha\beta}{\mu d_0},\ 0 \le y \le y^* \right\},$$

$$\Omega_4 = \left\{ (x, y) \in \Omega,\ 0 \le x < x^*,\ y^* < y \le \frac{\alpha}{\mu} \right\},$$

bu yerda

$$x^* = \frac{\alpha(\beta - \mu)}{\mu d_0} - 1,\ y^* = \frac{\alpha(\beta - \mu) - \mu d_0}{\mu(\beta - \mu)}. \tag{13}$$

3-tasdiq. Ω_1, Ω_2 *va* Ω *to‘plamlar* W *operatorga nisbatan invariant to‘plamlardir.*

4-tasdiq. *Parametrlar* β, μ *((12) shartlar ostida)ning har qanday qiymatida*

$$\varphi(z) = \mu x + \beta y$$

funksiya W *operator uchun Lyapunov funksiyasi bo‘ladi.*

1-Izoh. *Agar* $\beta \le \mu\left(1 + \frac{d_0}{\alpha}\right)$ *bo‘lsa, u holda* $\varphi(z)$ *Lyapunov funksiyasi* Ω *to‘plamda aniqlanadi, agar* $\beta > \mu\left(1 + \frac{d_0}{\alpha}\right)$ *bo‘lsa, u holda* $\varphi(z)$ *Lyapunov funksiyasi* $\Omega_1 \cup \Omega_2$ *to‘plamda aniqlanadi.*

6-teorema. *(10) orqali berilgan* W *operator ((12) shartlar ostida) va ixtiyoriy* $(x^{(0)}, y^{(0)}) \in \mathbb{R}_+^2 \setminus Fix(W)$ *boshlang‘ich nuqta uchun quyidagilar o‘rinli:*

1) *agar har qanday* n *natural sonlar uchun* $y^{(n)} > \frac{\alpha}{\mu}$ *bo‘lsa, u holda*

$$\lim_{n\to\infty} x^{(n)} = +\infty,\ \lim_{n\to\infty} y^{(n)} = \frac{\alpha}{\mu};$$

2) *agar shunday* n_0 *natural son topilib* $y^{(n_0)} \le \frac{\alpha}{\mu}$ *bo ‘lib* $\beta \le \mu\left(1 + \frac{d_0}{\alpha}\right)$ *bo‘lsa, u holda* $\lim_{n\to\infty} x^{(n)} = 0, \lim_{n\to\infty} y^{(n)} = 0,$

boshqa hollarda, ya’ni agar $\beta > \mu\left(1 + \frac{d_0}{\alpha}\right)$ *bo‘lsa, u holda* $\lim_{n\to\infty} x^{(n)} = x^*, \lim_{n\to\infty} y^{(n)} = y^*.$

Dissertatsiyaning **“Chivin populyatsiyasi modeli tomonidan hosil qilingan normallangan operatorning dinamikasi va epidemik modellar”** deb nomlangan uchinchi bobida, normallangan operator hosil qilgan dinamik sistema tadqiq qilingan. Bundan tashqari kasallikning tarqalishining SIS (susceptible-infected-susceptible) va SIR (susceptible-infected-recovered) modeli bilan bog‘liq diskret vaqtli dinamik sistemalar o‘rganilgan.

Quyidagi (10) orqali berilgan W operatorning ((12) shartlar ostida) normallashtirilgan ko‘rinishini qaraylik:

$$U:\begin{cases} x'=\dfrac{(\beta y+(1-d_0)x)(1+x)-\alpha x}{(1+x)((\beta-\mu+1)y+(1-d_0)x)}, \\ y'=\dfrac{\alpha x+(1+x)(1-\mu)y}{(1+x)((\beta-\mu+1)y+(1-d_0)x)}. \end{cases} \tag{14}$$

(10) shartlardan $x'\geq 0$ va $y'\geq 0$ bo‘ladi, bundan tashqari $x'+y'=1$. Shuning uchun $U:S_1\to S_1$.

5-tasdiq. *(14) orqali berilgan U operator yagona $z^*=\left(x^*,1-x^*\right)$ tortuvchi qo‘zg‘almas nuqtaga ega,*
bu yerda

$$x^*=\begin{cases} \dfrac{\sqrt{\alpha^2+4\beta^2}-\alpha}{2\beta}, & agar\ \beta+d_0=\mu, \\ \dfrac{\sigma(\beta^2-3(\alpha-\mu+\beta+d_0)(\mu-\beta-d_0))+1/\sigma-\beta}{3(\mu-\beta-d_0)}, & agar\ \beta+d_0\neq\mu, \end{cases}$$

$$\sigma=\sqrt[3]{2/(9\beta e(\alpha+2e)-2\beta^3+\sqrt{4(-\beta^2+3e(\alpha-e))^3+(9\beta e(\alpha+2e)-2\beta^3)^2})},$$
$$e=\mu-\beta-d_0.$$

7-teorema. *$p\geq 2$ holatlar uchun (14) orqali berilgan U operator S_1 to‘plamda p-davrli davriy nuqtalarga ega emas.*

S_1 to‘plamdan olingan har qanday boshlang‘ich $z^{(0)}$ nuqta trayektoriyasining limit nuqtalarini tavsifi uchun quyidagi teorema o‘rinli.

8-teorema. *(14) orqali berilgan U operator ((12) shartlar ostida) va ixtiyoriy $z^{(0)}=\left(x^{(0)},y^{(0)}\right)\in S_1\setminus Fix(U)$ boshlang‘ich nuqta uchun quyidagilar o‘rinli:*

$$\lim_{n\to\infty}U^n\left(z^{(0)}\right)=\begin{cases} \left(\tilde{x},1-\tilde{x}\right), & agar\ \ \mu=\beta+d_0, \\ \left(\hat{x},1-\hat{x}\right), & agar\ \ \mu\neq\beta+d_0, \end{cases}$$

bu yerda

$$\tilde{x}=\frac{\sqrt{\alpha^2+4\beta^2}-\alpha}{2\beta},\quad \hat{x}=\frac{\sigma(\beta^2-3(\alpha-\mu+\beta+d_0)(\mu-\beta-d_0))+1/\sigma-\beta}{3(\mu-\beta-d_0)}$$

qo‘zg‘almas nuqtalar.

Uchinchi bobning so‘nggi paragrafida epidemik SIS modelining quyidagi diskret ko‘rinishi uchun qo‘zg‘almas nuqtalar, ularning turlari, har qanday boshlang‘ich nuqta trayektoriyasining limit nuqtalarini tavsifi masalalari qaralgan:

$$\begin{cases} S'=S-\beta SI+\nu I, \\ I'=I+\beta SI-\nu I. \end{cases} \tag{15}$$

bu yerda $\beta > 0, \nu > 0$.

Ushbu $S' + I' = S + I = N$ tenglikdan foydalanib (15) sistemani quyidagicha yozish mumkin.

$$f(I) = I(1-\nu+\beta N) - \beta I^2 \tag{16}$$

Quyidagilarni belgilaymiz.

$$\Omega = [0;N],\ R_0 = \frac{\beta N}{\nu},$$

$$\Theta = \left\{ (\beta,\nu) \in \mathbb{R}^2 : \nu \in (0;1], \beta \in \left(0; \frac{(1+\sqrt{\nu})^2}{N} \right] \right\},$$

$$\Theta^* = \left\{ (\beta,\nu) \in \mathbb{R}^2 : \nu \in \left(\frac{1}{4};1 \right], \beta \in \left(\frac{2+\nu}{N}; \frac{(1+\sqrt{\nu})^2}{N} \right] \right\}.$$

9-teorema. *$I^{(0)} \in \Omega \setminus Fix(f)$ ixtiyoriy boshlang'ich nuqta bo'lsin. $\Theta \setminus \Theta^*$ to'plamdagi shartlar ostida (16) orqali berilgan f funksiya uchun quyidagilar o'rinli:*

$$\lim_{n\to\infty} f^n(I^{(0)}) = \begin{cases} 0, & agar\ \ R_0 \le 1, \\ \dfrac{\beta N - \nu}{\beta}, & agar\ \ R_0 > 1. \end{cases}$$

XULOSA

Dissertatsiya evolyutsion operatorlardan hosil qilingan dinamik sistemalarni tavsiflashga bag'ishlangan.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1. Ikki o'lchamli kvadratik operator yordamida boshqariladigan jinsga bog'liq evolyutsion operatorining diskret vaqtli dinamikasi to'liq tadqiq qilingan.
2. Parametrlarning ba'zi shatlarida yuqori o'lchamli evolyutsion operator uchun barcha trayektoriyalarning limit nuqtalar to'plami tavsiflangan.
3. Yovvoyi chivin populyatsiyasi bilan bog'liq bo'lgan modelning diskret vaqtli dinamik sistemasi uchun evolyutsion operatorining barcha qo'zg'almas nuqtalarini (besh parametrga bog'liq) topilgan va parametrlarga qarab ushbu operator bitta, ikkita qo'zg'almas nuqtalarga ega bo'lishi mumkinligini ko'rsatilgan va ularni tipi aniqlangan.
4. Parametrlarning alohida holatlari uchun mos keladigan modelning diskret vaqtli dinamikasi regulyar ekanligi isbotlangan.
5. Sog'lom, kasallangan va tuzalganlarni aks ettiruvchi epidemik modellar uchun diskret vaqtli dinamika o'rganilgan va uning Covid-19 virusi tarqalishiga qo'llanishi ko'rsatilgan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED AFTER V.I. ROMANOVSKIY

INSTITUTE OF MATHEMATICS

BOXONOV ZAFAR SAYDIMAXMUDOVICH

DYNAMICAL SYSTEMS OF RATIONAL MAPPINGS IN BIOLOGY

01.01.01-Mathematical analysis

ABSTRACT OF THESIS OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT-2023

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O'ZBEKISTON RESPUBLIKASI FANLAR AKADEMIYASI V.I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI

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INTRODUCTION

Actuality and demand of the theme of the dissertation. In the world, numerous scientific and applied research works are reduced to the study of dynamical systems of rational operators. The importance of the theory of dynamical systems is to describe the future state of the system from its current state, and it plays an important role in solving problems of mechanics, physics, biology, medicine, economics and others. This helps people to understand complex systems and make optimal decisions. It should be noted that dynamical systems are usually represented by difference equations, differential equations, and differential-difference equations. Dynamical systems of rational evolution operators are one of the main objects of research in the fields of biology and ecology. Also, results related to dynamical systems generated from rational evolutionary operators serve as a basis for researching population evolution in mathematical biology issues. Therefore, the research of dynamical systems of rational operators remains one of the important and urgent tasks in the theory of dynamical systems.

Currently, the theory of rational dynamical systems is used as the main tool in understanding the nature of many practical problems, analyzing them and finding the optimal solution. In the theory of dynamical systems, problems describing the set of limit points of the trajectory, determining the existence of fixed and periodic points and their types, finding invariant sets, checking the occurrence of bifurcations, and studying whether the system is stable or chaotic are widely used in analysis of many practical issues. In this regard, the creation of optimal mathematical models based on in-depth biological research and their application to population problems of mathematical biology are targeted scientific studies.

In our country much attention has been paid to medicine, biology, mathematics and physical sciences, which have scientific and practical application of fundamental sciences. In particular, in recent years, special attention has been given to the development of the theory of dynamic systems, which has wide application in economical, physical and biological systems. Significant results have been achieved on the dynamics of discrete-time continuous rational operators. Conducting scientific research at the level of international standards in the priority areas of “Functional analysis, mathematical physics and statistical physics” was defined as the main tasks and areas of activity of mathematics[1]. At present, the development of investigations on the dynamical systems of rational operators plays significant role in the implementation of this decree. Thus analyze the dynamical systems of rational operators is important in biology and related fields of science, to ensure the execution of decisions.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees of the President of the Republic of Uzbekistan UP-4947

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[1]Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May “On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan” No. 292 dated May 17, 2017.

of February 7, 2017 “On the strategy of action for the further development Of the Republic of Uzbekistan”, UP-60 dated January 28, 2022 “Development Strategy of New Uzbekistan for the period of 2022-2026”, UP-2789 dated April 20, 2017 “On measures to further develop the system of higher education”, UP-2789 dated April 20, 2017 “On measures to further develop the system of higher education”, PP-4387 from July 9, 2019 “On measures to further development of mathematical education and science, and also root improvement of the activity of the Uzbekistan Academy of Sciences V.I.Romanovsky Institute of Mathematics”, and PP-4708 of May 7, 2020 “On measures to improve the quality of education and research in the field of mathematics” as well as in other regulations related to basic science.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV, “Mathematics, Mechanics and Computer Science”.

The degree of scrutiny of the problem. In recent years, the understanding of mathematical models of many problems of natural science has aroused interest in the study of dynamical systems with discrete time. Late 19th-century biologists began to develop techniques in population modeling in order to understand the dynamics of growing and shrinking of all populations of living organisms. English scientist Thomas Malthus, while thinking about the fate of humanity, was the first to justify the geometric growth of populations by creating the simplest model. In 1838, the Belgian mathematician Pierre Francois Verhulst developed the logistic model, which is one of the most important models of population growth. Due to the limited food resources on our planet, interest in population modeling increased by the 20th century. Population models have also been used to control the spread of parasites, viruses, and diseases.

The main mathematical problem for a certain population is to study the evolution of the population, that is, the dynamics of states, depending on time. The mathematical methods used in the study of this problem are based on functional analysis, differential equations, probability theory, random processes and the theory of dynamical systems.

Nonlinear (in particular, rational and quadratic) evolutionary operators for the study of population dynamics were introduced by O. Reyersall, Yu. I. Lyubich, H. Kesten and others. Mathematical modeling of many processes is reduced to discrete time rational dynamical systems. Discrete time rational dynamical systems are considered an important class of dynamical systems, and the research conducted by mathematicians such as M. Kulenovich, O. Merino, E. Pilav, D. Clark, C.D. Lynd, E. Camouzis, A. Gilbert, M. Heissan, G. Ladas, E. Drymonis, E.A. Grove, Y. Kostrov, M. A. Radin, S. Schultz in recent years plays an important role in the development of the theory of rational dynamical systems.

The book “Population dynamics: algebraic and probabilistic approach” written by U.A. Rozikov in 2020, presents the models of population. As well as several dynamical systems of biological models (J.M. Casas, M. Ladra, U. Rozikov), sex-dependent population dynamics (Y.I. Lyubich, U.Jamilov,

U. Rozikov), dynamic system and evolutionary algebra of mosquito population (M. Velasco, R. Varro, U. Rozikov) and ocean ecosystems (S. Shoyimardonov, U. Rozikov) are given.

Although rational discrete time dynamical systems have a very simple form, due to the lack of a general theory, it is difficult to fully analyze the dynamics of operators, and the study of each operator requires a separate scientific approach from the researcher. Nowadays, despite the fact that many scientific studies of dynamical systems of rational operators have been carried out, it should be noted that a complete description of the set of limit points for dynamical systems formed by many rational operators has not been obtained.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation research is done in accordance with the planned theme of scientific research OT-F4-87 “The theory of global invariants of curves and surfaces in Euclidean and pseudo-Euclidean spaces and its applications in mechanics” (2017-2020), OT-F4-82 “Local derivations and automorphisms of operator and nonassociative algebras, phase transitions and chaos in nonlinear dynamical systems” (2017-2020) and scientific research “YoFA-Ftex-2018-78, Dynamical and thermodynamical systems on non-amenable graphs” (2018–2019 y) at the V.I. Romanovskiy Institute of Mathematics.

The aim of research work is for any initial point to give a full description of the set of limit points for discrete-time dynamical systems generated by concrete rational operators considered as evolution operators of mosquito population and sex-linked biological systems.

Research problems:

to find fixed points of evolution operators, which describe the equilibrium states of the corresponding mosquito population and sex-linked biological system;

to describe a set of periodic points and determine invariant sets;

to study the limit points of trajectories generated by evolution operators of a sex-linked population;

to study the limit points of trajectories generated by the operator of a wild mosquito population;

to study of discrete-time dynamics of SIS (susceptible-infected-susceptible) and SIR (susceptible-infected-recovered) epidemic models and apply them to the spread of the Covid-19 virus.

The research object: evolution operators of a sex-linked population, operator of a wild mosquito population, SIS and SIR epidemic models.

The research subject: Mathematical analysis, functional analysis, theory of nonlinear discrete-time dynamical systems.

Research methods: In the research the methods of mathematical analysis, functional analysis, linear algebra and theory of discrete-time dynamical systems are used.

Scientific novelty of the research work consists of the following:

the discrete-time dynamics of the sex-linked evolution operator given by the two-dimensional quadratic operator is fully studied. Under some conditions on parameters high-dimensional cases are studied;

for the discrete-time dynamical system of the model related to the wild mosquito population all fixed points (depending on five parameters) of the evolutionary operator are found, and it is shown that depending on the parameters, this operator can have one, two fixed points, and their type are determined;

the discrete-time dynamics of the corresponding model under some conditions on parameters are fully analyzed, and biological interpretations of the results are given;

the discrete-time dynamics of the SIS and SIR epidemic models are studied and its application to the spread of the Covid-19 virus is given.

Practical results of the research. The results obtained and the methods used in the dissertation can be used as a training course for undergraduates and basic doctoral students in higher education institutions. It can also be used to solve wild mosquito population control problems in mathematical biology.

The reliability of the results of the study. The results have been obtained by using the methods of mathematical and functional analysis, theory of discrete-time dynamical systems. The obtained results are proved mathematically correct.

Scientific and practical significance of the research results. The scientific significance of the research results is explained by the fact that the description of the set of limit points of the trajectory in the dynamics of rational reflection can be used in the theory of nonlinear dynamical systems with discrete time.

The practical significance of the research results is determined by their application to population and mathematical biology by determining the set of boundary points of the trajectory in dynamical systems built using rational reflections.

Implementation of the research results. Based on the obtained results on dynamical systems of rational mappings in biology:

the discrete-time dynamic system of the mathematical model of the mosquito population was used to investigate the dynamic systems generated by several quadratic stochastic operators defined on the standard simplex in the foreign grant project number G00003447 on the topic “Quantum Genetic Algebras and their applications” (Reference of United Arab Emirates University, October 11, 2023, UAE). The application of this scientific result enabled to describe the set of limit points of the trajectory for some nonlinear models;

the dynamics of evolution operators was used to research of nonlinear dynamical systems in the foreign grant project FRGS 17-027-0593 “The chaos implies the phase transition: p-adic dynamical system approaches to the quest for a new class of p-adic Gibbs measures” (Reference of the International Islamic University of Malaysia, 16 October 2023, Malaysia). The application of this scientific result enabled to determine fixed points and periods of nonlinear operators and to analyze trajectories.

Approbation of the research results. The main results of the research have been discussed at 4 international and 5 national scientific conferences.

Publications of the research results. On the topic of the dissertation 14 research papers have been published in the scientific journals, 5 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis, in addition 4 of them were published in international journals and 1 paper published in national mathematical journals.

The structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The general volume of the thesis is 103 pages.

THE MAIN CONTENT OF THE THESIS

The **introduction** of the thesis includes the motivation of the research, the relevance of the research to the priorities of science and technology, the review of foreign research on the topic, the degree of scrutiny of the problem, the aim, research problems, object and subject of research, scientific novelty and practical results, theoretical and practical significance of the results obtained, the statement of research results, published works and information on the structure of the thesis.

In the first chapter of the thesis, titled **"Discrete-time dynamical systems and trajectories of a sex-linked population"** we give main definitions and important notions necessary to cover the dissertation and research the subject. Also, the discrete-time dynamics of the sex-linked evolution operator has been fully studied in two dimensions. In addition, for some conditions of the parameters, the character of the trajectories of the evolution operator in the high-dimensional case was studied.

Let $X \subseteq \mathbb{R}$. To define a discrete-time dynamical system we consider a function $f : X \to X$.

For $x \in X$ denote by $f^n(x)$ the n-fold composition of f with itself (i.e. n time iteration of f to x):

$$f^n(x) = \underbrace{f(f(f\dots(f}_{n\ times}(x))\dots).$$

Definition 1. *For arbitrary given $x_0 \in X$ and $f : X \to X$ the discrete-time dynamical system (also called forward orbit or trajectory of x_0) is the sequence of points*

$$x_0,\ x_1 = f(x_0),\ x_2 = f^2(x_0),\ x_3 = f^3(x_0), \dots . \qquad (1)$$

Definition 2. *A point $x \in X$ is called a fixed point for $f : X \to X$ if $f(x) = x$. The point x is a periodic point of period p if $f^p(x) = x$. The least positive p for which $f^p(x) = x$ is called the prime period of x.*

Denote the set of all fixed points by $Fix(f)$ and the set of all periodic points of (not necessarily prime) period p by $\mathrm{P}er_p(f)$.

Consider a nonlinear mapping

$$F : x = (x_1, x_2, \dots, x_m) \in \mathbb{R}^m \to x' = F(x) = (x_1', x_2', \dots, x_m') \in \mathbb{R}^m$$

given by

$$x'_k = F_i(x_1, x_2, \ldots, x_m),\ k = 1,2,\ldots,m, \tag{2}$$

where $F_i : \mathbb{R}^m \to \mathbb{R}$, $i = 1,\ldots,m$ is a continuously differentiable single-value function.

Thus, the eigenvalues of the Jacobian matrix J_F determine the local behavior of the nonlinear dynamical system. Below we shall do this point more explicit:

Definition 3. *A fixed point x^* of a mapping F is called*

- *hyperbolic point if its Jacobian J_F at x^* has no eigenvalues on the unit circle;*
- *attracting point if all the eigenvalues of the Jacobi matrix $J_F(x^*)$ are less than 1 in absolute value;*
- *repelling point if all the eigenvalues of the Jacobi matrix $J_F(x^*)$ are greater than 1 in absolute value;*
- *a saddle point otherwise.*

The notion of topological conjugacy is important in the study of dynamical systems.

Definition 4. *Let $f : U \to U$ and $g : V \to V$ be two maps. If there exists a homeomorphism $h : U \to V$ such that $h \circ f = g \circ h$, then f and g are said to be topologically conjugate, the homeomorphism h is called a topologically conjugacy.*

Let $E = \{1,...,m\}$. By the $(m-1)$- simplex we mean the set of all probability measures on E which is given by

$$S^{m-1} = \{x = (x_1,...,x_m) \in \mathbb{R}^m : x_i \geq 0, \sum_{i=1}^{m} x_i = 1\}. \tag{3}$$

Definition 5. *The quadratic stochastic operator (QSO) is a mapping of the simplex (3) into itself, of the form*

$$W : x'_k = \sum_{i=1}^{m}\sum_{j=1}^{m} P_{ij,k} x_i x_j, \qquad k = 1,...,m, \tag{4}$$

where the coefficients $P_{ij,k}$ satisfy the following conditions

$$P_{ij,k} \geq 0,\ P_{ij,k} = P_{ji,k},\ \sum_{k=1}^{m} P_{ij,k} = 1, \quad (i,j,k = 1,...,m). \tag{5}$$

We call the partition into types hereditary if for each possible state $z \in S^{m-1}$ describing the current generation, the state $z' \in S^{m-1}$ is uniquely defined describing the next generation. This means that the association $z \to z'$ defines a map $W : S^{m-1} \to S^{m-1}$ called the evolution operator.

States of the population described by the following discrete-time dynamical system

$$z,\ z^{(1)} = s' = W(z),\ z^{(2)} = W^2(z),\ z^{(3)} = W^3(z),...$$

The main problem for a given dynamical system W is to describe the limit points of the trajectory $\{z^{(n)}\}_{n=0}^{\infty}$ for arbitrary given $z^{(0)}$.

For $a \in (0,1)$ we denote

$$S_a \equiv S_a^{2m-2} = \left\{ z = (x_1,...,x_m, y_1,..., y_m) \in S : \sum_{i=1}^m x_i = a, \sum_{j=1}^m y_j = 1-a \right\}.$$

In the first chapter, the following dynamic system defined the evolution operator $W_a : S_a \to S_a$ is considered:

$$W_a : \begin{cases} x_k' = (1-a)^{-1} \sum_{i=1}^m \sum_{p=1}^m \theta_{ipk} x_i y_p, \\ y_k' = a^{-1} \sum_{i=1}^m \sum_{p=1}^m \theta_{ipk} x_i y_p. \end{cases} \tag{6}$$

Lemma 1. *If $z = (x, y) \in S_a$ is a fixed point of W_a then*

$$y_k = \beta x_k,\ k = 1,\dots,m$$

where $\beta = (1-a)/a.$

By this lemma the problem of finding fixed points of W_a is reduced to solution of the following system:

$$x_k = a^{-1} \sum_{i=1}^m \sum_{p=1}^m \theta_{ipk} x_i x_p,\ k = 1,\dots,m. \tag{7}$$

Lemma 2. *If $z^{(0)} = (x_1^{(0)},\dots,x_m^{(0)}, y_1^{(0)},\dots, y_m^{(0)}) \in S_a$ is an initial point and $z^{(n)} = (x_1^{(n)},\dots,x_m^{(n)}, y_1^{(n)},\dots, y_m^{(n)}) = W_a^n(z^{(0)})$ then*

$$y_k^{(n)} = \beta x_k^{(n)},\ \textit{for any}\ k = 1,\dots,m,\ n \ge 1.$$

In general it is difficult to solve the system (7). Let us solve it for $m = 2$. In this case using $x_2 = a - x_1$ we can reduce the first equation to a quadratic equation with four parameters, which has the following two solutions

$$\begin{aligned} x_{1,1} &= \frac{a}{2} \cdot \frac{2\theta_3 - \theta_2 + 1 + \sqrt{(\theta_2 - 1)^2 + 4\theta_3(1-\theta_1)}}{\theta_1 + \theta_3 - \theta}, \\ x_{1,2} &= \frac{a}{2} \cdot \frac{2\theta_3 - \theta_2 + 1 - \sqrt{(\theta_2 - 1)^2 + 4\theta_3(1-\theta_1)}}{\theta_1 + \theta_3 - \theta}, \end{aligned} \tag{8}$$

where

$$\theta_1 = \theta_{111},\ \theta_2 = \theta_{121} + \theta_{211},\ \theta_3 = \theta_{221}.$$

Thus if the parameters satisfy the following condition

$$(\theta_2 - 1)^2 + 4\theta_3(1-\theta_1) \ge 0 \tag{9}$$

then the fixed points are

$$z_j = (x_{1,j}, x_{2,j}, y_{1,j}, y_{2,j}), j = 1, 2,$$

where $x_{2,i} = a - x_{1,i}$ and $y_{i,j} = \beta x_{i,j}$.

The following theorem describes the trajectory of any initial point $z^{(0)}$, in S_a^2.

Theorem 1. *Let $z^{(0)} = \left(x_1^{(0)}, x_2^{(0)}, y_1^{(0)}, y_2^{(0)}\right) \in S_a^2 \setminus Fix(W_a)$ be an initial point.*

1) *if $\theta_1 + \theta_3 = \theta_2$ and $\theta_1 \ne 1, \theta_3 \ne 0$ or $\theta_1 \ne 0, \theta_3 \ne 1$, then*

$$\lim_{n\to\infty} W_a^{(n)}\left(z^{(0)}\right) = (x_1^*, a - x_1^*, \beta x_1^*, \beta(a - x_1^*)).$$

2) *if* $\theta_1 \neq \theta_2, \theta_1 \in [0;1), \theta_2 \in [0;1], \theta_3 = 0$ *or* $\theta_1 = 1, \theta_2 \in [0;1), \theta_3 = 0$, *then*

$$\lim_{n\to\infty} W_a^{(n)}\left(z^{(0)}\right) = (0, a, 0, 1 - a).$$

3) *if* $\theta_1 + \theta_3 \neq \theta_2, \theta_1 = 1, \theta_2 \in [1;2], \theta_3 \in (0;1]$ *or* $\theta_1 = 1, \theta_2 \in (1;2], \theta_3 = 0$, *then*

$$\lim_{n\to\infty} W_a^{(n)}\left(z^{(0)}\right) = (a, 0, 1 - a, 0).$$

4) *if* $\theta_1 + \theta_3 \neq \theta_2, (\theta_2 - 1)^2 + 4\theta_3(1 - \theta_1) \leq 4$ *and* $\theta_1 \in (0;1), \theta_2 \in [0;2], \theta_3 \in (0;1]$ *or* $\theta_1 = 0, \theta_2 \in [0;2], \theta_3 \in (0;1)$, *then*

$$\lim_{n\to\infty} W_a^{(n)}\left(z^{(0)}\right) = (x_2^*, a - x_2^*, \beta x_2^*, \beta(a - x_2^*)).$$

5) *if* $\theta_1 = 1, \theta_2 \in [0;1), \theta_3 \in (0;1)$, *then*

$$\lim_{n\to\infty} W_a^{(n)}\left(z^{(0)}\right) = (x_3^*, a - x_3^*, \beta x_3^*, \beta(a - x_3^*)).$$

6) *if* $\theta_1 \in [0;1), \theta_2 \in (1;2], \theta_3 = 0$, *then*

$$\lim_{n\to\infty} W_a^{(n)}\left(z^{(0)}\right) = (x_4^*, a - x_4^*, \beta x_4^*, \beta(a - x_4^*)).$$

7) *if* $\theta_1 = 0, \theta_2 = 1, \theta_3 = 1$, *then*

$$\lim_{n\to\infty} W_a^{(n)}\left(z^{(0)}\right) = \begin{cases} (x_1^{(0)}, a - x_1^{(0)}, \beta x_1^{(0)}, \beta(a - x_1^{(0)})), & \textit{for } n = 2k,\ k = 0,1,2,... \\ (a - x_1^{(0)}, x_1^{(0)}, \beta(a - x_1^{(0)}), \beta x_1^{(0)}), & \textit{for } n = 2k + 1. \end{cases}$$

8) *if* $\theta_1 + \theta_3 \neq \theta_2, (\theta_2 - 1)^2 + 4\theta_3(1 - \theta_1) > 4$ *and* $\theta_1 \in (0;1), \theta_2 \in [0;2], \theta_3 \in (0;1]$ *or* $\theta_1 = 0, \theta_2 \in [0;2], \theta_3 \in (0;1)$ *or* $\theta_1 = 0, \theta_2 \in [0;1) \cup (1;2], \theta_3 = 1$ *then*

$$\lim_{n\to\infty} W_a^{(n)}\left(z^{(0)}\right) \subset \{\left(p_1, a - p_1, \beta p_1, \beta(a - p_1)\right), \left(p_2, a - p_2, \beta p_2, \beta(a - p_2)\right)\},$$

where

$$x_1^* = \frac{\theta_3 a}{\theta_3 - \theta_1 + 1},\ x_2^* = \frac{a}{2} \cdot \frac{2\theta_3 - \theta_2 + 1 - \sqrt{(\theta_2 - 1)^2 + 4\theta_3(1 - \theta_1)}}{\theta_1 + \theta_3 - \theta},\ x_3^* = \frac{\theta_3 a}{\theta_3 - \theta_2 + 1},$$

$$x_4^* = \frac{(1 - \theta_2)a}{\theta_1 - \theta_2},\ p_{1,2} = \frac{2\theta_3 - \theta_2 - 1 \pm \sqrt{(\theta_2 - 1)^2 + 4\theta_3(1 - \theta_1) - 4}}{2(\theta_1 + \theta_3 - \theta_2)} a.$$

We consider the following constraint on heredity coefficients θ_{ipk}

$$\theta_{ipk} = \begin{cases} c_{ik}, & if \quad p = j, \\ 1, & if \quad p \neq j, k = l, \qquad j, l = 1, \ldots, m. \\ 0, & if \quad p \neq j, k \neq l, \end{cases}$$

where

$$c_{ik} = \begin{cases} 1, & i = k, \\ 0, & i \neq k. \end{cases}$$

Theorem 2. *Let* $z^{(0)} = \left(x_1^{(0)}, ..., x_m^{(0)}, y_1^{(0)}, ..., y_m^{(0)}\right) \in S_a^{2m-2}$ *be an initial point.*

1) if $j = l$ *then*

$$\lim_{n\to\infty} W^{(n)}\left(z^{(0)}\right)=\lim_{n\to\infty} z^{(n)}=(\underbrace{0,...,0,a}_{j},\underbrace{0,...0}_{m-j},\underbrace{0,...,0,1-a}_{j},\underbrace{0,...,0}_{m-j}).$$

2) if $j\neq l$ then

$$\lim_{n\to\infty} W^{(n)}\left(z^{(0)}\right)=\lim_{n\to\infty} z^{(n)}=\begin{cases} z_{1,l}=(\underbrace{0,...,0,a}_{l},\underbrace{0,...0}_{m-l},\underbrace{0,...,0,1-a}_{l},\underbrace{0,...,0}_{m-l}), & \text{if } x_j^{(0)}\neq a, \\ z_{2,j}=(\underbrace{0,...,0,a}_{j},\underbrace{0,...0}_{m-j},\underbrace{0,...,0,1-a}_{j},\underbrace{0,...,0}_{m-j}), & \text{if } x_j^{(0)}=a. \end{cases}$$

The second chapter of the dissertation entitled **"A discrete-time system of mosquito population"** created a model of mosquito population consisting of two variables and five parameters and studied its dynamics.

Define the operator $W:\mathbb{R}^2\to\mathbb{R}^2$ by

$$\begin{cases} x'=\beta y-\dfrac{\alpha x}{1+x}-(d_0+d_1x)x+x, \\ y'=\dfrac{\alpha x}{1+x}-\mu y+y \end{cases} \tag{10}$$

where $\alpha>0,\beta>0,\mu>0,d_0\geq 0,d_1\geq 0.$

Let $\mathbb{R}_+^2=\{(x,y):x,y\in\mathbb{R},x\geq 0,y\geq 0\}$. Note that the operator W well defined on $\mathbb{R}^2\setminus\{x=-1\}$. But to define a dynamical system of continuous operator and interpret values of x and y as probabilities we assume $x\geq 0$ and $y\geq 0$. Therefore, we choose parameters of the operator W to guarantee that it maps $\mathbb{R}_+^2$ to itself.

We assume $\beta=\mu,\ d_0=d_1=0,$ and denote $S_1=\{(x,y):x,y\in\mathbb{R}_+,x+y=1\}.$

A set S_1 is called invariant with respect to W if $W\left(S_1\right)\subset S_1.$

We note that for each given $c>0$ the set $S_c=\{(x,y)\in\mathbb{R}_+^2:\ x+y=c\}$ is an invariant with respect to operator W given in (10). Moreover, the sum of all S_c over $c>0$ gives $\mathbb{R}_+^2$. Thus it suffices to study the dynamical system on each S_c. Observe that the restriction of the operator W to the set S_c has similar to the case S_1 dynamics for each $c>0.$

For case $\beta=\mu,\ d_0=d_1=0,$ the following theorem describes the trajectory of any initial point $z^{(0)}$, in S_1.

Theorem 3. *Let $z^{(0)}=\left(x^{(0)},1-x^{(0)}\right)\in S$ be an initial point*

***1)** If $\left(\alpha,\beta\right)\in\left(A\cup B\right)\setminus\left\{(2;1)\right\}$ then*

$$\lim_{n\to\infty} W_0^n\left(z^{(0)}\right)=\left(\frac{\sqrt{\alpha^2+4\beta^2}-\alpha}{2\beta},\frac{2\beta+\alpha-\sqrt{\alpha^2+4\beta^2}}{2\beta}\right).$$

***2)** If $\alpha=2,\beta=1$ then*

$$\lim_{n\to\infty} W_0^n\left(z^{(0)}\right)=\begin{cases} \left(x^{(0)},1-x^{(0)}\right) & \text{for } n=2k,k=0,1,2,... \\ \left(\dfrac{1-x^{(0)}}{1+x^{(0)}},\dfrac{2x^{(0)}}{1+x^{(0)}}\right) & \text{for } n=2k+1. \end{cases}$$

We assume $\beta \neq \mu,\ d_0 = d_1 = 0$.

It is easy to see that if

$$0 < \alpha \leq 1,\ \beta > 0,\ 0 < \mu \leq 1 \tag{11}$$

then operator (10) maps $\mathbb{R}^2_+$ to itself.

Theorem 4. *For the operator W given by (10) (i.e. under condition (11)) and for any initial point $(x^{(0)}, y^{(0)}) \in \mathbb{R}^2_+ \setminus Fix(W)$ the following hold*

$$\lim_{n\to\infty} x^{(n)} = \begin{cases} 0, & if\ \ \beta < \mu, \\ +\infty, & if\ \ \beta > \mu \end{cases}$$

$$\lim_{n\to\infty} y^{(n)} = \begin{cases} 0, & if\ \ \beta < \mu, \\ \dfrac{\alpha}{\mu}, & if\ \ \beta > \mu \end{cases}$$

where $\left(x^{(n)}, y^{(n)}\right) = W^n\left(x^{(0)}, y^{(0)}\right)$.

We assume $d_0 \neq 0$.

Proposition 1. *If*

$$\alpha > 0, \beta > 0, 0 < \mu \leq 1, d_0 > 0, \alpha + d_0 \leq 1, d_1 = 0 \tag{12}$$

then the operator (10) maps the set $\mathbb{R}^2_+$ to itself.

Proposition 2. *The type of the fixed points for (10) are as follows:*

1) *if $\beta \leq \mu\left(1 + \dfrac{d_0}{\alpha}\right)$, then the operator (10) (i.e. under condition (12)) has a unique fixed point $(0,0)$, the point*

$$(0,0)\ is\ \begin{cases} attracting, & if\ \ \beta < \mu\left(1 + \dfrac{d_0}{\alpha}\right), \\ non-hyperbolic, & if\ \ \beta = \mu\left(1 + \dfrac{d_0}{\alpha}\right). \end{cases}$$

2) *if $\beta > \mu\left(1 + \dfrac{d_0}{\alpha}\right)$, then the operator has two fixed points $(0,0)$, (x^*, y^*), and the point (x^*, y^*) is attracting, the point*

$$(0,0)\ is\ \begin{cases} repelling, & if\ \ \beta > \mu\left(1 + \dfrac{d_0}{\alpha}\right) + \alpha^*, \\ saddle, & if\ \ \mu\left(1 + \dfrac{d_0}{\alpha}\right) < \beta < \mu\left(1 + \dfrac{d_0}{\alpha}\right) + \alpha^*, \\ non-hyperbolic, & if\ \ \beta = \mu\left(1 + \dfrac{d_0}{\alpha}\right) + \alpha^*. \end{cases}$$

where $\alpha^* = \dfrac{1}{\alpha}\left(4 - 2(\alpha + \mu + d_0)\right)$.

Theorem 5. *For $p \geq 2$ the operator (10) (i.e. under condition (12)) does not have any p-periodic point (except fixed) in the set $\mathbb{R}_+^2$.*

Denote

$$\Omega = \left\{ (x, y) \in \mathbb{R}_+^2,\ 0 \leq x \leq \frac{\alpha\beta}{\mu d_0},\ 0 \leq y \leq \frac{\alpha}{\mu} \right\}.$$

We divide the set Ω into four parts as follows

$$\Omega_1 = \left\{ (x, y) \in \Omega,\ 0 \leq x \leq x^*,\ 0 \leq y \leq y^* \right\},$$

$$\Omega_2 = \left\{ (x, y) \in \Omega,\ x^* \leq x \leq \frac{\alpha\beta}{\mu d_0},\ y^* \leq y \leq \frac{\alpha}{\mu} \right\},$$

$$\Omega_3 = \left\{ (x, y) \in \Omega,\ x^* < x \leq \frac{\alpha\beta}{\mu d_0},\ 0 \leq y \leq y^* \right\},$$

$$\Omega_4 = \left\{ (x, y) \in \Omega,\ 0 \leq x < x^*,\ y^* < y \leq \frac{\alpha}{\mu} \right\},$$

where

$$x^* = \frac{\alpha(\beta - \mu)}{\mu d_0} - 1,\ y^* = \frac{\alpha(\beta - \mu) - \mu d_0}{\mu(\beta - \mu)}. \tag{13}$$

Proposition 3. *The sets Ω_1, Ω_2 and Ω are invariant with respect to W.*

Proposition 4. *For any β, μ (under condition (12)), the function*

$$\varphi(z) = \mu x + \beta y$$

is Lyapunov function for the operator W defined by (10).

Remark 2.4.1. *If $\beta \leq \mu\left(1 + \frac{d_0}{\alpha}\right)$, then $z = (x, y) \in \Omega$ and if $\beta > \mu\left(1 + \frac{d_0}{\alpha}\right)$, then $z = (x, y) \in \Omega_1 \bigcup \Omega_2$.*

Theorem 6. *For the operator W given by (10) (i.e. under condition (12)) and for any initial point $(x^{(0)}, y^{(0)}) \in \mathbb{R}_+^2 \setminus Fix(W_0)$ the following hold:*

***(1)** If $y^{(n)} > \frac{\alpha}{\mu}$ for any natural number n then*

$$\lim_{n\to\infty} x^{(n)} = +\infty,\ \lim_{n\to\infty} y^{(n)} = \frac{\alpha}{\mu};$$

***(2)** If there exists n_0 number such that $y^{(n_0)} \leq \frac{\alpha}{\mu}$ and $\beta \leq \mu\left(1 + \frac{d_0}{\alpha}\right)$ then*

$$\lim_{n\to\infty} x^{(n)} = 0,\ \lim_{n\to\infty} y^{(n)} = 0,$$

otherwise, i.e. if $\beta > \mu\left(1 + \frac{d_0}{\alpha}\right)$, then

$$\lim_{n\to\infty} x^{(n)} = x^*,\ \lim_{n\to\infty} y^{(n)} = y^*,$$

where $(x^{(n)}, y^{(n)}) = W_0^n (x^{(0)}, y^{(0)})$.

In the third chapter of the dissertation entitled **"Dynamics of normalized operator generated by model of mosquito population and epidemic models"** the dynamical systems generated by the normalized operator are considered. Besides, we studied the discrete-time dynamical systems associated with the SIS (susceptible-infected-susceptible) and SIR (susceptible-infected-recovered) model of disease spread.

We consider the normalized version of the operator (10) (under condition (12)), i.e.,

$$U:\begin{cases} x'=\dfrac{(\beta y+(1-d_0)x)(1+x)-\alpha x}{(1+x)((\beta-\mu+1)y+(1-d_0)x)}, \\ y'=\dfrac{\alpha x+(1+x)(1-\mu)y}{(1+x)((\beta-\mu+1)y+(1-d_0)x)}. \end{cases} \tag{14}$$

From conditions (10) one gets $x'\geq 0$ and $y'\geq 0$ moreover $x'+y'=1$. Hence $U:S_1\to S_1$.

Proposition 5. *The operator U given by (14) has unique attracting fixed point $z^*=\left(x^*,1-x^*\right)$, where*

$$x^*=\begin{cases} \dfrac{\sqrt{\alpha^2+4\beta^2}-\alpha}{2\beta}, & \text{if } \beta+d_0=\mu, \\ \dfrac{\sigma(\beta^2-3(\alpha-\mu+\beta+d_0)(\mu-\beta-d_0))+1/\sigma-\beta}{3(\mu-\beta-d_0)}, & \text{if } \beta+d_0\neq\mu, \end{cases}$$

$$\sigma=\sqrt[3]{2/(9\beta e(\alpha+2e)-2\beta^3+\sqrt{4(-\beta^2+3e(\alpha-e))^3+(9\beta e(\alpha+2e)-2\beta^3)^2})},$$
$$e=\mu-\beta-d_0.$$

Theorem 7. *For $p\geq 2$ the operator (14) does not have any p-periodic point in the set S_1.*

The following describes the trajectory of any initial point $z^{(0)}$ in S_1.

Theorem 8. *For the operator U given by (14), under condition (12), for any initial point $z^{(0)}=\left(x^{(0)},y^{(0)}\right)\in S_1\setminus Fix(U)$ the following holds*

$$\lim_{n\to\infty}U^n\left(z^{(0)}\right)=\begin{cases} \left(\tilde{x},1-\tilde{x}\right), & \text{if } \mu=\beta+d_0 \\ \left(\hat{x},1-\hat{x}\right), & \text{if } \mu\neq\beta+d_0 \end{cases}$$

where

$$\tilde{x}=\frac{\sqrt{\alpha^2+4\beta^2}-\alpha}{2\beta},\quad \hat{x}=\frac{\sigma(\beta^2-3(\alpha-\mu+\beta+d_0)(\mu-\beta-d_0))+1/\sigma-\beta}{3(\mu-\beta-d_0)}$$

are fixed points.

In the last paragraph of the third chapter the fixed points, their types and the description of limit points of the trajectory of any initial point for the following discrete version of the epidemic SIS model are considered:

$$\begin{cases} S' = S - \beta SI + \nu I, \\ I' = I + \beta SI - \nu I. \end{cases} \tag{15}$$

where $\beta > 0, \nu > 0.$

From $S' + I' = S + I = N$ system (15) can be written as follows:

$$f(I) = I(1 - \nu + \beta N) - \beta I^2 \tag{16}$$

We denote

$$\Omega = [0; N],$$

$$\Theta = \left\{ (\beta, \nu) \in \mathbb{R}^2 : \nu \in (0;1], \beta \in \left(0; \frac{(1+\sqrt{\nu})^2}{N} \right] \right\},$$

$$\Theta^* = \left\{ (\beta, \nu) \in \mathbb{R}^2 : \nu \in \left(\frac{1}{4}; 1 \right], \beta \in \left(\frac{2+\nu}{N}; \frac{(1+\sqrt{\nu})^2}{N} \right] \right\}.$$

Reproductive number as it relates to contact rate and infectious period. We define the basic reproductive number

$$R_0 = \frac{\beta N}{\nu}. \tag{17}$$

The following describes the trajectory of any point $I^{(0)}$ in Ω:

Theorem 9. *Let* $I^{(0)} \in \Omega \setminus Fix(f)$ *be an initial point. Under condition in set* $\Theta \setminus \Theta^*$*, for the function* f *given by (16) the following holds:*

$$\lim_{n\to\infty} f^n(I^{(0)}) = \begin{cases} 0, & if \quad R_0 \le 1, \\ \dfrac{\beta N - \nu}{\beta}, & if \quad R_0 > 1. \end{cases}$$

CONCLUSION

The thesis is devoted to investigation of the dynamical systems generated by evolution operators.

Basic results of the research are as follows:

1. The discrete-time dynamics of the sex-linked evolution operator given by the four-dimensional quadratic operator is fully studied.
2. Under some conditions on parameters, for the high-dimensional evolution operator the set of limit points of all trajectories was described.
3. For the discrete-time dynamical system of the model related to the wild mosquito population all fixed points (depending on five parameters) of the evolutionary operator are found, and it is shown that depending on the parameters, this operator can have one, two fixed points, and their type are determined.
4. The discrete-time dynamics of the corresponding model under some conditions on parameters are fully analyzed.
5. The discrete-time dynamics of the SIS and SIR epidemic models are studied and its application to the spread of the Covid-19 virus is given.

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ИНСТИТУТ МАТЕМАТИКИ

БОХОНОВ ЗАФАР САЙДИМАХМУДОВИЧ

ДИНАМИЧЕСКИЕ СИСТЕМЫ РАЦИОНАЛЬНЫХ ОТОБРАЖЕНИЙ В БИОЛОГИИ

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ВВЕДЕНИЕ (аннотация диссертации доктора философии(PhD))

Целью исследования является задание полного описания множества предельных точек траектории для динамической системы с дискретным временем, порождаемых конкретными рациональными операторами, рассматриваемыми как операторы эволюции некоторых биологических систем.

Объект исследования: операторы эволюции сцепленной с полом популяции, оператор популяции комаров, эпидемические модели SIS и SIR.

Научная новизна исследования состоит в следующем:

Полностью изучена дискретная динамика оператора эволюции, связанной с полом, задаваемой двумерным квадратичным оператором. При некоторых условиях по параметрам изучаются случаи большой размерности.

Для дискретной динамической системы модели, связанной с популяцией диких комаров, найдены все фиксированные точки (в зависимости от пяти параметров) эволюционного оператора и показано, что в зависимости от параметров этот оператор может иметь одну, две фиксированные точки, определяющие их типы.

Полностью проанализирована дискретная динамика соответствующей модели при некоторых условиях по параметрам и дана биологическая интерпретация результатов.

Изучена дискретная динамика эпидемических моделей SIS и SIR и дано ее применение к распространению вируса Covid-19.

Внедрение результатов исследования. На основе полученных результатов о динамические системы рациональных отображений в биологии:

на дискретной динамической системе математической модели популяции комаров исследованы динамические системы, порожденные несколькими квадратичными стохастическими операторами, определенными на стандартном симплексе в зарубежном грантовом проекте № G00003447 по теме «Квантовые генетические алгебры и их приложения». (Справка Университета Объединенных Арабских Эмиратов, 11 октября 2023 г., ОАЭ). Применение этого научного результата позволило описать множество предельных точек траектории для некоторых нелинейных моделей;

Динамика операторов эволюции использовалась для исследования нелинейных динамических систем в зарубежном грантовом проекте FRGS 17-027-0593 «Хаос вызывает фазовый переход: метод *p*-адических динамических систем для нового класса p-адических мер Гиббса» (Малайзия, справка Международного исламского университета Малайзии, 16 октября 2023 г.,). Применение этого научного результата позволило определить неподвижные точки и периоды нелинейных операторов и проанализировать траектории.

Структура и объем диссертации. Диссертация состоит из введения, трёх глав разбитых на одиннацать параграфов, заключения и списка использованной литературы. Объем диссертации составляет 103 страниц.

E’LON QILINGAN ILMIY ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo‘lim (part 1; часть 1)

1. Boxonov Z.S., Rozikov U.A., A dynamical system of temperature-dependent sex linked inheritance. // *Uzbek Mathematical Journal.* 2018. No. 2, p.17-31. (01.00.00; № 6).
2. Boxonov Z.S., Discrete dynamic system of mathematical model of coronavirus spread. // *Nonlinear Studies* 2020. Vol. 27, No. 3, p.823-830. (3.Scopus. IF=0.221.
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4. Boxonov Z.S., Rozikov U.A., A discrete-time dynamical system of stage-structured wild and sterile mosquito population. // *Nonlinear Studies.* 2021. Vol. 28, No. 2, p.413-425. (3.Scopus. IF=0.210).
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II bo‘lim (part 2; часть 2)

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