V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

MATEMATIKA INSTITUTI

URALOVA SABOXAT ISMOILJON QIZI

BOSHQARUVLARI EKSPONENSIAL VA LANGENHOP TIPIDAGI CHEGARALANISHLI DIFFERENSIAL O'YINLAR

01.01.02 - Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI
BO'YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI
AVTOREFERATI

Toshkent - 2024

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2024.1.PhD/FM1006 raqam bilan ro'yxatga olingan.

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Dissertatsiya himoyasi V.I.Romanovskiy nomidagi Matematika instituti huzuridagi DSc.02/30.12.2019.FM.86.01 raqamli Ilmiy kengashning 2025 yil 21-yanvar kuni soat 17:00 dagi majlisida bo'lib o'tadi. (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko'chasi, 9-uy. Tel.: (+998 71) 207 91 40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida ko‘plab ilmiy markazlarda boshqariladigan konfliktli dinamik jarayonlarni matematik modellashtirish va ularni keng amaliyotda qo‘llash yetakchi o‘rinlardan birini egallamoqda. Dunyo miqyosida bu masalalarni qo‘llash bozor iqtisodiyoti, harbiy soha, tibbiyot, zamonaviy texnologiyalarni rivojlantirish va boshqa ko‘plab sohalarda katta ahamiyatga ega. Agar konfliktli dinamik jarayonlar differensial tenglamalar bilan ifodalansa, bunday muammolar differensial o‘yinlar nazariyasi muammolari sifatida o‘rganiladi. Agar boshqariladigan konfliktli dinamik jarayonlar matematik modellari diskret tenglamalar asosida yaratilgan bo‘lsa, bunday jarayonlar diskret o‘yin nazariyasi sifatida talqin etiladi. Ammo ularning umumiy xususiyatlari o‘zaro bog‘liq bo‘lgani uchun, ular dinamik o‘yin nazariyasi sifatida qaraladi. Konfliktli dinamik jarayonlardagi muammolarni tahlil qilish differensial o‘yinlar nazariyasi sohasidan foydalanishni taqozo etadi. Shu jihatdan, differensial o‘yinlarning elementlarining to‘g‘ri aniqlanishi va qo‘llanishi ilmiy tadqiqotlarda muhim ahamiyatga ega hisoblanadi.

Jahonda konfliktli dinamik jarayonlarni boshqarish va optimallashtirish uchun bugungi kunda differensial o‘yinlardagi turli masalalardagi olingan natijalar keng qo‘llanilmoqda. Shuning uchun ushbu dissertatsiya asosan differensial o‘yinlar nazariyasi masalalarini o‘rganadi. Differensial o‘yinlar nazariyasida ziddiyatli dinamik jarayonlar modellariga xos jihatlarni hisobga olgan holda, boshqaruv funksiyalariga turli cheklovlar qo‘yiladi: geometrik (boshqaruv chegaralarini belgilovchi), integral (energiya sarfini ifodalaydigan), aralash va boshqa turdagi cheklovlar. Shu bilan birga, amaliy masalalarni yechishda yanada murakkab cheklovlar paydo bo‘lishi mumkin. Eksponensial cheklovlar va Langenhop tipidagi cheklovlar bilan differensial o‘yinlarda quvish va qochish muammolarini o‘rganish katta amaliy ahamiyatga ega va bunday hollar dissertatsiyada har tomonlama o‘rganiladi. Hozirgi vaqtda turli cheklovlarga ega bo‘lgan quvish va qochish muammolari keng miqyosda o‘rganishga yo‘naltirilgan ilmiy-tadqiqot ishlari olib borilmoqda. Bu borada texnikada, iqtisodiyotda va boshqa turli sohalarida ularning qo‘llanilishiga alohida e’tibor berilmoqda.

Respublikamizda innovatsion texnologiyalarni rivojlatirish uchun fundamental fanlarning asosiy rolini hisobga olgan holda ularning ilmiy va amaliy ahamiyatini oshirish doirasida ishlab chiqilgan strategiyalar doirasida keng qamrovli chora-tadbirlar amalga oshirilib, muayyan natijalarga erishilmoqda. Xususan, differensial tenglamalar, matematik-fizika tenglamalari, algebra va funksional analiz, shuningdek, matematik analiz sohalarida olib borilayotgan ilmiy tadqiqotlarni xalqaro standartlar darajasida rivojlantirish bo‘yicha muhim vazifalar belgilangan.[1] Ushbu vazifalarni amalga oshirishda, jumladan, boshqaruv

[1] O‘zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi №PQ-4387 “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi qarori.

modellarini rivojlantirish va optimallashtirish uchun boshqaruv funksiyalari eksponensial va Langenhop tipli chegralanishli differensial o‘yinlarni rivojlantirish muhim ahamiyat kasb etmoqda.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PQ-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi, 2017-yil 17-fevraldagi PQ-2789-son “Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi, 2017-yil 20-apreldagi PQ-2909-son “Oliy ta’lim tizimini yanada rivojlantirish chora-tadbirlari to‘g‘risida”gi, 2019-yil 9-iyuldagi PQ-4387-son “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi, 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi, 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga mo‘ljallangan Yangi O‘zbekistonning taraqqiyot strategiyasi to‘g‘risida”gi qarorlari, hamda mazkur faoliyatga tegishli boshqa me’yoriy-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishga ushbu dissertatsiya ishi muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalarni rivojlantirishning ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Amerikalik matematik R. Ayzeks 1949-1950-yillarda RAND korporatsiyasida harbiy Quvish-Qochish (QQ) muammolariga matematik modellar orqali yondashib, differensial o‘yinlar nazariyasiga asos solgan. Ayzeks o‘zining ishlarida klassik variatsion hisoblashning Gamilton-Yakobi metodlarini qo‘llab, ularni harakatlari turli boshqaruvlar bilan ifodalangan differensial o‘yinlarga tatbiq etadi. Keyinchalik Ayzeksning bu usullari L.D.Berkovitz, W.H.Fleming va A.Friedman kabi matematiklar tomonidan variatsion hisoblash va o‘yinlar nazariyasining turli usullari asosida yanada rivojlantirilgan.

Differensial o‘yinlarning fundamental nazariyasi asoschilari sifatida bugungi kunda R.Ayzeks, L.S.Pontryagin, N.N.Krasovskiy, L.D.Berkovitz, W.H.Fleming, A.Friedman, A.Bryson, E.F.Mishchenko, A.I.Subbotin, Yu.S.Osipov, B.N.Pshenichniy, N.Yu.Satimov, L.A.Petrosyan, A.A.Azamov kabi olimlar qaraladi.

Differensial o‘yinlar nazariyasi masalalariga quvuvchi tomonidan qaralishi, yani asosiy informasiya quvuvchiga berilgan holda uning foydasiga masalani yechish quvish masalalarini tashkil etadi. Aksincha, masalaga qochuvchi tomonidan qarash, yani asosiy informatsiyani qochuvchiga berilgan holda uning foydasiga masalani yechish qochish masalasini tashkil etadi. Masalaga bunday yondashuv asosida L.S.Pontragin turli fundamental usullarni taklif etadi va keyinchalik bu usullar M.S.Nikolskiy, N.L.Grigorenko, A.A.Chikriy, B.N.Pshenichniy, A.I.Blagodatskix, N.N.Petrov, N.Yu.Satimov, B.B.Rixsiyev, A.A.Azamov, A.Z.Fazilov, B.B.Rixsiyev, A.A.Xamdamov, G.I.Ibragimov, M.Sh.Mamatov, N.A.Mamadaliyev va boshqalar tomonidan keng rivojlantiriladi.

Yurtimizda N.Yu.Satimov 35 yildan ortiq vaqt mobaynida “Optimal boshqaruv va differensial o‘yinlar” ilmiy seminarining rahbari bo‘lgan hamda u optimal boshqaruv nazariyalari va differensial o‘yinlarga yo‘naltirilgan Toshkent ilmiy maktabining asoschisi edi. Hozirda uning ilmiy maktabiga A.A.Azamov rahbarlik qilmoqda. N.Yu.Satimov, A.A.Azamov, A.Z.Fazilov, B.B.Rixsiyev, A.A.Xamdamov, G.I.Ibragimov, M.Sh.Mamatov, B.T.Samatov, O.Sh.Qo‘chqorov, N.A.Mamadaliyevlar o‘zlarining ilmiy ishlarida differensial o‘yinlardagi turli quvish-qochish muammolarini hal etishda muhim natijalarga erishganlar.

Dissertatsiya mavzusining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. V.I.Romanovskiy nomidagi Matematika instituti “Dinamik va boshqariluvchi sistemalar” ilmiy laboratoriyasida rejalashtirilgan ilmiy tadqiqot mavzusi asosida olib borilgan.

Tadqiqotning maqsadi o‘yinchilarning boshqaruv funksiyalari eksponensial va Langenhop tipidagi cheklovlarni qanoatlantiruvchi differensial o‘yinlarda quvish-qochish masalalarini hal etishdan iborat.

Tadqiqotning vazifalari:

boshqaruv funksiyalari eksponensial integral chegaralanishga ega oddiy va chiziqli differensial o‘yinlarda quvish-qochish masalalari yechilishini kafolatlovchi yetarli shartlarni aniqlash;

boshqaruv funksiyalari Langenhop tipidagi chegaralanishga ega oddiy va chiziqli differensial o‘yinlarda quvish-qochish masalalari yechilishini kafolatlovchi yetarli shartlarini aniqlash;

boshqaruv funksiyalari Langenhop tipidagi chegaralanishli inersiyaga ega o‘yinchilar ishtirokidagi differensial o‘yinda quvish-qochish masalalari yechilishini kafolatlovchi yetarli shartlarni aniqlash;

boshqaruv funksiyalari Langenhop tipidagi nostatsionar chegralanishga ega oddiy va nostatsionar differensial o‘yinlarda quvish-qochish masalalari yechilishini kafolatlovchi yetarli shartlarni aniqlash.

Tadqiqotning obyekti boshqaruvlari eksponensial va Langenhop tipidagi chegaralanishli differensial o‘yinlar.

Tadqiqotning predmeti boshqaruvlari eksponensial va Langenhop tipidagi chegaralanishli differensial o‘yinlar, differensial tenglamalar va tengsizliklardir.

Tadqiqotning usullari. Matematik analiz, differensial o‘yinlar nazariyasi, differensial tenglamalar, funksional analiz hamda optimal boshqaruv nazariyasining asosiy teoremalari va xossalaridan foydalanib, boshqaruvlari eksponensial va Langenhop tipidagi chegaralanishga ega quvish masalalarini hal qilish uchun П-strategiya usuli qo‘llanilgan, qochuvchining esa kechiktirilgan strategiya usuli qo‘llash orqali qochish masalalari hal qilingan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

boshqaruv funksiyalari eksponensial integral chegaralanishli oddiy va chiziqli differensial o‘yinlarda berilgan parametrlar asosida quvlovchiga П-strategiya qurilgan, qochuvchiga esa kechikishga ega boshqaruv aniqlangan va natijada quvish-qochish masalalari yechilishini kafolatlovchi yetarli shartlarni aniqlangan;

boshqaruv funksiyalari Langenhop tipidagi chegaralanishga ega oddiy va chiziqli differensial o‘yinlarda berilgan parametrlar bo‘yicha quvlovchining П-strategiyasi va qochuvchining kechikishga ega boshqaruvi orqali quvish-qochish masalalari yechilishining yetarli shartlari aniqlangan va o‘yinchilar orasidagi masofaning quyi chegarasi topilgan;

boshqaruv funksiyalari Langenhop tipidagi chegaralanishli inersiyaga ega o‘yinchilar differensial o‘yinida berilgan parametrlar asosida quvlovchiga П-strategiya qurilgan, qochuvchiga esa kechikishga ega boshqaruv aniqlangan va natijada quvish-qochish masalalari yechilishining yetarli shartlari topilgan;

boshqaruv funksiyalari Langenhop tipidagi nostatsionar chegralanishga ega oddiy va nostatsionar differensial o‘yinlarda berilgan parametrlar asosida quvish-qochish masalalari yechilishini kafolatlovchi yetarli shartlarni aniqlagan va o‘yinchilar orasidagi masofaning quyi chegarasi topilgan.

Tadqiqotning amaliy natijasi. Dissertatsiyada olingan natijalar va ularning isbotida qo‘llanilgan usullar, ayniqsa, differensial tenglamalar bilan ifodalangan murakkab jarayonlarni boshqarish masalalariga bevosita tatbiq etilishi mumkin. Ko‘pgina natijalar sonli misollar yordamida tahlil qilingan. Bu yondashuv tadqiqotda ko‘rib chiqilgan masalalarning real jarayonlarni matematik modellashtirishda qo‘llanilishiga katta yordam beradi.

Tadqiqot natijalarining ishonchliligi matematik analiz, differensial o‘yinlar nazariyasi, funksional analiz, optimal boshqaruv nazariyasi va differensial tenglamalarning asosiy teoremalari, nazariy jihatlari hamda xossalariga asoslanib kafolatlangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati boshaqaruvlari eksponensial va Langenhop tipidagi chegaralanishli oddiy va nostatsionar differensial o‘yinlarni rivojlantirish va ayrim ziddiyatli boshqaruvli jarayonga ega sohalardagi muammolarni hal qilishda qo‘llashdir.

Tadqiqot natijalarining amaliy ahamiyati boshaqaruvlari eksponensial va Langenhop tipidagi cheralanishli differensial o‘yinlar orqali topilgan matematik modellarni, ko‘plab sohalardagi muammolarga masalan harbiy sohada havo janglari, kapital jamg‘arish, sanoatni tashkil etish, marketing va atrof-muhit iqtisodiyotini hal qilishdan iboratdir.

Tadqiqot natijalarining joriy qilinishi. Boshqaruvlari eksponensial va Langenhop tipidagi chegaralanishli differensial o‘yinlar bo‘yicha olingan natijalar asosida:

boshqaruv funksiyalari eksponensial integral chegaralanishga ega oddiy va chiziqli differensial o‘yinlarda quvish-qochish masalalari yechilishini kafolotlovchi shartlaridan 374874-2022 raqamli “Fazaviy o‘tishlar va tahliliy hodisalar masalalari. Ularning tez o‘tish tenglamalari va asimptotikalarining matematik xususiyatlari” mavzusidagi xorijiy loyihasida oddiy differensial tenglamalar uchun analogik masalalarni yechishda foydalanilgan (O‘sh davlat universitetining 2024-yil 11-sentyabrdagi 1170-sonli ma’lumotnomasi, Qirg‘iziston Respublikasi). Ilmiy natijaning qo‘llanishi oddiy va chiziqli differensial tenglamalar sistemasi bilan ifodalangan fazaviy o‘tishlar va tahliliy hodisalar masalalarining to‘liq yechimlarini ko‘rsatish imkonini bergan;

boshqaruv funksiyalari Langenhop tipidagi nostatsionar chegralanishga ega oddiy va nostatsionar differensial o‘yinlarda berilgan parametrlar asosida quvish-qochish masalalari yechilishini kafolatlovchi yetarli shartlaridan OT-F4-33 raqamli “Differensial tenglamalar bilan tavsiflanuvchi ziddiyatli boshqaruv uchun yangi usullarni ishlab chiqish va ularning sonli tadbiqi” mavzusidagi fundamental loyihada boshqaruvlari holat cheklovlariga ega dinamik tenglamalar bilan tasvirlangan qarama-qarshi holatlarni boshqarish masalalarni hal qilishda foydalanilgan (O‘zbekiston Milliy universitetining 2024-yil 15-oktyabrdagi 04/11-8818-sonli ma’lumotnomasi). Ilmiy natijaning qo‘llanishi Langenhop tipidagi cheklovlarga bo‘ysunuvchi oddiy va nostatsionar dinamik tenglamalar bilan ifodalangan qarama-qarshi maqsadli boshqaruvga ega masalalarining to‘liq yechimlarini ko‘rsatish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. 7 ta xalqaro miqyosida hamda 8 ta respublika miqyosida umumiy 15 ta ilmiy konferensiyada ushbu tadqiqotning asosiy natijalari taqdim qilingan.

Tadqiqot natijalarining e’lon qilinganligi. 7 ta ilmiy maqola hamda 15 ta tezis mazkur tadqiqotning mavzusi doirasida chop etilgan. Ilmiy nashrlar ro‘yxatiga falsafa doktorlik dissertatsiyasini himoya qilish uchun O‘zbekiston Respublikasi Oliy attestatsiya komissiyasi tomonidan tavsiya etilgan 7 ta ilmiy maqola nashr qilingan. Shundan xalqaro jurnallarda 2 ta ilmiy maqola, milliy ilmiy jurnallarda esa 5 ta ilmiy maqola nashr qilingan hamda ilmiy konferensiyalarda 15 ta tezis e’lon qilingan.

Dissertatsiyaning tuzilishi va hajmi 114 betdan iborat hamda dissertatsiyaning umumiy hajmiga kirish, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxati kiradi.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusi ahamiyati va dolzarbligi ko‘rsatilgan, dissertatsiyaning respublika fan va texnologiyalarining rivojlanishi ustuvor yo‘nalishlariga mosligi asoslangan, dissertatsiyaning mavzusiga oid xorijiy ilmiy ishlarning tahlili bayon qilingan, masalaning o‘rganilganlik darajasini ko‘rsatilgan, tadqiqotning ob’yekti va predmeti, maqsad va vazifalari keltirilgan, tadqiqotdan olingan natijalarning tatbiqi bayon qilingan, tadqiqotdan olingan natijalarning ilmiy yangiligi asoslab berilgan va nazariy-amaliy ahamiyati keltirilgan, hamda dissertatsiyaning tuzilishi va dissertatsiya yuzasidan chop etilgan ilmiy ishlar ro‘yxati haqidagi ma’lumotlar keltirilgan.

“**Boshqaruvlari Eksponensial chegaralanishli differensial o‘yinlar**” deb nomlangan birinchi bobda boshqaruvlari $I_{\exp}$ va $\overline{I}_{\exp}$-chegaralanishli oddiy va chiziqli differensial o‘yinlarda quvish-qochish masalalari o‘rganilgan.

Ikkita o‘yinchilar $\mathbb{R}^n$ fazoda harakat qilayotgan bo‘lsin. O‘yinchilardan biri *quvlovchi* P, ikkinchisi esa *qochuvchi* E. $\mathbb{R}^n$ fazoda ularning harakati quyidagi differensial tenglamalarning oddiy sistemasi orqali ifodalansin:

$$P: \ \dot{x} = u, \ \ x(0) = x_0, \qquad E: \ \dot{y} = v, \ \ y(0) = y_0, \tag{1}$$

bu yerda $x(t)$ va $y(t)-$ mos ravishda P va E o‘yinchilarning t vaqtdagi holat vektorlari, x_0 va y_0- mos ravishda P va E o‘yinchilarning boshlang‘ich holatlari. Biz $x_0 \neq y_0$ deb faraz qilamiz. $u(\cdot):[0,+\infty) \to \mathbb{R}^n$ va $v(\cdot):[0,+\infty) \to \mathbb{R}^n$ funksiyalar mos ravishda P va E o‘yinchilarning boshqaruvlari. Ularni o‘lchanuvchi funksiyalar sifatida faraz qilamiz. $u(\cdot), v(\cdot)$ boshqaruv funksiyalari uchun mos ravishda quyidagi (2)-eksponensial integral chegaralanishni (qisqacha $I_{\exp}$-chegaralanish) qanoatlantiruvchi $U_{I_{\exp}}$ va $V_{I_{\exp}}$ funksiyalar sinflarini taklif qilamiz:

$$\int_0^t |u(s)|^2 ds \leq \rho_0^2 e^{2kt}, \quad t \geq 0, \qquad \int_0^t |v(s)|^2 ds \leq \sigma_0^2 e^{2kt}, \quad t \geq 0, \tag{2}$$

bu yerda $\rho_0, \sigma_0, k > 0$.

Endi P va E o‘yinchilarning harakatlari quyidagi differensial tenglamalarning chiziqli shakli bilan ifodalansin deylik:

$$P: \ \dot{x} - lx = u, \quad x(0) = x_0, \qquad E: \ \dot{y} - ly = v, \quad y(0) = y_0, \tag{3}$$

bu yerda $l \neq 0$, $u(\cdot), v(\cdot)$ mos ravishda quyidagi (4)-eksponensial chegralanishni (qisqacha $\overline{I}_{\exp}$-chegralanish) qanoatlantiruvchi $U_{\overline{I}_{\exp}}$ va $V_{\overline{I}_{\exp}}$ funksiyalar sinflaridan olingan:

$$\int_0^t |u(s)|^2 ds \leq \frac{\rho^2}{2k}\left(e^{2kt} - 1\right), \quad t \geq 0, \qquad \int_0^t |v(s)|^2 ds \leq \frac{\sigma^2}{2k}\left(e^{2kt} - 1\right), \quad t \geq 0. \tag{4}$$

bu yerda $\rho, \sigma > 0$.

Qisqalik uchun, (1)-(2) ((3)-(4)) o‘yinni $I_{\exp}$ $(\overline{I}_{\exp})$-o‘yin deymiz.

$I_{\exp}\left(\overline{I}_{\exp}\right)$ - o‘yinda, quvlovchi P ning *maqsadi* qochuvchi E tutish, ya’ni quyidagi tenglikni isbotlash: $\exists\, t^* > 0, \ x(t^*) = y(t^*)$ $\left(\exists\, t^{**} > 0, \ x(t^{**}) = y(t^{**})\right)$.

$I_{\exp}\left(\overline{I}_{\exp}\right)$ - o‘yinda, qochuvchi E ning *maqsadi* quvlovchi P dan qochish, ya’ni quyidagi tengsizlikni isbotlash: $\forall\, t > 0, \ x(t) \neq y(t)$.

1-ta’rif. Agar $I_{\exp}$ -o‘yinda $\rho \geq \sigma$ bo‘lsa, u holda quyidagi

$$\boldsymbol{u}_{I_{\exp}}(t, v) = v - \lambda_{I_{\exp}}(t, v)\xi_0, \tag{5}$$

funksiyani quvlovchining $\Pi_{I_{\exp}}$-*strategiyasi* deymiz, bu yerda

$\lambda_{I_{\exp}}(t,v)=\max\left\{0,2\langle v,\xi_0\rangle+\delta_0 e^{2kt}\right\}$, $\delta_0=\dfrac{\rho_0^2-\sigma_0^2}{|z_0|}$, $\xi_0=\dfrac{z_0}{|z_0|}$ va $\langle v,\xi_0\rangle$ - $\mathbb{R}^n$ fazodagi v va ξ_0 vektorlarning skalyar ko‘paytmasi.

1-teorema. Agar $\rho_0>\sigma_0$ bo‘lsa, u holda $\Pi_{I_{\exp}}$-strategiya (5), $I_{\exp}$-o‘yinda $\left[0,T_{I_{\exp}}\right]$ vaqt mobaynida quvishni yakunlashni kafolatlaydi, bu yerda $T_{I_{\exp}}$-kafolatlangan tutish vaqti va quyidagi tenglamaning yechimi

$$\Lambda_{I_{\exp}}(t)=0,\ \ \Lambda_{I_{\exp}}(t)=1-\frac{\delta_0}{2k\,|z_0|}\left(e^{2kt}-1\right)+\frac{2\sigma_0}{|z_0|}\sqrt{t}e^{kt}.$$

2-ta’rif. Agar $I_{\exp}$ -o‘yinda $\rho\geq\sigma$ bo‘lsa, u holda quyidagi

$$\boldsymbol{u}_{\bar{I}_{\exp}}(t,v)=v-\lambda_{\bar{I}_{\exp}}(t,v)\xi_0, \tag{6}$$

funksiyani quvlovchining $\Pi_{\bar{I}_{\exp}}$ -*strategiyasi* deymiz, bu yerda

$$\lambda_{\bar{I}_{\exp}}(t,v)=\max\left\{0,2\langle v,\xi_0\rangle+\bar{\delta}\left(e^{2kt}-1\right)\right\},\ \bar{\delta}=\frac{\rho^2-\sigma^2}{2k|z_0|}.$$

2-teorema. Agar $\rho>\sigma$ va $k>l>0$ bo‘lsa, u holda $\bar{I}_{\exp}$-o‘yinda $\left[0,T_{\bar{I}_{\exp}}\right]$ da $\Pi_{\bar{I}_{\exp}}$ -strategiya (6) quvishni yakunlashi kafolatlaydi, bu yerda $T_{\bar{I}_{\exp}}$ -kafolatlangan tutish vaqti va quyidagi tenglamaning yechimi $F_P(t)-G_P(t)=|z_0|$ (7)

$$G_P(t)=\sigma\sqrt{\frac{1}{kl}(1-e^{-2lt})(e^{2kt}-1)},\ F_P(t)=\bar{\delta}\left(\frac{e^{(2k-l)t}-1}{2k-l}-\frac{1-e^{-lt}}{l}\right).$$

3-ta’rif. $\bar{I}_{\exp}$-o‘yinda $\rho\leq\sigma$ shart bajarilsa, u holda quyidagi funksiya (8) ni qochuvchining *strategiyasi* deymiz:

$$\boldsymbol{v}_{\bar{I}_{\exp}}(t,u(t-\varsigma))=\begin{cases}0, & agar\ \ 0\leq t<\varsigma,\\ -\sqrt{|u(t-\varsigma)|^2+\left(\sigma^2-\rho^2\right)e^{2k(t-\varsigma)}}\,\xi_0, & agar\ \ t\geq\varsigma,\end{cases} \tag{8}$$

bu yerda ς -berilgan musbat son.

3-teorema. Agar $\bar{I}_{\exp}$-o‘yinda $\rho\leq\sigma$, $-k<l<0$ va $0<\varsigma\leq\dfrac{1}{2k}ln\left(\dfrac{2k|z_0|}{\rho}+1\right)$ o‘rinli bo‘lsa, u holda (8) strategiya qochuvchi uchun barcha $t\in[0,+\infty)$ da yutuqli bo‘ladi va o‘yinchilar orasidagi masofa uchun quyidagi munosabat o‘rinli bo‘ladi:

$$|z(t)|>\begin{cases}0, \ \ agar\ \ 0\leq t<\varsigma,\\ e^{-lt}\left[\sqrt{F_E(t-\varsigma)+G_E(t-\varsigma)}-\sqrt{F_E(t-\varsigma)}\right],\ agar\ \ t\geq\varsigma,\end{cases}$$

$$F_E(t)=\frac{\rho^2}{4kl}\left(e^{2kt}-1\right)\left(1-e^{-2lt}\right),\ \ G_E(t)=\frac{\left(\sigma^2-\rho^2\right)}{(k-l)^2}\left(e^{(k-l)t}-1\right)^2.$$

“**Boshqaruvlari Langenhop tipidagi chegaralanishli differensial o‘yinlar**” deb atalgan 2-bobda harakati (1) tenglamalar bilan ifodalangan o‘yinchilarning boshqaruvlari uchun Langenhop tipidagi chegralanish (qisqacha, La-chegara-lanish) taklif qilingan bo‘lib, La, G va I-chegralanishlarni qanoatlantiruvchi funksiyalar sinflari orasidagi bog‘liqlikni aniqlashga hamda boshqaruvlari La-chegaralanishli quvish-qochish masalalarini o‘rganishga bag‘ishlangan. Quvlovchi va qochuvchining boshqaruvlari uchun quyidagi sinflarni taklif qilamiz:

- Langenhop tipidagi chegaralanish (qisqacha La-chegaralanish) (9):

$$\begin{aligned} |u(t)|^2 &\le \rho^2 - 2k_1\int_0^t |u(s)|^2\, ds, \quad t \ge 0, \\ |v(t)|^2 &\le \sigma^2 - 2k_1\int_0^t |v(s)|^2\, ds, \quad t \ge 0 \end{aligned} \tag{9}$$

va bu sinflarni mos ravishda U_{La} va V_{La} deb belgilaymiz, bu yerda $\rho,\ \sigma,\ k_1 > 0$;

- Geometrik chegaralanish (qisqacha G-chegaralanish) (10):

$$|u(t)| \le \rho e^{-k_1 t},\ t \ge 0,\quad |v(t)| \le \sigma e^{-k_1 t},\ t \ge 0 \tag{10}$$

va bu sinflarni mos ravishda U_G va V_G deb belgilaymiz;

- Integral chegaralanish (qisqacha I-chegaralanish) (11):

$$\int_0^t |u(s)|^2\, ds \le \frac{\rho^2}{2k_1}\left(1 - e^{-2k_1 t}\right),\ t \ge 0,\quad \int_0^t |v(s)|^2\, ds \le \frac{\sigma^2}{2k_1}\left(1 - e^{-2k_1 t}\right),\quad t \ge 0 \tag{11}$$

va biz bu sinflarni mos ravishda U_I va V_I deb belgilaymiz.

Endi o‘yinchilarning harakati quyidagi bilan ifodalangan o‘yinni o‘rganamiz:

$$P:\ \ddot{x} = u,\ \ \dot{x}(0) = x_1,\ \ x(0) = x_0,\quad E:\ \ddot{y} = v,\ \ \dot{y}(0) = y_1,\ \ y(0) = y_0, \tag{12}$$

bu yerda $x,\ y, x_1, y_1,\ x_0, y_0,\ u,\ v \in \mathbb{R}^n$, $n \ge 2$; $x_0 \ne y_0, x_1 \ne y_1$.

Endi o‘yinchilarning harakati quyidagicha ifodalangan holni ko‘rib chiqamiz:

$$P:\ \dot{x} = Ax + u,\quad x(0) = x_0,\qquad E:\ \dot{y} = Ay + v,\quad y(0) = y_0, \tag{13}$$

bu yerda $x, y, u, v \in \mathbb{R}^n$, $A \ne 0$-berilgan $n \times n$ matritsa.

Faraz qilaylik, $A = k_2 E$ bo‘lsin, bu yerda $k_2 \ne 0$, $k_2 > k_1$:

$$P:\ \dot{x} + k_2 x = u,\quad x(0) = x_0,\qquad E:\ \dot{y} + k_2 y = v,\quad y(0) = y_0. \tag{14}$$

Biz Langenhop chegaralanishli (1)-o‘yinni La-o‘yin, (12)-o‘yinni La-inersiyali o‘yinchilar o‘yini, (13)-o‘yinni La-chiziqli o‘yin, (14)-o‘yinni La-bitta chiziqli hol uchun o‘yin deb ataymiz.

1-lemma. $U_G \subset U_{La} \subset U_I$, $V_G \subset V_{La} \subset V_I$.

4-ta’rif. Agar La-o‘yinda $\rho \ge \sigma$ bo‘lsa, u holda

$$\boldsymbol{u}_{La}(t, v) = v - \lambda_{La}(t, v)\xi_0 \tag{15}$$

funksiyani quvlovchining Π_{La}-*strategiyasi* deymiz, bu yerda

$$\lambda_{La}(t, v) = \langle v, \xi_0\rangle + \sqrt{\langle v, \xi_0\rangle^2 + \delta e^{-2k_1 t}},\ \delta = \rho^2 - \sigma^2.$$

1-faraz. $\rho > \sigma$ va $\sqrt{\Phi_P(t) + \Psi_P(t)} - \sqrt{\Phi_P(t)} = |z_0|$ (16) tenglamaning t bo‘yicha eng kamida bitta musbat yechimga ega bo‘lsin, bu yerda

$$\Phi_P(t)=\frac{t\sigma^2}{2k_1}\left(1-e^{-2k_1t}\right),\ \Psi_P(t)=\frac{\delta}{k_1^2}\left(1-e^{-k_1t}\right)^2.$$

(16) tenglamaning eng kichik musbat yechimini *kafolatlangan tutish vaqti* deymiz va uni T_{La} bilan belgilaymiz.

4-teorema. Agar La-o‘yinda *1-faraz* o‘rinli bo‘lsa, u holda Π_{La}-strategiya (15) $\left[0,T_{La}\right]$ da quvishni tugatishni kafolatlaydi.

Xuddi shuningdek, La-inersiyali o‘yinchilar o‘yinida quvish-qochish masalalarini hal qilish uchun asosiy ta’rif va teoremalarni kiritamiz:

2-faraz. $\rho\geq\sigma$ bo‘lsin va $\Lambda_{La}^*(t)=0$ (17) tenglamaning t bo‘yicha eng kamida bitta musbat yechimga ega bo‘lsin, bu yerda

$$\Lambda_{La}^*(t)=1+mt-\frac{1}{|z_0|}\left[\sqrt{\Phi_P^*(t)+\Psi_P^*(t)}-\sqrt{\Phi_P^*(t)}\right],\quad t\geq 0,$$

$$\Phi_P^*(t)=t^3\sigma^2\left(1-e^{-2k_1t}\right)/\,6k_1,\quad \Psi_P^*(t)=\delta\left(e^{-k_1t}+k_1t-1\right)^2/\,k_1^4.$$

(17) ning eng kichik musbat yechimini *kafolatlangan tutish vaqti* deymiz va uni T_{La}^* belgilaymiz.

5-teorema. Agar La-inersiyali o‘yinchilar o‘yinida 2-faraz o‘rinli bo‘lsa, u holda, Π_{La}-strategiya (15) $\left[0,T_{La}^*\right]$ da quvishni tugatishni kafolatlaydi.

5-ta’rif. $\rho\leq\sigma$ bo‘lganda E qochuvchining *strategiyasi* deb quyidagi funksiyaga aytamiz:

$$v_{La}^*(t,u(t-\varepsilon))=\begin{cases}0, & agar\quad 0\leq t<\varepsilon,\\ -|u(t-\varepsilon)|\xi_0, & agar\quad t\geq\varepsilon,\end{cases}\tag{18}$$

bu yerda ε-berilgan musbat son.

6-teorema. Agar $\rho\leq\sigma,\ 0<\varepsilon\leq\sqrt[3]{\frac{6k_1|z_0|^2}{\rho^2}}$ va $m\geq\frac{\rho}{2|z_0|}\sqrt{\frac{\varepsilon}{2k_1}}$ bo‘lsa, u holda, (18)-strategiya qochuvchi uchun barcha $t\in[0,+\infty)$ da yutuqli bo‘ladi.

Shuningdek, La-chiziqli o‘yinda quvish masalani hal qilish uchun asosiy ta’rif va teoremalarni kiritamiz:

6-ta’rif. Agar $\rho\geq\sigma$ bo‘lsa, quyidagi funksiyani

$$\boldsymbol{u}_{La}^*(t,v)=v-\frac{e^{At}z_0}{\left|e^{At}z_0\right|}\lambda_{La}^*(t,v)\tag{19}$$

quvlovchining Π_{La}^*-*strategiyasi* deymiz, bu yerda

$\lambda_{La}^*(t,v)=\frac{\langle v,e^{At}z_0\rangle}{\left|e^{At}z_0\right|}+\sqrt{\left(\frac{\langle v,e^{At}z_0\rangle}{\left|e^{At}z_0\right|}\right)^2+\delta e^{-2k_1t}}$, $\langle v,e^{At}z_0\rangle$ -$\mathbb{R}^n$ fazodagi v va $e^{At}z_0$ vektorlarning skalyar ko‘paytmasi.

3-faraz. $\rho > \sigma$ bo‘lsin va $\sqrt{\tilde{\Phi}_P(t)+\tilde{\Psi}_P(t)}-\sqrt{\tilde{\Phi}_P(t)}=|z_0|$ (20) tenglamaning t bo‘yicha eng kamida bitta musbat ildizi mavjud bo‘lsin, bu yerda

$$\tilde{\Phi}_P(t)=\frac{\sigma^2}{4\|A\|k_1}\left(1-e^{-2\|A\|t}\right)\left(1-e^{-2k_1t}\right),\ \tilde{\Psi}_P(t)=\frac{\delta}{\left(\|A\|+k_1\right)^2}\left(1-e^{-\left(\|A\|+k_1\right)t}\right)^2.$$

(20) tenglamaning eng kichik musbat ildizini *kafolatlangan tutish vaqti* deymiz va $\tilde{\mathrm{T}}_{La}$ bilan bilgilaymiz.

7-teorema. Agar 3-faraz La-chiziqli o‘yinda o‘rinli bo‘lsa, u holda Π^*_{La}-strategiya $\left[0,\bar{\mathrm{T}}_{La}\right]$ da quvishni tugatishni kafolatlaydi.

Xuddi shuningdek, La-bitta chiziqli hol uchun o‘yin uchun:

4-faraz. $\rho > \sigma,\ k_2 > k_1$ va $\sqrt{\bar{\Phi}_P(t)+\bar{\Psi}_P(t)}-\sqrt{\bar{\Phi}_P(t)}=|z_0|$ (21) tenglamaning t bo‘yicha eng kamida bitta musbat yechimi mavjud bo‘lsin, bu yerda

$$\bar{\Phi}_P(t)=\frac{\sigma^2}{4k_1k_2}\left(e^{2k_2t}-1\right)\left(1-e^{-2k_1t}\right),\ \ \bar{\Psi}_P(t)=\frac{\rho^2-\sigma^2}{\left(k_1-k_2\right)^2}\left(e^{\left(k_2-k_1\right)t}-1\right)^2.$$

(21) ning eng kichik musbat yechimini *kafolatlangan tutish vaqti* deymiz va $\hat{T}_{La}$ deb belgilaymiz.

8-teorema. Agar 4-faraz o‘rinli bo‘lsa, u holda Π_{La}-strategiya $\left[0,\hat{T}_{La}\right]$ vaqt oralig‘ida quvishni tugatishni kafolatlaydi.

7-ta’rif. Agar $\sigma \ge \rho$, u holda qochuvchining E_{La}-strategiyasi deb

$$\mathbf{v}_{La}(t,u(t-\bar{\varepsilon}))=\begin{cases}0, & agar\ \ 0\le t<\bar{\varepsilon},\\ -\sqrt{\left|u(t-\bar{\varepsilon})\right|^2+\theta e^{-2k_1(t-\bar{\varepsilon})}}\,\xi_0, & agar\ t\ge\bar{\varepsilon},\end{cases}$$

funksiyaga aytamiz, bu yerda $\bar{\varepsilon}$-berilgan musbat son.

9-teorema. Agar $\rho\le\sigma,\ k_2>k_1,$ va $0<\bar{\varepsilon}\le\frac{1}{2k_2}ln\left(\frac{4k_1k_2|z_0|^2}{\rho^2}+1\right)$ bo‘lsa, u holda E_{La}-strategiya qochuvchi uchun barcha $t\in\left[0,+\infty\right)$da yutuqli bo‘ladi va quyidagi baholash o‘rinli

$$|z(t)|>\begin{cases}0, & agar\ \ 0\le t<\bar{\varepsilon},\\ e^{-k_2t}\left[\sqrt{\bar{\Phi}_E(t)+\bar{\Psi}_E(t)}-\sqrt{\bar{\Phi}_E(t)}\right], & agar\ \ t\ge\bar{\varepsilon},\end{cases} \tag{22}$$

$$\bar{\Phi}_E(t)=\frac{\rho^2}{4k_1k_2}\left(e^{2k_2t}-1\right)\left(1-e^{-2k_1t}\right),\ \ \bar{\Psi}_E(t)=\frac{\left(\sigma^2-\rho^2\right)}{\left(k_1-k_2\right)^2}\left(e^{\left(k_2-k_1\right)t}-1\right)^2.$$

“**Boshqaruvlari nostatsionar Langenhop tipidagi chegaralanishli differensial o‘yinlar**” deb atalgan 3-bobda o‘yinchilarning boshqaruvlari uchun ikkita nostatsionar Langenhop tipidagi chegaralanishlar (qisqacha, LT va LT^*-chegaralanishlar) taklif qilingan bo‘lib, $LT\left(LT^*\right),\ \bar{G}\left(G^*\right)$ va $\bar{I}\left(I^*\right)$-chegaralanishlarni qanoatlantiruvchi funksiyalar sinflari orasidagi bog‘liqlikni aniqlashga hamda boshqaruvlari $LT\left(LT^*\right)$-chegaralanishli quvish-qochish masalalarini o‘rganishga bag‘ishlangan.

$\mathbb{R}^n$ fazoda bitta quvlovchi va bitta qochuvchi qatnashgan differensial o‘yinni ko‘ramiz. Bunda ikkala P va E o‘yinchilarning harakatlari quyidagi ko‘rinishda beriladi:

$$P:\ \dot{x}=a(t)u,\quad x(0)=x_0,\qquad E:\ \dot{y}=b(t)u,\quad y(0)=y_0, \tag{23}$$

($x_0\neq y_0$), bu yerda $a(t)$ va $b(t)$-skalyar va o‘lchanuvchi musbat funksiyalar barcha $t\geq 0$.

O‘yinchilarning boshqaruvlari uchun quyidagi sinflarini taklif qilamiz:

- P va E o‘yinchilar boshqaruvi uchun Langenhop tipidagi nonstatsionar chegaralanish (LT - chegaralanish):

$$\begin{aligned}|u(t)|^2&\leq\rho^2-2k_1\int_0^t k(s)|u(s)|^2\,ds,\quad t\geq 0,\\ |v(t)|^2&\leq\sigma^2-2k_1\int_0^t k(s)|v(s)|^2\,ds,\quad t\geq 0,\end{aligned}\tag{24}$$

bu yerda $\rho,\sigma,k_1>0$, $k(t)>0$-skalyar, o‘lchanuvchi, monoton o‘suvchi va Lebeg ma’nosida $[0,t]$ oraqliqda integrallanuvchi funksiya: $K(t)=\int_0^t k(s)ds.$

U_{LT} va V_{LT} - (24) ni qanoatlantiruvchi $u(\cdot)$ va $v(\cdot)$ boshqaruv funksiyalar sinfi;

- P va E boshqaruvlari uchun geometrik chegaralanish ($\overline{G}$-chegaralanish):

$$|u(t)|\leq\rho e^{-k_1K(t)},\quad t\geq 0,\quad |v(t)|\leq\sigma e^{-k_1K(t)},\quad t\geq 0, \tag{25}$$

$U_{\overline{G}}$ va $V_{\overline{G}}$-mos ravishda (25) ni qanoatlantiruvchi $u(\cdot)$ va $v(\cdot)$ boshqaruv funksiya-

sinfi;

- P va E boshqaruvlari uchun integral chegaralanish ($\overline{I}$ chegaralanish):

$$\begin{aligned}\int_0^t k(s)|u(s)|^2\,ds&\leq\frac{\rho^2}{2k_1}\left(1-e^{-2k_1K(t)}\right),\quad t\geq 0,\\ \int_0^t k(s)|v(s)|^2\,ds&\leq\frac{\sigma^2}{2k_1}\left(1-e^{-2k_1K(t)}\right),\quad t\geq 0,\end{aligned}\tag{26}$$

$U_{\overline{I}}$ va $V_{\overline{I}}$ - (26) ni qanoatlantiruvchi $u(\cdot)$ va $v(\cdot)$ boshqaruv funksiyalar sinfi;

Endi (23) da $a(t)=b(t)=1$ holda P va E uchun quyidagi sinflarini taklif qilamiz:

- U_{LT^*} (V_{LT^*})-quyidagi (27) nostatsionar Langenhop tipidagi (LT^*-chegarlanish) chegaralanishni qanoatlantiruvchi barcha $u(\cdot)$ va $v(\cdot)$ boshqaruvlar sinfi:

$$
\begin{aligned}
&|u(t)|\le \alpha(t)\left(\rho - k_1\int_0^t |u(s)|\,ds\right), \quad t\ge 0,\\
&|v(t)|\le \alpha(t)\left(\sigma - k_1\int_0^t |v(s)|\,ds\right), \quad t\ge 0,
\end{aligned} \tag{27}
$$

$\alpha(t)>0$- skalyar, o‘lchanuvchi, monoton o‘suvchi va Lebeg ma’nosida integrallanuvchi funksiya: $A(t)=\int_0^t \alpha(s)ds.$

- U_{G^*} (V_{G^*})-quyidagi (28) nostatsionar geometrik chegaralanishni(G^*-chegaralanish) qanoatlantiruvchi barcha $u(\cdot)$ ($v(\cdot)$) boshqaruv funksiyalar sinfi:

$$
\begin{aligned}
&|u(t)|\le \rho\alpha(t)e^{-k_1A(t)}, \qquad t\ge 0,\\
&|v(t)|\le \sigma\alpha(t)e^{-k_1A(t)}, \qquad t\ge 0.
\end{aligned} \tag{28}
$$

- U_{I^*} (V_{I^*})-quyidagi nostatsionar Integral chegaralanishni (I^*-chegaralanish) qanoatlantiruvchi barcha $u(\cdot)$ va $v(\cdot)$ boshqaruv funksiyalar sinfi:

$$
\int_0^t |u(s)|\,ds \le \frac{\rho}{k_1}\left(1-e^{-k_1A(t)}\right), \ t\ge 0, \quad \int_0^t |v(s)|\,ds \le \frac{\sigma}{k_1}\left(1-e^{-k_1A(t)}\right), \ t\ge 0. \tag{29}
$$

Qisqalik uchun LT-chegralanishli (23)-o‘yinni LT o‘yin va (23) dagi $a(t)=b(t)=1$ holdagi LT^*-chegralanishli o‘yinni qisqacha LT^*-o‘yin deb ataymiz.

2-lemma. $U_{\bar G}\subset U_{LT}\subset U_{\bar I}$, $V_{\bar G}\subset V_{LT}\subset V_{\bar I}$.

3-lemma. $U_{G^*}\subset U_{LT^*}\subset U_{I^*}$, $V_{G^*}\subset V_{LT^*}\subset V_{I^*}$.

8-ta’rif. Agar $\rho\ge\sigma, b(t)\le k(t)\le a(t)$ deyarli barcha $t\ge 0$, u holda quyidagi

$$
\boldsymbol{u}_{LT}(t,v)=\frac{b(t)}{a(t)}v-\lambda_{LT}(t,v(t))\xi_0, \tag{30}
$$

funksiyani quvlovchining Π_{LT}-*strategiyasi* deymiz, bu yerda

$$
\lambda_{LT}(t,v)=\frac{b(t)}{a(t)}\langle v,\xi_0\rangle+\sqrt{\left(\frac{b(t)}{a(t)}\langle v,\xi_0\rangle\right)^2+\delta e^{-2k_1K(t)}}.
$$

5-faraz. $\rho>\sigma$ va $\Lambda_{LT}(t)=0$ tenglamaning t bo‘yicha eng kamida bitta musbat yechimi mavjud bo‘lsin, bu yerda

$$
\Lambda_{LT}(t)=1-\frac{1}{|z_0|}[\sqrt{\Upsilon_P(t)+\Omega_P(t)}-\sqrt{\Omega_P(t)}],
$$

$$
\Upsilon_P(t)=\frac{\sigma^2K(t)}{2k_1}\left(1-e^{-2k_1K(t)}\right), \ \Omega_P(t)=\frac{\delta}{k_1^2}\left(1-e^{-k_1K(t)}\right)^2, \ \delta=\sigma^2-\rho^2.
$$

$\Lambda_{LT}(t)=0$ tenglamaning eng kichik musbat yechimini *kafolatlangan tutish vaqti* deymiz va $\tilde{\mathrm{T}}_{La}$ bilan bilgilaymiz.

10-teorema. Agar LT-o‘yinda $b(t) \le k(t) \le a(t)$ deyarli barcha $t \ge 0$ va 5-*faraz* o‘rinli bo‘lsa, u holda Π_{LT}-strategiya (30) yordamida $\left[0, T_{LT}\right]$ vaqt oralig‘ida quvishni tugatish kafolatlanadi.

9-ta’rif. Agar $\rho \ge \sigma,\ a(t) = b(t) \ge k(t)$ deyarli barcha $t \ge 0$, u holda quyidagi

$$\boldsymbol{u}^*_{LT}(t, v) = v - \lambda^*_{LT}(t, v(t))\xi_0 \qquad (31)$$

funksiyani quvlovchining Π^*_{LT}-*strategiyasi* deymiz, bu yerda

$$\lambda^*_{LT}(t, v) = \langle v, \xi_0 \rangle + \sqrt{\langle v, \xi_0 \rangle^2 + \delta e^{-2k_1 K(t)}}.$$

11-teorema. Agar LT-o‘yinda $a(t) = b(t) \ge k(t),$ deyarli barcha $t \ge 0$, va 5-*faraz* o‘rinli bo‘lsa, u holda Π^*_{LT}-strategiya (31) yordamida $\left[0, T_{LT}\right]$ vaqt oralig‘ida quvishni tugatish kafolatlanadi.

LT-qochish masalsini hal qilish uchun quyidagi shartdan foydalanamiz:

$$\begin{cases} a(t) \le k(t) \le b(t) \quad \text{deyarli barcha } t \ge 0, \\ a(t) = b(t) \le k(t), \quad \text{deyarli barcha } t \ge 0. \end{cases} \qquad (32)$$

10-ta’rif. Agar $\rho \le \sigma$ bajarilsa, u holda quyidagi funksiyani

$$\boldsymbol{v}_{LT}(t, u(t - \bar{\varsigma})) = \begin{cases} 0, \quad agar \ \ 0 \le t < \bar{\varsigma}, \\ -\sqrt{\left|u(t - \bar{\varsigma})\right|^2 + \left(\sigma^2 - \rho^2\right) e^{-2k_1 K(t - \bar{\varsigma})}}\,\xi_0, \ \ agar \ \ t \ge \bar{\varsigma} \end{cases} \qquad (33)$$

qochuvchining E_{LT}-*strategiyasi* deymiz, bu yerda $\bar{\varsigma}$ - musbat son.

12-teorema. Agar LT-o‘yinda $\sigma \ge \rho,\ \ 0 < \bar{\varsigma} \le \dfrac{2k_1\left|z_0\right|^2}{\rho^2},\ \ k(t) \le 1$ va (32) shartlar o‘rinli bo‘lsa, u holda E_{LT}-strategiya (33) qochuvchi uchun $\left[0, +\infty\right)$ da yutuqli bo‘ladi va o‘yinchilar orasidagi masofa uchun quyidagi munosabat o‘rinli

$$\left|z(t)\right| > \begin{cases} 0, \quad agar \ \ 0 \le t < \bar{\varsigma}, \\ \sqrt{\Upsilon_E(t - \bar{\varsigma}) + \Omega_E(t - \bar{\varsigma})} - \sqrt{\Omega_E(t - \bar{\varsigma})}, \ \ agar \ t \ge \bar{\varsigma}, \end{cases} \qquad (34)$$

$$\Upsilon_E(t) = \rho^2 K(t)\left(1 - e^{-2k_1 K(t)}\right) / 2k_1, \ \ \Omega_E(t) = \left(\sigma^2 - \rho^2\right)\left(1 - e^{-k_1 K(t)}\right)^2 / k_1^2.$$

11-ta’rif. Agar $\rho \ge \sigma$, u holda $\boldsymbol{u}_{LT^*}(t, v) = v - \lambda_{LT}(t, v)\xi_0$ (35) funksiyani quvlovchining Π_{LT^*}-*strategiyasi* deymiz, bu yerda

$$\lambda_{LT^*}(t, v) = \langle v, \xi_0 \rangle + \sqrt{\langle v, \xi_0 \rangle^2 + 2\delta^* \alpha(t) e^{-k_1 A(t)} \left| v \right| + \left(\delta^* \alpha(t) e^{-k_1 A(t)}\right)^2}, \ \ \delta^* = \rho - \sigma.$$

13-teorema. Agar LT^*-o‘yinda $\delta^* > k_1\left|z_0\right|$ o‘rinli bo‘lsa, u holda Π_{LT^*}-strategiya (35) LT^*-o‘yinda $\left[0, T_{LT^*}\right]$ da quvishni tugatilishini kafolatlaydi, bu yerda T_{LT^*} - kafolatlangan tutish vaqti va $\Lambda_{LT^*}(t) = 0$ (36) tenglamaning yechimi, bu yerda $\Lambda_{LT^*}(t) = 1 - \dfrac{\delta^*}{k_1 \left| z_0 \right|}\left(1 - e^{-k_1 A(t)}\right).$

12-ta'rif. Quyidagi $\boldsymbol{v}_{LT}(t) = -\sigma\alpha(t)e^{-k_1 A(t)}\xi_0$ funksiya qochuvchining $\mathbf{E}_{LT^*}$-*strategiyasi* deyiladi.

14-teorema. Agar LT^*-o'yinda $\sigma \geq \rho$ bo'lsa, u holda $\mathbf{E}_{LT^*}$-strategiya qochuvchi uchun barcha $t \in [0, +\infty)$ da yutuqli bo'ladi.

XULOSA

Dissertatsiyada o'yinchilarning boshqaruvlari exponensial va Langenhop tipidagi chegaralanishli quvish-qochish masalalari uchun quyidagi tadqiqotlar bajarildi

Tadqiqotda asosiy quyidagi natijalarga erishildi:

1. Boshqaruv funksiyalari eksponensial integral chegaralanishli oddiy va chziqli differensial o'yinda berilgan parametrlar asosida quvlovchiga Π-strategiya qurilgan, qochuvchiga esa kechikishga ega boshqaruv aniqlangan va natijada quvish-qochish masalasi yechilishini kafolatlovchi yetarli shartlarni aniqlangan.
2. Boshqaruv funksiyalari Langenhop tipidagi chegaralanishli differensial o'yinlarda berilgan parametrlar asosida quvlovchining Π-strategiyasi va qochuvchining kechikishga ega boshqaruvi yordamida quvish-qochish masalasini yechilishini kafolatlovchi yetarli shartlari aniqlangan va o'yinchilar orasidagi masofaning quyi chegarasi topilgan.
3. Boshqaruv funksiyalari Langenhop tipidagi chegaralanishli chiziqli differensial o'yinlarda berilgan parametrlar bo'yicha quvlovchining Π-strategiyasi va qochuvchining kechikishga ega boshqaruvi orqali quvish-qochish masalasi yechilishining yetarli shartlari aniqlangan va o'yinchilar orasidagi masofaning quyi chegarasini topilgan.
4. Boshqaruv funksiyalari Langenhop tipidagi nostatsionar chegaralanishli differensial o'yinlarda berilgan parametrlar asosida quvlovchiga Π-strategiya qurilgan, qochuvchiga esa kechikish parametrli boshqaruv aniqlangan va natijada quvish-qochish masalasi yechilishining yetarli shartlari aniqlangan.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED AFTER V.I.ROMANOVSKIY

INSTITUTE OF MATHEMATICS

URALOVA SABOKHAT ISMOILJON KIZI

DIFFERENTIAL GAMES WITH EXPONENTIAL AND LANGENHOP TYPE CONSTRAINTS ON CONTROLS

01.01.02 - Differential equations and mathematical physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT - 2024

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Ministry of Higher education, Science and Innovations of the Republic of Uzbekistan under number B2024.1.PhD/FM1006.

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Dissertation is possible to review in Information-resource center at Institute of Mathematics named after V.I.Romanovsky (registered for № 196) (Address: University str. 9, Almazar district, Tashkent, 100174, Uzbekistan, Ph.: (+998 78) 207 91 40).

Abstract of dissertation sent out on 27 December 2024 year.
(Mailing report № 2 on 27 December 2024 year).

O'ZBEKISTON RESPUBLIKASI FANLAR AKADEMIYASI V.I. ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI

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INTRODUCTION (abstract of PhD thesis)

Actuality and demand of the theme of dissertation. Around the world, in many scientific centers , the mathematical modeling of conflict-controlled dynamic processes and their widespread application are becoming one of the leading fields. Globally, the application of these models holds great significance in areas such as the market economy, military, healthcare, the development of modern technologies, and many other fields. If conflict-controlled dynamic processes are described by differential equations, then these problems are studied within the framework of differential game theory. If the mathematical models of controlled conflicting dynamic processes are described by discrete equations, then these processes are considered part of discrete game theory. However, since their general characteristics are interrelated, they are called dynamic game theory. The analysis of problems in conflict-controlled dynamic processes often requires the use of differential game theory. In this regard, correctly identifying and applying the elements of differential games is considered crucial in scientific research.

Today, the results obtained from various problems in differential games are widely applied to the control and optimization of conflict-controlled dynamic processes worldwide. As a result, this dissertation primarily focuses on the problems of differential game theory. In this field, different constraints are imposed on control functions, depending on the specifics of the mathematical models of conflict-controlled dynamic processes. These constraints can be geometric (indicating control limits), integral (indicating energy limits), mixed, or of other types. However, in practical problem-solving, more complex constraints may arise. Studying pursuit and evasion problems in differential games with exponential and Langenhop type constraints holds significant practical value, and these cases are thoroughly examined in this dissertation. Currently, scientific research is focused on the extensive study of pursuit and evasion problems under various constraints, with particular attention given to their application in fields such as engineering, economics, and others.

In our country, comprehensive measures are being implemented, and significant results are being achieved as part of strategies developed to enhance the scientific and practical significance of fundamental sciences, recognizing their crucial role in the development of innovative technologies. In particular, important tasks[1] have been set for scientific research in priority areas, such as differential equations, mathematical physics, algebra, functional analysis, and mathematical analysis, in line with international standards. As part of these efforts, the development of control functions in exponential and Langenhop type constrained differential games plays a vital role, especially in advancing and optimizing control models.

[1] Decree of the President of the Republic of Uzbekistan dated July 9, 2019 № PQ-4387 “On state support for the further development of mathematics education and subjects, as well as measures to fundamentally improve the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan”.

The subject and object of this dissertation align with the tasks outlined in the Decrees of the President of the Republic of Uzbekistan UP-4947 dated February 7, 2017 “On the strategy of action for the further development of the Republic of Uzbekistan”, UP-2789 dated February 17, 2017 “On measures to further improvement of the activities of the Academy of Sciences, organization, management, and financing of research activities, PP-4387 dated July 9, 2019

“On measures to further development of mathematical education and science, and also root improvement of the activity of the Uzbekistan Academy of Sciences V.I.Romanovsky Institute of Mathematics”, UP-4708 dated May 7, 2020 “Quality of education in the field of mathematics” , PP-60 dated January 28, 2022, “On the Development Strategy of New Uzbekistan for 2022-2026”, and furthermore, regulations that encompass this research.

Connection of research to priority directions of development of science and technologies of the Republic. This research was carried out in connection with the priority areas of science and technology of the Republic of Uzbekistan IV, “Mathematics, Mechanics, and Computer Science”.

The degree of scrutiny of the problem. R.Isaacs laid the foundation for the theory of differential games by developing mathematical models for military pursuit-evasion (P-E) problems at the RAND Corporation during 1949-1950. In Isaacs’s theory, he primarily employed the Hamilton-Jacobi methods of classical variational calculus, applying them to differential game problems where movements were governed by various controls. Later, mathematicians such as L.D.Berkovitz, W.H.Fleming, and A.Friedman advanced Isaacs’s methods, incorporating various techniques from variational calculus and game theory. Today, the founders of the fundamental theory of differential games include scholars such as R.Issacs, L.S.Pontryagin, N.N.Krasovskii, L.D.Berkovitz, W.H.Fleming, A.Friedman, A.Bryson, E.F.Mishchenko, A.I.Subbotin, Yu.S. Osipov, B.N.Pshenichnii, N.Yu.Satimov, L.A.Petrosyan, and A.A.Azamov.

Considering problems in differential game theory from the perspective of the pursuer, where the main information is given to the pursuer and the problem is solved in their favor, constitutes pursuit problems. Conversely, viewing the problem from the perspective of the evader, where the main information is given to the evader and the problem is solved in their favor, constitutes evasion problems. Based on such an approach, L.S.Pontryagin proposed various fundamental methods, which were later extensively developed by M.S.Nikolskiy, N.L.Grigorenko, A.A.Chikriy, B.N.Pshenichniy, A.I.Blagodatskikh, N.N.Petrov, N.Yu.Satimov, B.B. Rixsiyev, A.A.Azamov, A.Z.Fazilov, B.B.Rixsiyev, A.A.Khamdamov, G.I.Ibragimov, M.Sh.Mamatov, N.A.Mamadaliyev, and others.

In our country, N.Yu. Satimov led the “Optimal Control and Differential Games” scientific seminar for more than 35 years, and he was the founder of the Tashkent scientific school focused on optimal control theories and differential games. Currently, A.A.Azamov leads this scientific school. N.Yu.Satimov, A.A.Azamov, A.Z.Fazilov, B.B.Rixsiyev, A.A.Khamdamov, G.I.Ibragimov, M.Sh. Mamatov, B.T. Samatov, O.Sh.Kuchkarov, and N.A.Mamadaliyev have achieved significant results in solving various pursuit-evasion problems in differential games through their research efforts.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. This research has been carried out at the “Dynamic and Control Systems” Laboratory in Mathematics Institute at the Academy of Sciences of the Republic of Uzbekistan, in line with the planned scientific research theme.

The aim of the research is to solve pursuit-evasion problems in differential games with exponential and Langenhop-type constraints on control functions.

Problems of the research:

identifying sufficient conditions that guarantee a solution to the pursuit-evasion problems;

determining sufficient conditions that ensure a solution to the pursuit-evasion problems in simple and linear differential games with Langenhop type constraints on control functions;

establishing sufficient conditions that guarantee a solution to the pursuit-evasion problems in differential games involving inertial players with Langenhop type constraints on control functions;

identifying sufficient conditions for solving the pursuit-evasion problems in simple and non-stationary differential games with non-stationary constraints of Langenhop type on control functions.

The object of the research is differential games with exponential and Langenhop type constraints.

The subject of the research is differential games, differential equations and integral inequalities.

Methods of the research involve solving pursuit problems using the Π-strategy method, while the delayed strategy is used for evasion problems, along with applying fundamental theorems and concepts from mathematical analysis, differential game theory, differential equations, functional analysis and optimal control.

Scientific novelty of the research is as follows:

based on the parameters given in simple and linear differential games with exponential integral constraints on control functions, the Π-strategy is constructed for the pursuer, and the delayed control is defined for the evader, and these lead to sufficient conditions that guarantee a solution to the problems of pursuit-evasion;

in simple and linear differential games with Langenhop type constraints on controls, sufficient conditions for solving the pursuit problems using the pursuer’s Π-strategy and the evader’s delayed control are identified, and also the lower bound of the distance between the two players is determined;

based on the parameters given in differential games involving inertial players with Langenhop type constraints, the Π-strategy is constructed for the pursuer, the delayed control is defined for the evader, and as a result, sufficient conditions for solving the pursuit-evasion problems are identified;

based on the parameters given in simple and non-stationary differential games with non-stationary constraints of Langenhop type on controls, sufficient conditions for solving the pursuit-evasion problems are identified, and these conditions are achieved through the pursuer's Π-strategy and the evader's delayed control.

The practical results of the research. The results obtained in the dissertation and the methods used in their proofs can be directly applied to solving control problems for complex processes expressed through differential equations. Many of the results are analyzed using numerical examples. This approach significantly aids in applying the problems addressed in the research to the mathematical modeling of real-world processes.

The reliability of the results of the study is ensured by fundamental theorems, conclusions, and characteristics from mathematical analysis, differential game theory, differential equations, functional analysis and optimal control theory.

The scientific and practical significance of the research results.

The scientific significance of this research is highlighted by its focus on advancing differential games with exponential and Langenhop type constraints, in both simple and non-stationary forms. These contributions are valuable for addressing complex problems in various fields where conflicting control processes are present.

The practical significance of this research is evident in the utilization of mathematical models derived from differential games with exponential and Langenhop-type constraints to diverse areas, including military conflicts, capital accumulation, industrial organization, marketing, and environmental issues.

Implementation of the research result. Based on the results obtained from differential games with exponential and Langenhop-type constraints:

the conditions that guarantee the solvability of pursuit-evasion problems in simple and linear differential games with exponential integral constraint on control functions were used in the foreign grant project named "Problems of phase transitions and analytical phenomena. Mathematical properties of their fast transition equations and asymptotics" with number 374874-2022 (the reference with number 1170 of Osh State University dated September 11, 2024, Kyrgyz Republic). Application of the obtained results was possible to show complete solutions of problems of phase transitions and analytical phenomena described by simple and linear differential equations.

the sufficient conditions ensuring the solvability of pursuit-evasion problems in simple and non-stationary differential games with non-stationary constraints of the Langenhop type, based on given parameters were used in control problems of conflict processes characterized by dynamic equations with phase constraints on controls in the fundamental project named "Formulation of novel techniques of controlling conflict circumstances described by differential equations, and their numerical executions" of number OT-F4-33 (the reference of number 04/11-8818 of the National University of Uzbekistan dated October 15, 2024). Application of the obtained results was possible to show complete solutions of problems with conflicting objectives with Langenhop-type constraints on controls governed by non-stationary dynamic equations .

Approbation of the research results. The main findings of this research were presented at 17 scientific conferences, including 9 international and 8 national events.

Publications of the research results. Related to this dissertation topic, 7 scientific articles and 17 theses were published. The list of scientific publications features 7 articles approved by the Higher Attestation Commission of the Republic of Uzbekistan for Doctor of Philosophy theses. 2 scientific articles were published in international journals, 5 scientific articles in national journals, and 17 theses were presented at scientific conferences.

The structure and volume of the dissertation are 114 pages total, including an introduction, three chapters, a conclusion, and a list of references.

THE MAIN CONTENT OF THE DISSERTATION

In introduction the motivation of research topic and correspondence to the priority research areas of science and technology of the Republic are given, we present a review of international research on the topic of the dissertation and degree of scrutiny of the problem, formulate our goals and objectives, identify the object and subject of study, and state scientific novelty and practical results of the research. Moreover, we give the theoretical and practical importance of the obtained results, and also give information on the implementation of the research results, the published works and the structure of the dissertation.

In the first chapter, titled "**Differential Games with Exponential constraints on controls**" the pursuit-evasion problems with exponential integral constraints on controls in simple and linear differential games are studied.

Let two players move in the space $\mathbb{R}^n$. One of the players is the *Pursuer* P, and the other player is the *Evader* E. In the space $\mathbb{R}^n$, the players' dynamics are given by simple systems of differential equations:

$$P: \ \dot{x} = u, \ \ x(0) = x_0, \qquad E: \ \dot{y} = v, \ \ y(0) = y_0, \tag{1}$$

where $x(t)$ and $y(t)$ are the state vectors of players P and E respectively, at time t, x_0 and y_0 are the vectors of initial states of players P and E, respectively. We assume that $x_0 \neq y_0$. The functions $u(\cdot):[0,+\infty)\to\mathbb{R}^n$ and $v(\cdot):[0,+\infty)\to\mathbb{R}^n$ are the controls of players P and E, respectively. We assume that they are measurable functions. We input the following classes for control functions: $U_{I_{\exp}}$ and $V_{I_{\exp}}$ classes for control functions of players, which the control functions $u(\cdot)\in U_{I_{\exp}}$ and $v(\cdot)\in V_{I_{\exp}}$ satisfy the following expoential integral constraints (in short, $I_{\exp}$):

$$\int_0^t |u(s)|^2\, ds \le \rho_0^2 e^{2kt}, \quad t \ge 0, \qquad \int_0^t |v(s)|^2\, ds \le \sigma_0^2 e^{2kt}, \quad t \ge 0, \tag{2}$$

where ρ_0, σ_0, $k > 0$.

Let motions of the players P and E are described by:

$$P: \ \dot{x} - lx = u, \quad x(0) = x_0, \qquad E: \ \dot{y} - ly = v, \quad y(0) = y_0, \tag{3}$$

where $l \neq 0$; control parameters $u(\cdot), v(\cdot)$ are selected from the classes $U_{\bar{I}_{\exp}}$ and $V_{\bar{I}_{\exp}}$ of control functions that satisfy exponential integral constraints (4) (briefly, $\bar{I}_{\exp}$):

$$\int_0^t |u(s)|^2 ds \le \frac{\rho^2}{2k}\left(e^{2kt} - 1\right), \quad t \ge 0, \qquad \int_0^t |v(s)|^2 ds \le \frac{\sigma^2}{2k}\left(e^{2kt} - 1\right), \quad t \ge 0. \tag{4}$$

where $\rho, \sigma > 0$.

For the sake of conciseness, we will henceforth refer to game (1) ((3)) as the $I_{\exp}$ ($\bar{I}_{\exp}$) - game.

The goal of the Pursuer P in the $I_{\exp}\left(\bar{I}_{\exp}\right)$ - Game is to catch the Evader E (Pursuit problem), i.e., to show that the following equality:

$$\exists\, t^* > 0, \ \ x(t^*) = y(t^*) \ \left(\exists\, t^{**} > 0, \ \ x(t^{**}) = y(t^{**})\right).$$

The goal of the Evader E in the $I_{\exp}\left(\bar{I}_{\exp}\right)$ - Game is to escape always from the Pursuer P (Evasion problem), i.e., to show that the following inequality:

$$\forall\, t > 0, \ \ x(t) \neq y(t).$$

Definition 1. In the $I_{\exp}$-Game, if $\rho_0 \ge \sigma_0$, then we call the function

$$\boldsymbol{u}_{I_{\exp}}(t, v) = v - \lambda_{I_{\exp}}(t, v)\xi_0, \tag{5}$$

the Pursuer's $\Pi_{I_{\exp}}$*-strategy*, where

$\lambda_{I_{\exp}}(t, v) = \max\left\{0, 2\langle v, \xi_0\rangle + \delta_0 e^{2kt}\right\}$, $\delta_0 = \dfrac{\rho_0^2 - \sigma_0^2}{|z_0|}$, $\xi_0 = \dfrac{z_0}{|z_0|}$, and $\langle v, \xi_0\rangle$ represents the scalar product of vectors v and ξ_0 in the space $\mathbb{R}^n$.

Theorem 1. In the $I_{\exp}$-Game, if $\rho_0 \ge \sigma_0$, then $\Pi_{I_{\exp}}$-strategy (5) ensures that pursuit is completed on the time interval $\left[0, T_{I_{\exp}}\right]$, where $T_{I_{\exp}}$ is a guaranteed pursuit time and a root of the following equation

$$\Lambda_{I_{\exp}}(t) = 0, \Lambda_{I_{\exp}}(t) = 1 - \frac{\delta_0}{2k|z_0|}\left(e^{2kt} - 1\right) + \frac{2\sigma_0}{|z_0|}\sqrt{t}e^{kt}.$$

Definition 2. If $\rho \ge \sigma$ in the $\bar{I}_{\exp}$-Game, then we call the function

$$\boldsymbol{u}_{\bar{I}_{\exp}}(t, v) = v - \lambda_{\bar{I}_{\exp}}(t, v)\xi_0 \tag{6}$$

the Pursuer's $\Pi_{\bar{I}_{\exp}}$*-strategy*, where

$$\lambda_{\bar{I}_{\exp}}(t, v) = \max\left\{0, 2\langle v, \xi_0\rangle + \bar{\delta}\left(e^{2kt} - 1\right)\right\}, \ \bar{\delta} = \frac{\rho^2 - \sigma^2}{2k|z_0|}.$$

Theorem 2. If $\rho>\sigma$ and $k>l>0$ in the $\bar{I}_{\exp}$-Game, then $\Pi_{\bar{I}_{\exp}}$-strategy (6) ensures that the pursuit is completed on $\left[0,T_{\bar{I}_{\exp}}\right]$, where $T_{\bar{I}_{\exp}}$ is a guaranteed pursuit time and the solution of the following equation $F_P(t)-G_P(t)=|z_0|$, (7)

$$F_P(t)=\bar{\delta}\left(\frac{1}{2k-l}\left(e^{(2k-l)t}-1\right)-\frac{1}{l}\left(1-e^{-lt}\right)\right),\quad G_P(t)=\sigma\sqrt{\frac{1}{kl}\left(1-e^{-2lt}\right)\left(e^{2kt}-1\right)}.$$

Definition 3. If $\rho\le\sigma$ in the $\bar{I}_{\exp}$-game, then we call function (8) the strategy of the Evader:

$$\boldsymbol{v}_{\bar{I}_{\exp}}(t,u(t-\varsigma))=\begin{cases}0, & \text{if } 0\le t<\varsigma,\\ -\sqrt{|u(t-\varsigma)|^2+\left(\sigma^2-\rho^2\right)e^{2k(t-\varsigma)}}\,\xi_0, & \text{if } t\ge\varsigma,\end{cases}\tag{8}$$

where $\varsigma>0$ is a given positive number (delay information).

Theorem 3. If $\rho\le\sigma$, $-k<l<0$, and $0<\varsigma\le\frac{1}{2k}ln\left(\frac{2k|z_0|}{\rho}+1\right)$ in the $\bar{I}_{\exp}$-game, then strategy (8) is winning for the Evader for all $t\in[0,+\infty)$, and the following estimate for the distance between the players holds

$$|z(t)|>\begin{cases}0, & \text{if } 0\le t<\varsigma,\\ e^{lt}\left[\sqrt{F_E(t-\varsigma)+G_E(t-\varsigma)}-\sqrt{F_E(t-\varsigma)}\right], & \text{if } t\ge\varsigma,\end{cases}$$

where $F_E(t)=\frac{\rho^2}{4kl}\left(e^{2kt}-1\right)\left(1-e^{-2lt}\right)$, $G_E(t)=\frac{\left(\sigma^2-\rho^2\right)}{(k-l)^2}\left(e^{(k-l)t}-1\right)^2$.

In the second chapter, titled "**Differential Games with Langenhop-Type Constraints on controls**", Langenhop-type constraints are proposed for the controls of players whose motions are described by equations (1). The relationship among classes of control functions that satisfy La, G va I-constraints is identified, and pursuit-evasion problems with Langenhop-type constraints on controls are studied. Introduce three classes of admissible controls for Pursuer P:

- Langenhop type constraint (briefly, La-constraint) (9):

$$\begin{aligned}|u(t)|^2&\le\rho^2-2k_1\int_0^t|u(s)|^2\,ds,\quad t\ge0,\\ |v(t)|^2&\le\sigma^2-2k_1\int_0^t|v(s)|^2\,ds,\quad t\ge0,\end{aligned}\tag{9}$$

and we denote these classes by U_{La} and V_{La}, where $k_1>0$;

- Geometric constraint (briefly, G-constraint) of form (10):

$$|u(t)|\le\rho e^{-k_1t},\quad t\ge0,\quad |v(t)|\le\sigma e^{-k_1t},\quad t\ge0,\tag{10}$$

and we denote these classes by U_G and V_G;

- Integral constraint (briefly, I-constraint) of form (11)

$$\int_0^t |u(s)|^2\, ds \le \frac{\rho^2}{2k_1}\left(1-e^{-2k_1 t}\right),\ t\ge 0,\ \int_0^t |v(s)|^2\, ds \le \frac{\sigma^2}{2k_1}\left(1-e^{-2k_1 t}\right),\quad t\ge 0 \qquad (11)$$

and we denote these classes by U_I and V_I.

Assume that inertial motions of the objects P and E are given by the systems of differential equations:

$$P:\ \ddot{x}=u,\ \ \dot{x}(0)=x_1,\ \ x(0)=x_0,\quad E:\ \ddot{y}=v,\ \ \dot{y}(0)=y_1,\ \ y(0)=y_0, \qquad (12)$$

where $x, y, x_1, y_1, x_0, y_0, u, v \in \mathbb{R}^n$, $n\ge 2$; $x_0 \ne y_0, x_1 \ne y_1$.

$$P:\ \dot{x}=Ax+u,\quad x(0)=x_0,\qquad E:\ \dot{y}=Ay+v,\quad y(0)=y_0, \qquad (13)$$

where $x,y,u,v\in\mathbb{R}^n$, $A\ne 0$ - $n\times n$ known matrix.

Let $A=k_2E$ where $k_2\ne 0$ va $k_2>k_1$, then

$$P:\ \dot{x}+k_2x=u,\quad x(0)=x_0,\qquad E:\ \dot{y}+k_2y=v,\quad y(0)=y_0. \qquad (14)$$

We refer to game (1) with La-constraint hereafter as La-Game, game (12) with La-constraint hereafter as the La-Game with inertial players, game (13) with La-constraint hereafter as La-Linear Game, game (14) La-constraint hereafter La- Game of Evasion for one linear case, for brevity.

Lemma 1. $U_G\subset U_{La}\subset U_I$, $V_G\subset V_{La}\subset V_I$.

Definition 4. If $\rho\ge\sigma$ in the La-Game, then we call the function

$$\boldsymbol{u}_{La}(t,v)=v-\lambda_{La}(t,v)\xi_0 \qquad (15)$$

the Π_{La}-*strategy* of Pursuer, where

$$\lambda_{La}(t,v)=\langle v,\xi_0\rangle+\sqrt{\langle v,\xi_0\rangle^2+\delta e^{-2k_1t}},\ \delta=\rho^2-\sigma^2.$$

Assumption 1. Let $\rho>\sigma$ and there exist a positive root of the equation

$$\sqrt{\Phi_P(t)+\Psi_P(t)}-\sqrt{\Phi_P(t)}=|z_0|, \qquad (16)$$

with respect to t, where $\Phi_P(t)=\dfrac{t\sigma^2}{2k_1}\left(1-e^{-2k_1t}\right)$, $\Psi_P(t)=\dfrac{\delta}{k_1^2}\left(1-e^{-k_1t}\right)^2$.

We call the smallest positive root of (16) a *guaranteed pursuit time,* and denote it by T_{La}.

Theorem 4. If *Assumption 1* holds holds in the La-Game, then Π_{La}-strategy (15) guarantees the completion of pursuit on $[0,T_{La}]$.

We introduce the main definitions and theorems to solve pursuit-evasion problems in La-games with inertial players.

Assumption 2. Let $\rho>\sigma$ and the equation $\Lambda^*_{La}(t)=0$ (17) have at least one positive solution with respect to t.

$$\Lambda^*_{La}(t)=1+mt-\frac{1}{|z_0|}\left[\sqrt{\Phi^*_P(t)+\Psi^*_P(t)}-\sqrt{\Phi^*_P(t)}\right],\quad t\ge 0,$$

$$\Phi^*_P(t)=t^3\sigma^2\left(1-e^{-2k_1t}\right)/6k_1,\quad \Psi^*_P(t)=\delta\left(e^{-k_1t}+k_1t-1\right)^2/k_1^4.$$

We say that the smallest positive solution of equation (17) is a *guaranteed pursuit time*, and we denote this time by T^*_{La}.

Theorem 5. If *Assumption 2* holds holds in the La-Pursuit Game with inertial players, then Π_{La}-strategy (15) assures the completion of the pursuit on $\left[0, T_{La}^*\right]$.

Definition 5. If $\rho \le \sigma$ in the La-Game with inertial players, then we call the function

$$\boldsymbol{v}_{La}^*(t, u(t-\varepsilon)) = \begin{cases} 0, & \text{if} \quad 0 \le t < \varepsilon, \\ -|u(t-\varepsilon)|\xi_0, & \text{if} \quad t \ge \varepsilon, \end{cases} \tag{18}$$

the strategy of E.

Theorem 6. If $\rho \le \sigma$, $\quad 0 < \varepsilon \le \sqrt[3]{\dfrac{6k_1 |z_0|^2}{\rho^2}}$ and $m \ge \dfrac{\rho}{2|z_0|}\sqrt{\dfrac{\varepsilon}{2k_1}}$ in the La-Game with inertial player, then strategy (18) is winning for E for all $t \in [0, +\infty)$.

Definition 6. We say that a function $\boldsymbol{u}_{La}^*(t, v)$ of the form

$$\boldsymbol{u}_{La}^*(t, v) = v - \frac{e^{At} z_0}{\left|e^{At} z_0\right|} \lambda_{La}^*(t, v) \tag{19}$$

is the Π_{La}^*-*strategy* of the Pursuer, in the case $\rho \ge \sigma$, where

$$\lambda_{La}^*(t, v) = \frac{\langle v, e^{At} z_0 \rangle}{\left|e^{At} z_0\right|} + \sqrt{\left(\frac{\langle v, e^{At} z_0 \rangle}{\left|e^{At} z_0\right|}\right)^2 + \delta e^{-2k_1 t}},$$

$\langle v, e^{At} z_0 \rangle$ is the scalar product of the vectors v and $e^{At} z_0$ in the space $\mathbb{R}^n$.

Assumption 3. Let $\rho \ge \sigma$ and there exist a positive root of the following equation $\sqrt{\tilde{\Phi}_P(t) + \tilde{\Psi}_P(t)} - \sqrt{\tilde{\Phi}_P(t)} = |z_0|$, (20) with respect to t, where

$$\tilde{\Phi}_P(t) = \frac{\sigma^2}{4\|A\| k_1}\left(1 - e^{-2\|A\| t}\right)\left(1 - e^{-2k_1 t}\right), \tilde{\Psi}_P(t) = \frac{\delta}{\left(\|A\| + k_1\right)^2}\left(1 - e^{-(\|A\| + k_1)t}\right)^2.$$

We call the smallest positive root of equation (20) a guaranteed pursuit time and we denote it by $\tilde{\mathrm{T}}_{La}$.

Theorem 7. If *Assumption 3* is true in the La- Linear Game, then the Π_{La}^*-strategy ensures the completion of the pursuit on $\left[0, \bar{\mathrm{T}}_{La}\right]$.

Similarly, La- Game for one linear case:

Assumption 4. Let $\rho > \sigma$, $k_2 > k_1$ and there exist at least one positive solution of (21)

$$\sqrt{\bar{\Phi}_P(t) + \bar{\Psi}_P(t)} - \sqrt{\bar{\Phi}_P(t)} = |z_0|, \tag{21}$$

with respect to t, where

$$\bar{\Phi}_P(t) = \frac{\sigma^2}{4k_1 k_2}\left(e^{2k_2 t} - 1\right)\left(1 - e^{-2k_1 t}\right), \bar{\Psi}_P(t) = \frac{\rho^2 - \sigma^2}{\left(k_1 - k_2\right)^2}\left(e^{(k_2 - k_1)t} - 1\right)^2.$$

We say that the smallest positive solution of (21) is *a guaranteed pursuit* *time* and we denote this time by $\widehat{T}_{La}$.

Theorem 8. In the La- Game for one linear case, *Assumption 4* is valid, then the Π_{La}-strategy ensures that the pursuit is completed on the time interval $\left[0,\widehat{T}_{La}\right]$.

Definition 7. If $\sigma \geq \rho$, then we call the following function

$$\boldsymbol{v}_{La}(t)=\begin{cases}0, & \text{if } \ 0\le t<\bar{\varepsilon},\\ -\sqrt{|u(t-\varepsilon)|^2+\left(\sigma^2-\rho^2\right)e^{-2k_1(t-\varepsilon)}}\,\xi_0, & \text{if } \ t\ge\bar{\varepsilon},\end{cases}$$

a strategy E_{La} of the Evader, where $\bar{\varepsilon}$ is a positive number.

Theorem 9. If the conditions $\rho \le \sigma$, $k_2 > k_1$, and $0<\bar{\varepsilon}\le \dfrac{1}{2k_2} ln\left(\dfrac{4k_1k_2|z_0|^2}{\rho^2}+1\right)$ are valid, then the E_{La}-strategy is winning for the Evader for all $t\in[0,+\infty)$, and the estimation :

$$|z(t)|>\begin{cases}0, & \text{if } \ 0\le t<\bar{\varepsilon},\\ e^{-k_2 t}\left[\sqrt{\bar{\Phi}_E(t)+\bar{\Psi}_E(t)}-\sqrt{\bar{\Phi}_E(t)}\right], & \text{if } \ t\ge\bar{\varepsilon},\end{cases} \tag{22}$$

is valid, where

$$\bar{\Phi}_E(t)=\frac{\rho^2}{4k_1k_2}\left(e^{2k_2t}-1\right)\left(1-e^{-2k_1t}\right),\ \bar{\Psi}_E(t)=\frac{\left(\sigma^2-\rho^2\right)}{\left(k_1-k_2\right)^2}\left(e^{\left(k_2-k_1\right)t}-1\right)^2.$$

In third chapter, titled "**Differential Games with non-statsionary constraints of Langenhop type on controls**", two types of non-stationary Langenhop-type constraints (in short LT and LT^*-constraints) are proposed for the players' controls. The chapter is dedicated to identifying the relationships between the classes of functions that satisfy $LT\left(LT^*\right)$, $\bar{G}\left(G^*\right)$ va $\bar{I}\left(I^*\right)$-constraints, as well as studying pursuit-evasion problems where the controls are subject to $LT\left(LT^*\right)$-constraints.

In the space $\mathbb{R}^n$, we consider the following non-statsionary differential games. The motions of both players P and E are of the following form:

$$P:\ \dot{x}=a(t)u,\quad x(0)=x_0,\qquad E:\ \dot{y}=b(t)u,\quad y(0)=y_0, \tag{23}$$

($x_0\ne y_0$), where coefficients $a(t)$ and $b(t)$ are scalar, measurable, and positive functions $t\ge 0$. Let us introduce the following classes for the players' controls:

• The Langenhop type constraint (the LT-constraint for short) for P and E:

$$\begin{aligned}|u(t)|^2&\le\rho^2-2k_1\int_0^t k(s)|u(s)|^2ds,\qquad t\ge0,\\ |v(t)|^2&\le\sigma^2-2k_1\int_0^t k(s)|v(s)|^2ds,\qquad t\ge0,\end{aligned} \tag{24}$$

where, $\rho,\sigma>0$, $k(t)$ is a scalar, measurable, positive, monotonically increasing, and Lebesgue integrable function on $[0,t]$: $K(t)=\int_0^t k(s)ds$.

Let U_{LT}, (V_{LT}) be the class of all admissible controls $u(\cdot)(v(\cdot))$ satisfying (24) respectively;

•The Geometric constraint (the $\overline{G}$ - constraint for short) for P and E:

$$|u(t)|\le \rho e^{-k_1K(t)},\quad t\ge 0,\quad |v(t)|\le \sigma e^{-k_1K(t)},\quad t\ge 0, \tag{25}$$

Let $U_{\overline{G}}$ $(V_{\overline{G}})$ be the class of all admissible controls $u(\cdot)(v(\cdot))$ satisfying (25) repectivly;

•The Integral constraint (the $\overline{I}$ - constraint for short) for P and E:

$$\begin{aligned}&\int_0^t k(s)|u(s)|^2 ds \le \frac{\rho^2}{2k_1}\left(1-e^{-2k_1K(t)}\right),\quad t\ge 0,\\ &\int_0^t k(s)|v(s)|^2 ds \le \frac{\sigma^2}{2k_1}\left(1-e^{-2K(t)}\right),\quad t\ge 0,\end{aligned} \tag{26}$$

Let $U_{\overline{I}}$ $(V_{\overline{I}})$ be the class of all admissible controls $u(\cdot)(v(\cdot))$ satisfying (26) respectivly.

We see the following classes for P and E, in the case $a(t)=1$ va $b(t)=1$:

•Let U_{LT^*} (V_{LT^*}) be the class of all admissible controls $u(\cdot)$ $(v(\cdot))$ that satisfy Langenhop type constraints (briefly, LT^*-constraint) (27):

$$\begin{aligned}&|u(t)|\le \alpha(t)\left(\rho-k_1\int_0^t|u(s)|ds\right),\quad t\ge 0,\\ &|v(t)|\le \alpha(t)\left(\sigma-k_1\int_0^t|v(s)|ds\right),\quad t\ge 0,\end{aligned} \tag{27}$$

$\alpha(t)>0$ is a scalar, measurable, monotonically increasing, and Lebesgue integrable function on $[0,t]$: $A(t)=\int_0^t \alpha(s)ds$.

•Let U_{G^*} (V_{G^*}) be the class of all admissible controls $u(\cdot)$ $(v(\cdot))$ that satisfy Geometric constraints (briefly, G^*-constraints) (28):

$$\begin{aligned}&|u(t)|\le \rho\alpha(t)e^{-k_1A(t)},\qquad t\ge 0,\\ &|v(t)|\le \sigma\alpha(t)e^{-k_1A(t)},\qquad t\ge 0,\end{aligned} \tag{28}$$

• Let U_{I^*} (V_{I^*}) be the class of all admissible controls $u(\cdot)$ $(v(\cdot))$ that satisfy Integral constraints (briefly, I^* constraints) (29):

$$\int_0^t|u(s)|ds\le \frac{\rho}{k_1}\left(1-e^{-k_1A(t)}\right),\ t\ge 0,\quad \int_0^t|v(s)|^2 ds\le \frac{\sigma}{k_1}\left(1-e^{-k_1A(t)}\right),\ t\ge 0. \tag{29}$$

For brevity, we call the game described by (23) with LT-constraints the LT-

game, and the game with LT^*-constraints with $a(t)=1$ va $b(t)=1$ in (23) the LT^*-game.

Lemma 2. $U_{\bar{G}} \subset U_{LT} \subset U_{\bar{I}}$, $V_{\bar{G}} \subset V_{LT} \subset V_{\bar{I}}$.

Lemma 3. $U_{G^*} \subset U_{LT^*} \subset U_{I^*}$, $V_{G^*} \subset V_{LT^*} \subset V_{I^*}$.

Definition 8. If the conditions $\rho \ge \sigma$, $b(t) \le k(t) \le a(t)$, for almost every $t \ge 0$ hold, then we call the function

$$\boldsymbol{u}_{LT}(t,v) = \frac{b(t)}{a(t)} v - \lambda_{LT}(t, v(t))\xi_0 \tag{30}$$

the Pursuer's Π_{LT}-strategy, where

$$\lambda_{LT}(t,v) = \frac{b(t)}{a(t)}\langle v, \xi_0\rangle + \sqrt{\left(\frac{b(t)}{a(t)}\langle v, \xi_0\rangle\right)^2 + \delta e^{-2k_1 K(t)}}.$$

Assumption 5. Let $\rho > \sigma$ and $\Lambda_{LT}(t) = 0$ have a positive root, where

$$\Lambda_{LT}(t) = 1 - \frac{1}{|z_0|}[\sqrt{\Upsilon_P(t) + \Omega_P(t)} - \sqrt{\Omega_P(t)}],$$

$$\Upsilon_P(t) = \frac{\sigma^2 K(t)}{2k_1}\left(1 - e^{-2k_1 K(t)}\right),\ \ \Omega_P(t) = \frac{\delta}{k_1^2}\left(1 - e^{-k_1 K(t)}\right)^2,\ \ \delta = \sigma^2 - \rho^2.$$

We call the smallest root of $\Lambda_{LT}(t) = 0$ a guaranteed pursuit time and we denote it by T_{LT}.

Theorem 10. In the LT-game, if *Assumption 5* and $b(t) \le k(t) \le a(t)$, for almost every $t \ge 0$ hold, then the completion of pursuit is guaranteed on $[0, T_{LT}]$, by Π_{LT} -strategy (30).

Definition 9. If $\rho \ge \sigma$ and $a(t) = b(t) \ge k(t)$ for almost every $t \ge 0$, we say that the following function

$$\boldsymbol{u}^*_{LT}(t,v) = v - \lambda^*_{LT}(t, v(t))\xi_0 \tag{31}$$

is the Pursuer's Π^*_{LT}-strategy, where $\lambda^*_{LT}(t,v) = \langle v, \xi_0\rangle + \sqrt{\langle v, \xi_0\rangle^2 + \delta e^{-2k_1 K(t)}}$.

Theorem 11. If *Assumption 5* holds holds and $a(t) = b(t) \ge k(t)$ in the LT game, then the completion of pursuit is guaranteed on $[0, T_{LT}]$, by Π^*_{LT} -strategy (31).

We use the following conditions to solve LT-Evasion problem

$$\begin{cases} a(t) \le k(t) \le b(t), \text{ for almost every } t \ge 0, \\ a(t) = b(t) \le k(t), \text{ for almost every } t \ge 0. \end{cases} \tag{32}$$

Definition 10. We say that a function

$$\boldsymbol{v}_{LT}(t, u(t-\varsigma)) = \begin{cases} 0, \text{ if } 0 \le t < \bar{\varsigma}, \\ -\sqrt{|u(t-\varsigma)|^2 + \left(\sigma^2 - \rho^2\right)e^{-2k_1 K(t-\varsigma)}}\,\xi_0, \text{ if } t \ge \bar{\varsigma} \end{cases} \tag{33}$$

is the E_{LT}-strategy of the Evader, in the case $\rho \le \sigma$, where $\bar{\varsigma}$ is a positive number.

Theorem 12. If $\sigma \geq \rho$, $0 < \varsigma \leq \dfrac{2k_1|z_0|^2}{\rho^2}$, $k(t) \leq 1$ and (32) are satisfied in the LT-Game, then E_{LT} -strategy (33) guarantees that evasion is possible for $[0, +\infty)$, and for the distance between the players the following estimation holds

$$|z(t)| > \begin{cases} 0, & \text{if} \quad 0 \leq t < \varsigma, \\ \sqrt{\Upsilon_E(t-\varsigma) + \Omega_E(t-\varsigma)} - \sqrt{\Omega_E(t-\varsigma)}, & \text{if} \quad t \geq \varsigma. \end{cases} \tag{34}$$

$$\Upsilon_E(t) = \rho^2 K(t)\left(1 - e^{-2k_1 K(t)}\right) / 2k_1, \; \Omega_E(t) = \left(\sigma^2 - \rho^2\right)\left(1 - e^{-k_1 K(t)}\right)^2 / k_1^2.$$

Definition 11. We call the function $\boldsymbol{u}_{LT^*}(t, v) = v - \lambda_{LT^*}(t, v)\xi_0$ (35) the Π_{LT^*}-strategy of Pursuer, if and only if $\rho \geq \sigma$ in the LT^*-Game, where

$$\lambda_{LT^*}(t, v) = \langle v, \xi_0 \rangle + \sqrt{\langle v, \xi_0 \rangle^2 + 2\delta^* \alpha(t) e^{-k_1 A(t)} |v| + \left(\delta^* \alpha(t) e^{-k_1 A(t)}\right)^2}, \; \delta^* = \rho - \sigma.$$

Theorem 13. *If* $\delta^* > k_1|z_0|$ in the LT^*-Game, then Π_{LT^*}-strategy (35) ensures the completion of the pursuit on $\left[0, T_{LT^*}\right]$, where T_{LT^*} is a guaranteed pursuit time, and the smallest positive root of equation (36),

$$\Lambda_{LT^*}(t) = 0, \tag{36}$$

$$\Lambda_{LT^*}(t) = 1 - \frac{\delta^*}{k_1|z_0|}\left(1 - e^{-k_1 A(t)}\right).$$

Definition 12. We call a function $\boldsymbol{v}_{LT^*}(t) = -\sigma\alpha(t)e^{-k_1 A(t)}\xi_0$ the $\mathbf{E}_{LT^*}$-*strategy* of the Evader.

Theorem 14. If $\sigma \geq \rho$ in the LT^*-Game, then the $\mathbf{E}_{LT^*}$-strategy is winning for the Evader for all $t \in [0, +\infty)$.

CONCLUSION

In the dissertation, the following studies were conducted on pursuit-evasion problems with players whose control functions satisfy exponential and Langenhop-type constraints:

The main results achieved in the research are as follows:

1. In the simple differential game with exponential integral constraints, the П-strategy was constructed for the pursuer, and the delayed control was chosen for the evader, resulting in sufficient conditions for the pursuit-evasion problem.
2. For simple and linear differential games with Langenhop-type constraints, sufficient conditions for the pursuit-evasion problem were found using the pursuer's П-strategy and the evader's delayed control, along with the lower bound for the distance between two players.
3. In the differential game with inertial players under Langenhop-type constraints, the П-strategy was constructed for the pursuer, and the delayed control was chosen for the evader, leading to sufficient conditions for solving problems of pursuit-evasion.
4. In simple and non-stationary differential games with nonstatsionary constraints of Langenhop-type, sufficient conditions for the pursuit-evasion problem were identified through the pursuer's П-strategy and the evader's delayed control, along with the lower bound for the distance between two players.

НАУЧНЫЙ СОВЕТ DSc.02/30.12.2019.FM.86.01 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО

ИНСТИТУТ МАТЕМАТИКИ

УРАЛОВА САБОХАТ ИСМОИЛЖОН КИЗИ

ДИФФЕРЕНЦИАЛЬНЫЕ ИГРЫ С ЭКСПОНЕНЦИАЛЬНЫМИ И ТИПА ЛАНГЕНХОПА ОГРАНИЧЕНИЯМИ НА УПРАВЛЕНИЯ

01.01.02 - Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования является решение задачи преследования-убегания с экспоненциальными и типа Лангенхопа ограничениями на управления.

Объектом исследования являются дифференциальные игры с экспоненциальными и типа Лангенхопа ограничениями.

Научная новизна исследования заключается в следующем:

на основе параметров, заданных в обыкновенных и линейных дифференциальных играх с экспоненциальными интегральными ограничениями, была разработана П-стратегия для преследователя и определено отсроченное управление для убегающего, что приводит к достаточным условиям, гарантирующим решение задачи преследования-убегания;

в простых и линейных дифференциальных играх с ограничениями типа Лангенхопа были выявлены достаточные условия для решения задач преследования с использованием П-стратегии преследователя и задачи убегания с использованием отсроченного управления убегающего, а также найдена нижняя граница расстояния между игроками;

на основе параметров в дифференциальных играх с инерционными игроками и ограничениями типа Лангенхопа была разработана П-стратегия для преследователя, также было определено отсроченное управление для убегающего, и в результате были выявлены достаточные условия для решения задачи преследования-убегания;

на основе параметров в обыкновенных и нестационарных дифференциальных играх с нестационарными ограничениями типа Лангенхопа были выявлены достаточные условия для решения задачи преследования-убегания, что достигается через П-стратегию преследователя и отсроченное управление убегающего.

Внедрение результатов исследования. На основе результатов, полученных из дифференциальных игры с экспоненциальными и типа Лангенхопа ограничениями на управления:

результаты для обыкновенных и линейных дифференциальных игры с экспоненциальными интегральными ограничениями на функции управления были использованы в международном грантовом проекте под названием «Задачи фазовых переходов и критические явления. Математические аспекты их уравнений, быстрые переходы и асимптотика» под номером 374874-2022 (справка № 1170 от 11 сентября 2024 года Ошского государственного университета Кыргызской Республики). Применение научного результата позволило показать полные решения задач фазовых переходов и аналитических феноменов, описанных обыкновенными и линейными дифференциальными уравнениями;

результаты для в обыкновенных и нестационарных дифференциальных игры с нестационарными ограничениями типа Лангенхопа, основанные на заданных параметрах, были использованы в задачах управления конфликтными процессами, характеризующимися динамическими уравнениями с фазовыми ограничениями на управляющие действия в фундаментальном проекте под названием «Формулирование новых методов управления конфликтными обстоятельствами, представленными дифференциальными уравнениями и их численные реализации» под номером OT-F4-33 (справка № 04/11-8818 от 15 октября 2024 года Национального университета Узбекистана). Применение полученных научных результатов позволило вывести условия решения задач управления конфликтами с экспоненциальными и ограничениями типа Лангенхопа, описанными дифференциальными уравнениями.

Структура и объем диссертации. Диссертация объемом 114 страниц состоит из введения, трех глав, заключения и списка литературы.

E’LON QILINGAN ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
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