V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

NAMANGAN DAVLAT UNIVERSITETI

SOYIBBOYEV O‘LMASJON BOYQO‘ZI O‘G‘LI

BOSHQARUVLARI TURLI CHEGARALANISHGA EGA IKKINCHI TARTIBLI DIFFERENSIAL O‘YINLAR UCHUN QUVISH-QOCHISH MASALALARI

01.01.02 – Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI
bo‘yicha falsafa doktori (PhD) dissertatsiyasi
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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida ilm-fanda qo‘lga kiritilayotgan natijalarni amaliy sohalarga qo‘llash bo‘yicha tadqiqotlarning keskin o‘sib borayotganligi tufayli, ko‘plab jarayonlarning matematik modellarini differensial tenglamalar yordamida ifodalanadigan boshqariluvchi sistemalar bilan bog‘liq holda ishlab chiqish va to‘g‘ridan-to‘g‘ri amaliyotga tatbiq qilish yetakchi o‘rinlardan birini egallamoqda. Dunyo miqyosida bunday masalalarni hal qilish uchun bevosita differensial o‘yinlar nazariyasiga murojaat qilishni taqozo etadi. Shu jihatdan differensial o‘yinlar usullaridan foydalangan holda ikkinchi tartibli differensial tenglamalar uchun quriladigan matematik modellarning texnika, muhandislik, iqtisodiyot, biologiya kabi sohalarga joriy qilinishi muhim ahamiyatga ega hisoblanadi.

Jahonda differensial o‘yinlarning xilma-xilligini oshirish va ularga aloqador sohalarni kengaytirish maqsadida boshqaruvlari turli chegaralanishli yuqori tartibli differensial tenglamalar uchun o‘yin masalalarini o‘rganishga yo‘naltirilgan ilmiy-tadqiqot ishlari olib borilmoqda. Bu borada, o‘yindagi obyektlar ikkinchi tartibli differensial tenglamalar orqali ifodalanuvchi dinamikaga va boshqaruvlari konstruktiv cheklov (tezlanish) yoki manba chekloviga (yoqilg‘i, energiya) ega bo‘lganda quvish, qochish va R.Ayzeksning “Qutulish chizig‘i”ga ega sifat masalalarining yangi yechilish shartlarini topish va obyektlarning uchrashish sohasini qurish bo‘yicha ilmiy tadqiqotlarga alohida e’tibor berilmoqda.

Respublikamizda nazariy tadqiqotlarning ilmiy va amaliy joriy qilinishiga qaratilgan dolzarb yo‘nalishlariga e’tibor berish yuzasidan keng qamrovli chora-tadbirlar amalga oshirilib, muayyan natijalarga erishilmoqda. Jumladan, “differensial tenglamalar nazariyasi, algebra va uning tatbiqlari, funksional analiz va uning tatbiqlari, dinamik sistemalar nazariyasi, matematik statistika, matematik modellashtirish” kabi ustuvor yo‘nalishlar bo‘yicha muhim vazifalar belgilab berilgan[1]. Ushbu vazifalarni amalga oshirishda, jumladan, dinamik sistemalar, differensial tenglamalar, matematik boshqaruv va dinamik o‘yinlar nazariyalarini takomillashtirishda turli chegaralanishlarga ega differensial o‘yin masalalari uchun optimal yechimlarni ta’minlaydigan boshqaruv modellarini qurish muhim ahamiyat kasb etmoqda.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PQ-4947-son “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi va 2022-yil 28-yanvardagi PF-60-son “2022-2026-yillarga mo‘ljallangan Yangi O‘zbekistonning Taraqqiyot strategiyasi to‘g‘risida”gi Farmonlari, 2017-yil 17-fevraldagi PQ-2789-son “Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi, 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-

[1] O‘zbekiston Respublikasi Prezidentining 2019-yil 9-iyuldagi №PQ-4387 “Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi qarori.

tadbirlari to‘g‘risida”gi qarorlar hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalarni rivojlantirishning ustivor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Differensial o‘yinlar nazariyasi 50-yillarning boshlarida rivojlana boshladi. Birinchi jiddiy tadqiqotlardan ba’zilari “differensial o‘yin” atamasini kiritgan amerikalik matematik R.Ayzeksning ishlaridir. O‘tgan asr oltmishinchi yillarining boshidan boshlab ziddiyatli dinamik masalalar tadqiq qilindi va bu tadqiqotlar bevosita rus matematiklari L.S.Pontryagin, N.N.Krasovskiy, E.F.Mishchenko, Yu.S.Osipov, L.A.Petrosyan, B.N.Pshenichniy, A.I.Subbotinlar nomlari bilan bevosita bog‘liq. O‘tgan asrning oltmishinchi yillari oxiri-yetmishinchi yillari boshida amerikalik matematik olimlar orasida S.Baron, L.D.Bercovitz, J.V.Breakwell, O.Hajekning ishlari diqqatga sazovordir. Ular tegishli strategiyalar sinfida qiymat funksiyasi uchun mavjudlik teoremalarini ko‘rib chiqishdi va R.Ayzeksning birlik sirtlarni qurish orqali differensial o‘yinlarni yechish usulini shakllantirishdi. Differensial o‘yinlar nazariyasining keyingi rivojiga A.A.Azamov, F.L.Chernousko, A.A.Chikriy, A.V.Kryajimskiy, A.A.Melikyan, M.S.Nikolskiy, N.N.Petrov, N.Yu.Satimov, N.N.Subbotina, V.I.Uxobotov, V.N. Ushakov va yana boshqa ko‘plab matematiklar katta hissa qo‘shishdi.

L.S.Pontryagin differensial o‘yinni ikki nuqtai nazardan o‘rganish g‘oyasini ilgari surdi va takomillashtirdi. A.A.Azamov, A.A.Chikriy, A.N.Dar’in, N.L. Grigorenko, G.I.Ibragimov, D.V.Kornev, N.Yu.Lukoyanov, N.A.Mamadaliyev, M.Sh.Mamatov, Yu.N.Onopchuk, A.G.Pashkov, S.V.Pashko, N.Nikandr.Petrov, B.N.Pshenichniy, O.Sh.Qo‘chqarov, I.S.Rappoport, B.B.Rixsiyev, N.Yu.Satimov, B.T.Samatov, J.Shiyuan, S.D.Terexov, M.Tuxtasinov, L.P.Yugay, Q.Zhihua va bosh-qalar geometrik va integral chegaralanishlar uchun ko‘p ishtirokchilarga ega diskret va differensial o‘yinlarida quvish masalalarini tadqiq qilishdi va bunday masa-lalarni yechish usullarini kengaytirishdi.

L.A.Petrosyan o‘zi ishlab chiqqan parallel quvish strategiyasi (Π-strategiya) orqali fazaviy chegaralanishlar bilan guruhli quvish muammosi va R.Ayzeksning “Qutulish chizig‘i” muammosiga o‘zining yechimlarini ko‘rsatdi. A.A.Azamov birinchi bo‘lib ko‘p qiymatli akslantirishning tayanch funksiyasini qo‘llash orqali ko‘p quvlovchi va bitta qochuvchili “Qutulish chizig‘i” o‘yinining analitik yechimini taklif qildi. B.T.Samatov integral, chiziqli, nostatsionar, nochiziq, Gronuoll, Langenhop chegaralanishlar va ularning aralash ko‘rinishlariga ega sodda harakatli differensial o‘yinlar uchun Π-strategiyaning alohida-alohida analoglarini qurdi. G.I.Ibragimov Gilbert fazosidagi differensial o‘yinlar bo‘yicha tadqiqotlarni boshlab berdi va ko‘p quvlovchi va bitta qochuvchining cheksiz differensial tenglamalari sistemasi bilan tasvirlangan ham sodda, ham inersion harakatli o‘yinlar uchun quvish va qochish masalalarini hal qildi. N.A.Mamadaliyev o‘yinchilar boshqaruvlariga geometrik, integral va turli cheklovlar kiritilgan hollar

uchun kechikish argumentli differensial tenglama bilan tasvirlangan sistemaga nisbatan differensial o‘yinlar nazariyasining quvish va qochish masalalari bo‘yicha tadqiqotlar olib bordi va muhim natijalarga erishdi. Sodda quvish masalasining umumlashmasi L.S.Pontryaginning boshqaruv misoli hisoblanadi. A.S.Bannikov, A.I.Blagodatskix, N.N.Petrov, N.A.Solov’eva, D.A.Vaginlarning bir qator tadqiqot ishlarida Pontryagin misolida quvlovchilar guruhi va bitta qochuvchi holida quvish-qochish masalalari atroflicha o‘rganilgan.

Dissertatsiya mavzusining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti Namangan davlat universitetining ilmiy-tadqiqot ishlari rejasiga muvofiq “Differensial tenglamalar va uning turdosh matematik sohalarining dolzarb muammolari” dasturi doirasida bajarilgan.

Tadqiqotning maqsadi obyektlarning boshqaruv parametrlari geometrik cheklovlarga bo‘ysinuvchi o‘lchovli funksiyalar oilasidan tanlab olingan holda ikkinchi tartibli differensial o‘yinlar uchun quvish, qochish va “Qutulish chizig‘i” masalalarini hal qilishdan iborat.

Tadqiqotning vazifalari quyidagilardan iborat:

obyektlarning bir xil boshlang‘ich tezliklari holidagi inersion harakatli differensial o‘yinda quvish, qochish va “Qutulish chizig‘i” masalalarining yechilish shartlarini topish va obyektlarning uchrashish sohasining analitik formulasini qurish;

obyektlarning boshlang‘ich vaziyatlari ayirmasini ularning boshlang‘ich tezliklari ayirmasiga chiziqli bog‘liqligi holida inersion harakatli differensial o‘yinda quvish masalasining yechilish shartlarini aniqlash va obyektlarning uchrashish sohasining analitik formulasini topish;

obyektlarning boshlang‘ich vaziyatlari ayirmasini ularning boshlang‘ich tezliklari ayirmasiga chiziqsiz bog‘liqligi holida inersion harakatli differensial o‘yinda quvish masalasining yechilish shartlarini aniqlash va obyektlarning uchrashish sohasining analitik formulasini qurish;

bir xil boshlabg‘ich tezliklar va ishqalanish koeffitsiyentlari bilan berilgan obyektlarning bir xil tipdagi harakat tenglamalari orqali ifodalangan Pontryagin misoli uchun quvish, qochish va “Qutulish chizig‘i” masalalarining yechilish shartlarini aniqlash va obyektlarning uchrashish sohasining analitik formulasini topish.

Tadqiqotning obyekti matematik boshqaruvning chiziqli nazariyasi elementlari, chiziqli differensial tenglamalar, boshqaruv parametrlari geometrik cheklovlarga bo‘ysinuvchi o‘lchovli funksiyalar oilasiga mansub bo‘lgan holda ikkinchi tartibli differensial o‘yinlarning quvish, qochish va “Qutulish chizig‘i” masalalaridan iborat.

Tadqiqotning predmeti boshqaruv parametrlari geometrik cheklovlarga bo‘ysinuvchi qarama-qarshi boshqaruvli ikkinchi tartibli dinamik sistemalarda quvish, qochish va “Qutulish chizig‘i” masalalari yechimlarini topishdan iborat.

Tadqiqotning usullari. Tadqiqot ishida quvish masalalari uchun hal qiluvchi funksiyalar metodi yordamida parallel quvish strategiyasi tatbiq qilingan, qochish

masalalari uchun esa qarama-qarshi yo‘nalishli boshqaruv metodidan foydalanilgan va bundan tashqari, chiziqli boshqaruv nazariyasi, differensial o‘yinlar, variatsion hisob usullari qo‘llanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

obyektlarning bir xil boshlang‘ich tezliklari holidagi inersion harakatli differensial o‘yinda quvish, qochish va “Qutulish chizig‘i” masalalarining yechilish shartlari topilgan va quvlovchining parallel quvish strategiyasi optimal ekanligi isbotlangan;

obyektlarning boshlang‘ich vaziyatlari ayirmasini ularning boshlang‘ich tezliklari ayirmasiga chiziqli bog‘liqligi holida inersion harakatli differensial o‘yinda quvish masalasining yechilishi va obyektlarning uchrashish sohasi ichma-ich joylashishga nisbatan monotonligi isbotlangan;

obyektlarning boshlang‘ich vaziyatlari ayirmasini ularning boshlang‘ich tezliklari ayirmasiga chiziqsiz bog‘liqligi holida inersion harakatli differensial o‘yinda quvish masalasining yechilish shartlari aniqlangan va obyektlarning uchrashish sohasi uchun oshkor ko‘rinishdagi formula hosil qilingan;

bir xil boshlang‘ich tezliklar va ishqalanish koeffitsiyentlari bilan berilgan obyektlarning bir xil tipdagi harakat tenglamalari orqali ifodalangan Pontryagin misoli uchun quvish, qochish va “Qutulish chizig‘i” masalalarining yechilish shartlari topilgan va obyektlarning uchrashish sohasi uchun oshkor ko‘rinishdagi formula aniqlangan.

Tadqiqotning amaliy natijasi dissertatsiyada keltirilgan natijalar va ularning isbotini yoritishda foydalanilgan metodlar ayniqsa, ikki va undan yuqori tartibli differensial tenglamalar bilan ifodalanadigan ziddiyatli jarayonlarni boshqarish masalalariga bevosita tatbiq qilinadi. Aksariyat natijalar sonli misollar orqali tahlil qilingan va geometrik tasvirlari ham ko‘rsatib berilgan. Bu esa tadqiqot ishida hal etilgan masalalarning hayotiy jarayonlarni matematik modellashtirishda qo‘llanilishiga yordam beradi.

Tadqiqot natijalarining ishonchliligi. Matematik boshqaruvning chiziqli nazariyasi, chiziqli differensial tenglamalar nazariyasi, funksional va matematik tahlil, variatsion hisob, differensial o‘yinlar nazariyasining quvish-qochish muammolariga oid teorema va lemmalar qo‘llanilgan. Tadqiqot ishida keltirilgan natijalar matematikadagi qat’iy yondashuvlar asosida isbotlanganligi bilan izohlanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati inersiyaga ega dinamik sistemalar bilan ifodalangan ziddiyatli boshqaruv masalalarining optimal yechimlarini ta’minlaydigan strategiyalarning qurilishi va qo‘llanishidadir. Keltirilgan natijalar boshqaruv va differensial o‘yinlar nazariyalaridagi klassik metodlarni takomillashtirish va ularning amaliyotdagi tadbiqlari bilan bog‘liq muammolarning yechimlarini aniqlashda foydalanishi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati ikkinchi tartibli differensial tenglamalar ko‘plab murakkab real jarayonlar, jumladan, atomlar yoki molekulalar orasidagi bog‘lanish, sun’iy yo‘ldosh vositalarining harakati, elektr zanjiri qonun-

larining matematik modeli sifatida fizika, texnika, tibbiyot, muhandislik va ekologiya sohalaridagi ziddiyatli boshqaruv masalalari uchun optimal strategiyaning aniqlanishi hamda hisoblash dasturlarini qurish mumkinligi bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Turli chegaralanishli boshqaruvlarga ega ikkinchi tartibli differensial o‘yinlar bo‘yicha olingan natijalar asosida:

obyektlarning boshqaruvlari geometrik cheklovlar ostidagi inersion harakatli differensial o‘yinda quvish, qochish va Ayzeksning “Qutulish chizig‘i” masalalari yechilish shartlaridan OT-F4-33 raqamli “Differensial tenglamalar bilan tavsiflanuvchi ziddiyatli boshqaruv uchun yangi usullarni ishlab chiqish va ularning sonli tadbiqi” mavzusidagi fundamental loyihada boshqaruvlari holat cheklovlariga ega dinamik tenglamalar bilan tasvirlangan qarama-qarshi holatlarni boshqarish masalalarni yechishda foydalanilgan (O‘zbekiston Milliy universitetining 2024-yil 6-fevraldagi 04/11-1004-sonli ma’lumotnomasi). Ilmiy natijaning qo‘llanishi geometrik cheklovlarga bo‘ysinuvchi va ikkinchi tartibli dinamik tenglamalar bilan ifodalangan qarama-qarshi holatlarni boshqarish masalalarining to‘liq yechimlarini ko‘rsatish imkonini bergan;

bir xil boshlang‘ich tezliklar va ishqalanish koeffitsiyentlari bilan berilgan obyektlarning bir xil tipdagi harakat tenglamalari orqali ifodalangan Pontryagin boshqaruv misoli uchun quvish, qochish va “Qutulish chizig‘i” masalalarining yechilish shartlaridan OT-F4-28 raqamli “Giperbolik sistemalar uchun adekvat hisoblash modellarini qurish” mavzusidagi fundamental loyihada boshqaruvlari turli cheklovlarga ega bir xil tipdagi ikkinchi tartibli xususiy hosilali tenglamalar bilan tavsiflanuvchi ziddiyatli jarayonlarni optimal boshqarish masalalarini hal qilishda foydalanilgan (O‘zbekiston Milliy universitetining 2024-yil 28-fevraldagi 04/11-1578-sonli ma’lumotnomasi). Ilmiy natijaning qo‘llanishi ikkinchi tartibli giperbolik sistemalar uchun adekvat hisoblash modellarini qurish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Dissertatsiyaning asosiy natijalari 16 ta ilmiy-amaliy anjumanlarda, jumladan 8 tasi xalqaro va 8 tasi respublika miqyosidagi anjumanlarda muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha jami 23 ta ilmiy ish chop etilgan, shulardan, 7 tasi O‘zbekiston Respublikasi Oliy attestatsiya komissiyasining doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda, jumladan 3 tasi xorijiy va 4 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning hajmi 116 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida tadqiqot mavzusining dolzarbligi va zaruriyligi asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan, dissertatsiya mavzusi bo‘yicha xorijiy ilmiy tadqiqotlarning tahlili berilgan, muammoning o‘rganilganlik darajasi yoritilgan, tadqiqotning maqsad va vazifalari, obyekti va predmeti ko‘rsatilgan, tadqiqot natijalarining ilmiy yangiligi ochib berilgan, olingan natijalarning nazariy va amaliy ahamiyati ko‘rsatilgan, tadqiqot natijalarining tatbiqi, shuningdek nashr etilgan ilmiy ishlar va dissertatsiyaning tuzilishi haqida ma’lumotlar keltirilgan.

Birinchi bob “**Bir xil boshlang‘ich tezliklarga ega inersion o‘yinchilar uchun “Qutulish chizig‘i” ga ega differensial o‘yin**” deb nomlanadi va bir xil boshlang‘ich tezliklarga ega obyektlarning boshqaruv funksiyalari geometrik chegaralanishlarga bo‘ysinuvchi inersion harakatli differensial o‘yinida quvish, qochish va “Qutulish chizig‘i” masalalari tadqiq qilinadi.

Faraz qilaylik, $\mathbb{R}^n$ fazoda *quvlovchi* deb nomlangan boshqariluvchi $\boldsymbol{P}$ obyekt *qochuvchi* deb nomlangan boshqa bir boshqariluvchi $\boldsymbol{E}$ obyektni ta’qib qilsin. x – quvlochining holat vektori, y – qochuvchining holat vektori bo‘lsin va ushbu obyektlarning harakat dinamikasi quyidagi ko‘rinishda bo‘lsin:

$$\boldsymbol{P}: \ \ddot{x} = u, x(0) = x_{10}, \dot{x}(0) = x_{11}; \ \boldsymbol{E}: \ \ddot{y} = v, y(0) = y_{10}, \dot{y}(0) = y_{11}, \qquad (1)$$

bu yerda $x, y, x_{10}, y_{10}, x_{11}, y_{11}, u, v \in \mathbb{R}^n$, $n \geq 2$; x_{10} va y_{10} (mos holda, x_{11} va y_{11}) – obyektlarning boshlang‘ich vaziyat vektorlarini (mos holda, boshlang‘ich tezlik vektorlarini) ifodalaydi. Hozir va bundan keyin, $x_{10}, y_{10}, x_{11}, y_{11}$ vektorlar $x_{10} \neq y_{10}$ va $x_{11} = y_{11}$ munosabatlarni qanoatlantiradi deb qaraymiz.

u va v vektorlarning vaqt bo‘yicha o‘zgarishi mos holda $u(\cdot): [0, +\infty) \to \mathbb{R}^n$ va $v(\cdot): [0, +\infty) \to \mathbb{R}^n$ o‘lchovli funksiyalar bo‘lishi talab etiladi va ularga quyidagi *geometrik chegaralanishlar* (qisqa qilib, $\boldsymbol{G}$*-chegaralanishlar*) belgilanadi:

$$|u(t)| \leq \alpha, \ |v(t)| \leq \beta \text{ deyarli barcha } t \geq 0, \qquad (2)$$

bu yerda α va β – berilgan musbat sonlar va shu bilan birgalikda, ular $\boldsymbol{P}$ va $\boldsymbol{E}$ obyektlar tezlanishlarining maksimal qiymatlarini bidiradi.

Aytib o‘tish joizki, hozir va bundan keyin, (2) dagi $u(t)$ va $v(t)$ boshqaruv vektorlarining normalari sifatida odatdagi Yevklid normasini nazarda tutiladi.

Hozirdan e’tiboran, $\boldsymbol{U}_\alpha$ (mos holda, $\boldsymbol{V}_\beta$) orqali (2) $\boldsymbol{G}$- chegaralanishlarning birinchisiga (mos holda, ikkinchisiga) mos keluvchi barcha $u(\cdot)$ (mos holda, $v(\cdot)$) boshqaruv funksiyalar oilasini tasvirlaymiz.

(1) orqali $(x_{10}, x_{11}, u(\cdot))$ va $(y_{10}, y_{11}, v(\cdot))$ uchliklar, bu yerda $u(\cdot) \in \boldsymbol{U}_\alpha$ va $v(\cdot) \in \boldsymbol{V}_\beta$, $\boldsymbol{P}$ va $\boldsymbol{E}$ obyektlarning quyidagi trayektoriyalarini hosil qiladi:

$$x(t) = x_{10} + x_{11}t + \int_0^t (t-s)u(s)ds, \ y(t) = y_{10} + y_{11}t + \int_0^t (t-s)v(s)ds \qquad (3)$$

Endi $\mathbb{R}^n$ fazoda “Qutulish chizig‘i” deb ataluvchi $\boldsymbol{M}$ yopiq qism soha ko‘rsatilgan deb qaraymiz. $\boldsymbol{P}$ obyektning asosiy maqsadi $\boldsymbol{E}$ obyekt bilan ustma-ust tushish, aniqrog‘i, $\boldsymbol{E}$ obyekt $\mathbb{R}^n \setminus \boldsymbol{M}$ sohani tark etishidan oldin $\tilde{t} > 0$ chekli vaqtda $x(\tilde{t}) = y(\tilde{t})$ tenglik bajarilishini amalga oshirish. $\boldsymbol{E}$ obyektning maqsadi esa $\boldsymbol{P}$ obyekt bilan to‘qnashishdan oldinroq $\boldsymbol{M}$ sohaga yetib borish yoki har qanday $t \in [0, +\infty)$ qiymatlari uchun $x(t) \neq y(t)$ munosabatning o‘zgarmasligiga erishish. Aytib o‘tish kerakki, $\boldsymbol{M}$ soha $\boldsymbol{P}$ obyektning harakatini cheklamaydi. Bundan tashqari, x_{10} va y_{10} boshlang‘ich vaziyat vektorlari $x_{10} \neq y_{10}$ va $y_{10} \notin \boldsymbol{M}$ shartlar bilan berilgan deb qaraladi.

$$z(t) = x(t) - y(t),\ z(0) = z_{10} = x_{10} - y_{10},\ \dot{z}(0) = z_{11} = x_{11} - y_{11} \tag{4}$$

ifodalashlarni yozib olaylik. U holda, (1)–(3) ga asoslanib, quyidagi boshlang‘ich qiymat masalasiga kelamiz:

$$\ddot{z} = u - v,\ z(0) = z_{10},\ \dot{z}(0) = 0. \tag{5}$$

Buning natijasida, (1)–(2) o‘yinga muqobil sifatida, (2), (5) o‘yin yoki qisqa qilib, $(\boldsymbol{U}_\alpha, \boldsymbol{V}_\beta)$ o‘yinni hosil qilamiz.

$\boldsymbol{S}_r$ – $\mathbb{R}^n$ dagi markazi koordinatalar boshidagi r radiusli sharni ifodalasin.

1-ta’rif. Faraz qilaylik, $\alpha \geq \beta$ bo‘lsin. U holda, $(\boldsymbol{U}_\alpha, \boldsymbol{V}_\beta)$ o‘yinda

$$\boldsymbol{u}(z_{10}, v) = v - \lambda(z_{10}, v)\xi_{10} \tag{6}$$

funksiyani $\boldsymbol{P}$ obyektning $\boldsymbol{\Pi}$-*strategiyasi* deb ataymiz, bu yerda

$$\lambda(z_{10}, v) = \langle v, \xi_{10} \rangle + \sqrt{\langle v, \xi_{10} \rangle^2 + \alpha^2 - |v|^2},\ \ \xi_{10} = \frac{z_{10}}{|z_{10}|},$$

va $\langle v, \xi_{10} \rangle$ – v va ξ_{10} vektorlarning $\mathbb{R}^n$ fazodagi skalyar ko‘paytmasi. Bu holda, $\lambda(z_{10}, v)$ – *hal qiluvchi funksiya* deb ataladi.

2-ta’rif. $\boldsymbol{\Pi}$-strategiya $[0, t_g]$ vaqt oralig‘ida $\boldsymbol{E}$ *obyektni tutishni kafolatlaydi* deb aytamiz, agar har qanday $v(\cdot) \in \boldsymbol{V}_\beta$ boshqaruv uchun: *i*) $z(\tilde{t}) = 0$ tenglik hosil bo‘ladigan $\tilde{t} \in [0, t_g]$ vaqt qiymati mavjud bo‘lsa; *ii*) $[0, \tilde{t}]$ vaqt oralig‘ida ushbu $\boldsymbol{u}(z_{10}, v(\cdot)) \in \boldsymbol{U}_\alpha$ tegishlilik bajarilsa. Bunday holatda, t_g qiymatni *kafolatlangan quvish* (yoki *tutish*) *vaqti* deyiladi.

1-teorema. $(\boldsymbol{U}_\alpha, \boldsymbol{V}_\beta)$ o‘yinda $\alpha > \beta$ bajarilsin. U holda, (6) $\boldsymbol{\Pi}$-strategiya $[0, t_g]$ vaqt oralig‘ida $\boldsymbol{E}$ obyektni tutishni kafolatlaydi, bu yerda $t_g = \sqrt{\dfrac{2|z_{10}|}{\alpha - \beta}}$.

3-ta’rif. $(\boldsymbol{U}_\alpha, \boldsymbol{V}_\beta)$ o‘yinda

$$\boldsymbol{v}_{\boldsymbol{E}}(t) = -\beta\xi_{10} \tag{7}$$

boshqaruv funksiyani $\boldsymbol{E}$ *obyektning strategiyasi* deb ataymiz.

4-ta'rif. $\boldsymbol{v_E}(\cdot)\in \boldsymbol{V}_\beta$ strategiya *qochishni kafolatlaydi* deyiladi, agar har qanday $u(\cdot)\in \boldsymbol{U}_\alpha$ boshqaruv uchun $\ddot{z}=u(t)-\boldsymbol{v_E}(t),\ z(0)=z_{10},\ \dot{z}(0)=0$ Koshi masalasining $z(t)$ yechimi barcha $t\in[0,+\infty)$ qiymatlarda nol bo'lmasa, ya'ni $z(t)\neq 0$ bo'lsa.

2-teorema. *i*) $\alpha>\beta$ bo'lsin. U holda, $(\boldsymbol{U}_\alpha,\boldsymbol{V}_\beta)$ o'yinda (7) strategiya $[0,\,t_g)$ vaqt oralig'ida qochishni kafolatlaydi, bu yerda t_g – kafolatlangan tutish vaqti; *ii*) $\alpha\le\beta$ bo'lsin. U holda, $(\boldsymbol{U}_\alpha,\boldsymbol{V}_\beta)$ o'yinda (7) strategiya $[0,+\infty)$ vaqt oralig'ida qochishni kafolatlaydi va obyektlar orasidagi masofa quyidagicha baholanadi:

$$|z(t)|\ge\begin{cases}|z_{10}| & \text{agar } \alpha=\beta \text{ bo'lsa,}\\ |z_{10}|+\dfrac{(\beta-\alpha)t^2}{2} & \text{agar } \alpha<\beta \text{ bo'lsa.}\end{cases}$$

Har qanday $v(\cdot)\in\boldsymbol{V}_\beta$ boshqaruv va $\boldsymbol{u}(z_{10},v(\cdot))\in\boldsymbol{U}_\alpha$ strategiya orqali $[0,\,\tilde{t}\,]$, bu yerda $0\le\tilde{t}<t_g$ va $x(\tilde{t})=y(\tilde{t})$ o'rinli, vaqt oralig'idagi $(x(t),y(t))$ uchun $\boldsymbol{W}(x(t),y(t))=\left\{w:\ |w-x(t)|\ge\dfrac{\alpha}{\beta}|w-y(t)|\right\}$ ko'p qiymatli akslantirish aniqlanadi va $t=0$ da $\boldsymbol{W}(x_{10},y_{10})=\left\{w:\ |w-x_{10}|\ge\dfrac{\alpha}{\beta}|w-y_{10}|\right\}$ to'plamga ega bo'lamiz.

3-teorema. $\boldsymbol{W}(x(t),y(t))-x_{11}t$ ko'p qiymatli akslantirish $t\in[0,\,\tilde{t}\,]$ bo'yicha ichma-ich joylashishga nisbatan monoton kamayuvchidir, ya'ni agar har qanday $t_1,\,t_2\in[0,\,\tilde{t}\,]$ uchun $t_1<t_2$ bo'lsa, $\boldsymbol{W}(x(t_2),y(t_2))-x_{11}t_2\subset\boldsymbol{W}(x(t_1),y(t_1))-x_{11}t_1$ munosabat o'rinli.

"Qutulish chizig'i" o'yinini $\boldsymbol{P}$ obyektning foydasiga hal qilish uchun ushbu qavariq va yopiq $\boldsymbol{W}_{\boldsymbol{P}}(x_{10},y_{10},x_{11},t_g)=\bigcup\limits_{t=0}^{t_g}\{\boldsymbol{W}(x_{10},y_{10})+x_{11}t\}$ to'plamni aniqlaymiz.

$\boldsymbol{W}_{\boldsymbol{P}}(x_{10},y_{10},x_{11},t_g)$ to'plamga $\boldsymbol{P}$ *obyektning quvish masalasidagi yetishish sohasi* deyiladi.

5-ta'rif. (1)–(2) "Qutulish chizig'i" o'yinida (6) $\boldsymbol{\Pi}$-strategiya $[0,\,t_g\,]$ vaqt oralig'ida $\boldsymbol{P}$ *obyekt uchun yutishni ta'minlaydi* deyiladi, agar shunday $t_*\in[0,\,t_g\,]$ vaqt mavjud bo'lsaki, quyida keltirilgan shartlar bajarilsa: *i*) $x(t_*)=y(t_*)$ bo'lsa; *ii*) har bir $t\in[0,\,t_*\,]$ uchun $y(t)\notin\boldsymbol{M}$ bo'lsa.

4-teorema. Agar: *i*) $\alpha>\beta$; *ii*) $\boldsymbol{P}$ obyektning yetishish sohasi $\boldsymbol{M}$ soha bilan kesishmasa, ya'ni $\boldsymbol{W}_{\boldsymbol{P}}(x_{10},y_{10},x_{11},t_g)\cap\boldsymbol{M}=\varnothing$ bo'lsa, u holda (6) $\boldsymbol{\Pi}$-strategiya $\boldsymbol{P}$ obyekt uchun (1)–(2) "Qutulish chizig'i" o'yinida yutishni ta'minlaydi.

6-ta'rif. (1)–(2) "Qutulish chizig'i" o'yinida $v_{\boldsymbol{M}}(\cdot)\in\boldsymbol{V}_\beta$ boshqaruv $\boldsymbol{E}$ *obyekt uchun yutishni ta'minlaydi* deyiladi, agar har qanday $u(\cdot)\in\boldsymbol{U}_\alpha$ boshqaruv uchun

quyida keltirilgan shartlardan kamida bittasi o‘rinli bo‘lsa, ya’ni: *i*) shunday t_M chekli vaqt topilsaki, barcha $t \in [0, t_M)$ qiymatlar uchun $y(t_M) \in M$ va $x(t) \neq y(t)$ munosabatlar bajarilsa; *ii*) barcha $t \in [0, +\infty)$ qiymatlar uchun $x(t) \neq y(t)$ bo‘lsa.

$W_E(x_{11}, y_{10}, w) = \bigcup_{w \in W(x_{10}, y_{10})} \left\{ w_E : \; w_E = x_{11} \sqrt{\frac{2|w - y_{10}|}{\beta}} + w \right\}$ to‘plamni qaraylik.

$W_E(x_{11}, y_{10}, w)$ to‘plamga (1)–(2) “Qutulish chizig‘i” o‘yinida E *obyektning yetishish sohasi* deyiladi.

5-teorema. Agar: *i*) $\alpha > \beta$ bo‘lsa; *ii*) E obyektning yetishish sohasi M soha bilan kesishsa, ya’ni $W_E(x_{11}, y_{10}, w) \cap M \neq \varnothing$ bo‘lsa, u holda (1)–(2) “Qutulish chizig‘i” o‘yinida E obyekt uchun yutishni ta’minlaydigan $v_M(\cdot) \in V_\beta$ boshqaruv mavjud.

6-teorema. $\alpha \leq \beta$ bo‘lsin. U holda, (1)–(2) “Qutulish chizig‘i” o‘yinida E obyekt uchun yutishni ta’minlaydigan $v_M(\cdot) \in V_\beta$ boshqaruv mavjud.

Ikkinchi bob “**Turli boshlang‘ich tezliklarga ega inersion o‘yinchilar differensial o‘yinida Π-strategiyaning tatbiqlari**” deb nomlanadi va o‘zaro teng bo‘lmagan boshlang‘ich tezliklarga ega obyektlar uchun inersion harakatli differensial o‘yinda yangicha metod asosida quvish masalasi tadqiq qilinadi.

Dastlabki bobdagi kab, $\mathbb{R}^n$ fazoda ikkita boshqariluvchi P va E obyektlarning inersion harakatli differensial o‘yinini o‘rganamiz va bunda, ulardan birinchisi (2) G-chegaralanishlarning birinchisiga bo‘ysinuvchi $u(\cdot)$ boshqaruv bilan (1) dinamikaning birinchi tenglamasiga asosan harakatlanadi, ikkinchisi esa (2) G-chegaralanishlarning ikkinchisiga bo‘ysinuvchi $v(\cdot)$ boshqaruv yordamida (1) dinamikaning ikkinchi tenglamasiga asosan harakatlanadi.

Yuqoridagi bobdan farqli ravishda, bu yerda $x_{11} \neq y_{11}$ deb qaraladi.

Ravshanki, (4) ifodalashlar tufayli (1) dinamikadan

$$\ddot{z} = u - v, \; z(0) = z_{10}, \; \dot{z}(0) = z_{11}. \tag{8}$$

boshlang‘ich qiymat masalasi hosil qilinadi. Har qanday tanlangan $u(\cdot) \in U_\alpha$ va $v(\cdot) \in V_\beta$ joiz boshqaruvlar uchun (8) masalaning yechimi quyidagicha yoziladi:

$$z(t) = z_{10} + z_{11} t + \int_0^t (t - s)\big(u(s) - v(s)\big) ds. \tag{9}$$

(9) funksiyaga muvofiq, P obyekt berilgan z_{10} va z_{11} boshlang‘ich vaziyat vektorlari uchun eng qisqa t vaqtda $z(t) = 0$ tenglikka erishishni maqsad qilgan bo‘lsin va shuning barobarida, E obyekt esa barcha $t \geq 0$ vaqt qiymatlari uchun $z(t) \neq 0$ tengsizlikning o‘zgarmasdan saqlanishiga intiladi.

Biz z_{10} va z_{11} boshlang‘ich vaziyat vektorlariga bog‘liq holda, quvish masalasining yechimini ko‘rsatamiz va quyidagi ikkita holatni ko‘rib chiqamiz:

1-hol. z_{10} va z_{11} vektorlar kollinear, aniqrog‘i, chekli k son uchun $z_{11} = k z_{10}$ o‘rinli.

2-hol. z_{10} va z_{11} vektorlar nokollinear, aniqrog‘i, ular chiziqli bog‘liqsiz.

Xuddi birinchi bobdagi kabi, $\boldsymbol{P}$ obyekt 1-holda (6) $\boldsymbol{\Pi}$-strategiyani qo‘llasin.

7-teorema. Quyida berilgan shartlardan kamida bittasi o‘rinli bo‘lsin: *i*) $\alpha > \beta$ va $k \in \mathbb{R}$; *ii*) $\alpha = \beta$ va $k < 0$. U holda (6) $\boldsymbol{\Pi}$-strategiya 1-holda $[0, T_{10}]$ vaqt oralig‘ida yutuqlidir, bu yerda

$$T_{10} = \begin{cases} \left(k|z_{10}| + \sqrt{k^2|z_{10}|^2 + 2(\alpha-\beta)|z_{10}|}\right)\Big/(\alpha-\beta) & \text{agar } \alpha > \beta,\ k \in \mathbb{R} \text{ bo‘lsa,} \\ -1/k & \text{agar } \alpha = \beta,\ k < 0 \text{ bo‘lsa.} \end{cases}$$

1-bobdagi kabi $\big(x(t), y(t)\big)$ juftlik uchun $\left[0, t_{10}^*\right]$ vaqt oralig‘idagi qiymatlar uchun $\boldsymbol{A}\big(x(t), y(t)\big) = \left\{ w_{\boldsymbol{A}} : \left|w_{\boldsymbol{A}} - x(t)\right| \le \dfrac{\alpha}{\beta}\left|w_{\boldsymbol{A}} - y(t)\right| \right\}$ ko‘p qiymatli akslantirishni aniqlay olamiz.

1-xossa. 1-holda $\boldsymbol{E}$ obyektning yetishish sohasi $\boldsymbol{A}_{\boldsymbol{E}} = \boldsymbol{A}_{10} + \bigcup\limits_{t=0}^{T_{10}} t\boldsymbol{A}_{11}$ to‘plam bo‘ladi, ya’ni, $\boldsymbol{E}$ obyektning $y_{T_{10}}(\cdot) = \{y(s) : 0 \le s \le T_{10}\}$ trayektoriyasi $\boldsymbol{A}_{\boldsymbol{E}}$ yopiq qavariq to‘plamdan chiqib ketmaydi, bu yerda $\boldsymbol{A}_{10} = \dfrac{\alpha^2 y_{10} - \beta^2 x_{10}}{\alpha^2 - \beta^2} + \dfrac{\alpha\beta|z_{10}|}{\alpha^2 - \beta^2}\boldsymbol{S}$.

Endi boshlang‘ich nol bo‘lmagan z_{10} va z_{11} vektorlar nokollinear bo‘lsin va $\alpha > \beta$ bo‘lsin, ya’ni 2-hol tadqiq qilinadi.

Dastlab, quyidagi qo‘shimcha masalani hal qilamiz.

1-masala. $\boldsymbol{E}$ obyektning har qanday $v(\cdot) \in \boldsymbol{V}_\beta$ boshqaruvi uchun

$$\dot{x}(\theta) = \dot{y}(\theta) \tag{10}$$

tezliklar tengligiga erishiladigan $\boldsymbol{\Pi}$-strategiyani qurish talab etiladi, bu yerda θ – biror chekli vaqt.

1-masalani yechish uchun, (1) ikkinchi tartibli differensial o‘yin birinchi tartibli differensial o‘yinga keltiriladi. Buning uchun $x_1 = \dot{x}$ va $y_1 = \dot{y}$ yangi o‘zgaruvchilarni kiritamiz. U holda, (1) dinamikadan mos ravishda

$$\dot{x}_1 = u,\ x_1(0) = x_{11};\ \ \dot{y}_1 = v,\ y_1(0) = y_{11}$$

tenglamalarga ega bo‘lamiz, bu yerda u (mos holda, v) x_1 (mos holda, y_1) vektorning t vaqtga bog‘liq holda o‘zgarish ko‘rsatkichi bo‘lib xizmat qiladi. Ravshanki, $x_{11} \ne y_{11}$, aks holda, yana 1-holga qaytamiz, bu yerda x_{11}, y_{11} – mos ravishda x_1, y_1 vektorlarning boshlang‘ich vaziyatlaridir.

7-ta’rif. 1-masala uchun

$$\boldsymbol{u}_1(z_{11}, v) = v - \lambda_1(z_{11}, v)\xi_{11} \tag{11}$$

funksiyaga $\boldsymbol{P}$ obyektning $\boldsymbol{\Pi}_1$-*strategiyasi* deb ataladi, bu yerda

$$\lambda_1(z_{11}, v) = \langle v, \xi_{11}\rangle + \sqrt{\langle v, \xi_{11}\rangle^2 + \alpha^2 - |v|^2},\ \xi_{11} = \frac{z_{11}}{|z_{11}|},\ z_{11} = x_{11} - y_{11}$$

va $\boldsymbol{u}_1\big(z_{11}, v(t)\big)$, $t \ge 0$ – har qanday $v(\cdot) \in \boldsymbol{V}_\beta$ uchun $\boldsymbol{u}_1(z_{11}, v)$ ning joriy qilinishi.

8-teorema. $\alpha > \beta$ bo‘lsin and $\boldsymbol{P}$ obyekt (11) Π_1-strategiyani qo‘llasin. U holda, $\boldsymbol{E}$ obyektning har qanday $v(\cdot) \in \boldsymbol{V}_\beta$ boshqaruvi uchun biror $\theta \in [\theta_{10}, \theta_{11}]$ vaqtda (10) tenglikka erishish mumkin, bu yerda $\theta_{10} = \dfrac{|z_{11}|}{\alpha + \beta}$, $\theta_{11} = \dfrac{|z_{11}|}{\alpha - \beta}$.

9-teorema. $\alpha > \beta$ bo‘lsin and $\boldsymbol{P}$ obyekt (11) Π_1-strategiyani qo‘llasin. U holda, $\boldsymbol{E}$ obyektning har qanday $v(\cdot) \in \boldsymbol{V}_\beta$ boshqaruvi uchun $|z(t)|$ obyektlar orasidagi masofa $[0, \theta]$ vaqt oralig‘ida $L_{10} \le |z(t)| \le L_{11}$ baholashni qanoatlantiradi, bu yerda

$$L_{10} = \begin{cases} |z_{10}| & \text{agar } \langle z_{10}, z_{11} \rangle > 0 \text{ bo‘lsa,} \\ \left| z_{10} - \dfrac{\langle z_{10}, z_{11} \rangle}{|z_{11}|^2} z_{11} \right| & \text{agar } \langle z_{10}, z_{11} \rangle < 0 \text{ bo‘lsa,} \end{cases}$$

$$L_{11} = \begin{cases} \left| z_{10} + \dfrac{|z_{11}|}{2(\alpha - \beta)} z_{11} \right| & \text{agar } \langle z_{10}, z_{11} \rangle > 0 \text{ bo‘lsa,} \\ \max\left\{ |z_{10}|, \left| z_{10} + \dfrac{|z_{11}|}{2(\alpha - \beta)} z_{11} \right| \right\} & \text{agar } \langle z_{10}, z_{11} \rangle < 0 \text{ bo‘lsa.} \end{cases}$$

Shunday qilib, 9-teoremada ta’kidlanganidek, $\boldsymbol{P}$ va $\boldsymbol{E}$ obyektlar $[0, \theta]$ vaqt oralig‘ida ustma-ust tushmaydi. Shunga qaramasdan, ularning tezlik vektorlari θ vaqt paytida ustma-ust tushadi. Endi esa θ vaqtdan boshlab, quyidagi differensial o‘yin bilan shug‘ullanamiz:

$$\ddot{x} = u,\ x(\theta) = x_*,\ \dot{x}(\theta) = x_{**};\ \ \ddot{y} = v,\ y(\theta) = y_*,\ \dot{y}(\theta) = y_{**}, \tag{12}$$

bu yerda x_*, y_* – obyektlarning θ vaqtdagi vaziyatlari va x_{**}, y_{**} – obyektlar tezliklarining θ vaqtdagi vaziyatlari. Aytish joizki, (12) sistemada $x_{**} = y_{**}$ tenglikka erishildi, aniqrog‘i, (8) umumiy o‘yin yana 1-holga, ya’ni $k = 0$ ga keltirildi.

8-ta’rif. $\alpha > \beta$ va $t \ge \theta$ bo‘lsin. U holda, (12) o‘yinda

$$\boldsymbol{u}_2(z_*, v) = v - \lambda_2(z_*, v)\xi_* \tag{13}$$

funksiya $\boldsymbol{P}$ obyektning Π_2-*strategiyasi* deb ataladi, bu yerda

$$\lambda_2(z_*, v) = \langle v, \xi_* \rangle + \sqrt{\langle v, \xi_* \rangle^2 + \alpha^2 - |v|^2},\ \xi_* = \frac{z_*}{|z_*|},\ z_* = x_* - y_*,$$

va $\boldsymbol{u}_2(z_*, v(t))$, $t \ge \theta$ – har qanday $v(\cdot) \in \boldsymbol{V}_\beta$ boshqaruv uchun $\boldsymbol{u}_2(z_*, v)$ ning joriy qilinishi.

10-teorema. $\alpha > \beta$ bo‘lsin hamda z_{10} va z_{11} vektorlar nokollinear bo‘lsin. U holda (8) o‘yinda $\boldsymbol{E}$ obyektning har qanday $v(\cdot) \in \boldsymbol{V}_\beta$ boshqaruvi uchun $\boldsymbol{P}$ obyekt

(11) va (13) Π_1 va Π_2-strategiyalardan ketma-ket foydalanib, $\boldsymbol{E}$ obyektni $[T_{21}, T_{22}]$ oraliqda tutadi, bu yerda $T_{21} = \frac{|z_{11}|}{\alpha+\beta} + \sqrt{\frac{2L_{10}}{\alpha+\beta}}$, $T_{22} = \frac{|z_{11}|}{\alpha-\beta} + \sqrt{\frac{2L_{11}}{\alpha-\beta}}$.

11-teorema. Agar $\alpha > \beta$ bo‘lsa, u holda, (8) differensial o‘yinning 2-holida har qanday $v(\cdot) \in \boldsymbol{V}_\beta$ boshqaruv uchun va barcha $t \in [0, T_{22}]$ vaqt qiymatlari uchun $y(t) \in y_{10} + t\boldsymbol{A}_{11} + \boldsymbol{B}_{11}$ tegishlilik bajariladi.

9-ta’rif. (8) o‘yinni 2-holida $\boldsymbol{Q}_{\boldsymbol{E}} = y_{10} + \boldsymbol{B}_{11} + \bigcup_{t=0}^{T_{22}} t\boldsymbol{A}_{11}$ to‘plam $\boldsymbol{E}$ *obyektning yetishish sohasi* deb ataladi.

Uchinchi bob “**O‘yinchilarning ishqalanishli harakatlariga ega differensial o‘yinlarda Π-strategiyaning tatbiqlari**” deb nomlanadi va bir xil boshlang‘ich tezliklarga va ishqalanish ta’sirlariga ega ikkita inersion obyektlarning bir xil tipdagi harakatlari dinamikasi orqali ifodalangan Pontryagin boshqaruv misoli uchun quvish, qochish va “Qutulish chizig‘i” masalalari tadqiq qilinadi.

$\mathbb{R}^n$ fazoda boshqariluvchi $\boldsymbol{X}$ o‘yinchi, ya’ni *quvlovchi* boshqa bir boshqariluvchi $\boldsymbol{Y}$ o‘yinchi, ya’ni *qochuvchini* ta’qib qilsin. Faraz qilaylik, $\boldsymbol{X}$ va $\boldsymbol{Y}$ o‘yinchilar holatlari mos holda x va y bilan ifodalansin va ularning harakatlari

$$\boldsymbol{X}:\ \ddot{x} + a\dot{x} = u,\ x(0) = x_{10},\ \dot{x}(0) = x_{20};\ \ \boldsymbol{Y}:\ \ddot{y} + a\dot{y} = v,\ y(0) = y_{10},\ \dot{y}(0) = y_{20} \quad (14)$$

tenglamalarga asoslangan bo‘lsin, bu yerda $x, y, x_{10}, y_{10}, x_{20}, y_{20}, u, v \in \mathbb{R}^n$, $n \geq 2$, $a > 0$; x_{10} va y_{10} vektorlar $\boldsymbol{X}$ va $\boldsymbol{Y}$ o‘yinchilarning boshlang‘ich vaziyatlari, x_{20} va y_{20} vektorlar esa ularning boshlang‘ich tezlik vektorlaridir. Bunda, $x_{10} \neq y_{10}$ va $x_{20} = y_{20}$ shart bajarilishi talab etiladi.

Xuddi birinchi bobdagi singari, o‘yinchilarning u va v boshqaruv vektorlari $u(\cdot):[0,+\infty) \to \mathbb{R}^n$ va $v(\cdot):[0,+\infty) \to \mathbb{R}^n$ sifatida qaraladi va bu vektor funksiyalar

$$|u(t)| \leq \rho,\ |v(t)| \leq \sigma \text{ deyarli barcha } t \geq 0 \quad (15)$$

$\boldsymbol{G}$-chegaralanishlarga bo‘ysinadi, bu yerda ρ va σ – $\boldsymbol{X}$ va $\boldsymbol{Y}$ o‘yinchilar tezlanishlarining maksimal qiymatlarini tasvirlaydigan musbat parametrik sonlar.

$$z(t) = x(t) - y(t),\ z(0) = z_{10} = x_{10} - y_{10},\ \dot{z}(0) = z_{20} = x_{20} - y_{20} \quad (16)$$

ifodalashlarni kiritaylik. U holda, (14), (16) asosida

$$\ddot{z} + a\dot{z} = u - v,\ z(0) = z_{10},\ \dot{z}(0) = z_{20} = 0 \quad (17)$$

boshlang‘ich qiymat masalasini aniqlaymiz.

1-bobdagi kabi, (15), (17) o‘yin yoki qisqacha, $(\boldsymbol{U}_\rho, \boldsymbol{V}_\sigma)$ o‘yinga keldik.

$p(t) = e^{-at} + at - b$ funksiyani qaraylik, bu yerda $b = 1 + \frac{a^2|z_{10}|}{\rho - \sigma}$.

1-tasdiq. Faraz qilaylik, $\rho > \sigma$ bo‘lsin. U holda, $p(t) = 0, t \geq 0$ tenglamaning yagona musbat ildizi mavjud va uni T orqali ifodalaymiz.

10-ta’rif. $(\boldsymbol{U}_\rho, \boldsymbol{V}_\sigma)$ o‘yinda

$$\boldsymbol{u}_{\boldsymbol{\Pi}}(v, z_{10}) = v - \gamma(v, z_{10})\xi_{10} \tag{18}$$

vektor funksiyani $\boldsymbol{\Pi}$-*strategiya* deb ataymiz, bu yerda

$$\gamma(v, z_{10}) = \langle v, \xi_{10} \rangle + \sqrt{\langle v, \xi_{10} \rangle^2 + \rho^2 - |v|^2}, \ \xi_{10} = \frac{z_{10}}{|z_{10}|}$$

va $\langle v, \xi_{10} \rangle$ – v va ξ_{10} vektorlarning $\mathbb{R}^n$ dagi skalyar ko‘paytmasi.

12-teorema. Agar $\rho > \sigma$ bo‘lsa, u holda, (18) $\boldsymbol{\Pi}$-strategiya $t \in \left[0, T\right]$ vaqt oralig‘ida $\boldsymbol{X}$ o‘yinchi uchun yutuqlidir.

11-ta’rif. $(\boldsymbol{U}_\rho, \boldsymbol{V}_\sigma)$ o‘yinda

$$\boldsymbol{v}_{\boldsymbol{Y}}(t) = -\sigma\xi_{10} \tag{19}$$

boshqaruv funksiyani $\boldsymbol{Y}$ *o‘yinchining strategiyasi* deb ataladi.

13-teorema. *i*) $\rho > \sigma$ bo‘lsin. U holda, (19) strategiya $\left[0, T\right)$ vaqt oralig‘ida $\boldsymbol{Y}$ o‘yinchi uchun yutuqlidir; *ii*) $\rho \le \sigma$ bo‘lsin. U holda, (19) strategiya $[0, +\infty)$ vaqt oralig‘ida $\boldsymbol{Y}$ o‘yinchi uchun yutuqlidir hamda $\boldsymbol{X}$ va $\boldsymbol{Y}$ o‘yinchilar orasidagi masofa quyidagi ko‘rinishda baholanadi:

$$|z(t)| \ge \begin{cases} |z_{10}| & \text{agar } \rho = \sigma \text{ bo‘lsa,} \\ |z_{10}| + \dfrac{\sigma - \rho}{a^2}\left(e^{-at} + at - 1\right) & \text{agar } \rho < \sigma \text{ bo‘lsa.} \end{cases}$$

Har qanday $v(\cdot) \in \boldsymbol{V}_\sigma$ boshqaruv va $\boldsymbol{u}_{\boldsymbol{\Pi}}(v(\cdot), z_{10}) \in \boldsymbol{U}_\rho$ strategiya orqali $[0, T_*]$, bu yerda $0 < T_* \le T$ va $x(T_*) = y(T_*)$ o‘rinli, vaqt oralig‘idagi $(x(t), y(t))$ juftlik uchun $\boldsymbol{\Delta}(x(t), y(t)) = \left\{\delta : \ |\delta - x(t)| \ge \dfrac{\rho}{\sigma}|\delta - y(t)|\right\}$ ko‘p qiymatli akslantirishni va $t = 0$ dagi $\boldsymbol{\Delta}(x_{10}, y_{10}) = \left\{\delta : \ |\delta - x_{10}| \ge \dfrac{\rho}{\sigma}|\delta - y_{10}|\right\}$ to‘plamni aniqlay olamiz.

1-lemma. $\boldsymbol{\Psi}(x(t), y(t), x_{20}) = \boldsymbol{\Delta}(x(t), y(t)) - \dfrac{x_{20}}{a}\left(1 - e^{-at}\right)$ bo‘lsin. U holda, har qanday $t_1, t_2 \in [0, T_*]$ uchun $t_1 < t_2$ da, $\boldsymbol{\Psi}(x(t_2), y(t_2), x_{20}) \subset \boldsymbol{\Psi}(x(t_1), y(t_1), x_{20})$ bajariladi.

12-ta’rif. Ushbu $\boldsymbol{\Delta}_{\boldsymbol{X}}(x_{10}, x_{20}, y_{10}, T) = \bigcup\limits_{t=0}^{T}\left\{\boldsymbol{\Delta}(x_{10}, y_{10}) + \dfrac{x_{20}}{a}\left(1 - e^{-at}\right)\right\}$ to‘plamni $(\boldsymbol{U}_\rho, \boldsymbol{V}_\sigma)$ o‘yinda $\boldsymbol{X}$ *o‘yinchining yetishish sohasi* deyiladi.

$\mathbb{R}^n$ fazoda “Qutulish chizig‘i” deb ataluvchi $\boldsymbol{L}$ yopiq qism soha ko‘rsatilgan deb qaraymiz. $\boldsymbol{Y}$ o‘yinchining boshlang‘ich vaziyati $y_{10} \notin \boldsymbol{L}$ shart ostida berilgan deb qaraladi.

Quyidagilarni olaylik: $\boldsymbol{T}_{\boldsymbol{L}}(\delta, y_{10}) = \bigcup\limits_{\delta \in \boldsymbol{\Delta}(x_{10}, y_{10})}\left\{t_{\boldsymbol{L}} : \ e^{-at_{\boldsymbol{L}}} + at_{\boldsymbol{L}} = 1 + \dfrac{a^2|\delta - y_{10}|}{\sigma}\right\}$,

$$\Delta_{\boldsymbol{Y}}\left(\delta, x_{20}, t_{\boldsymbol{L}}\right) = \bigcup_{\delta \in \boldsymbol{\Delta}(x_{10}, y_{10})} \left\{ \delta_{\boldsymbol{Y}} :\ \delta_{\boldsymbol{Y}} = \delta + \frac{x_{20}}{a}\left(1 - e^{-at_{\boldsymbol{L}}}\right) \right\}. \qquad (20)$$

(20) to‘plam (14)–(15) “Qutulish chizig‘i” o‘yinida $\boldsymbol{Y}$ *o‘yinchining yetishish sohasi* deb yuritiladi.

14-teorema. Agar $\rho > \sigma$ va $\Delta_{\boldsymbol{Y}}\left(\delta, x_{20}, t_{\boldsymbol{L}}\right) \cap \boldsymbol{L} \neq \varnothing$ bo‘lsa, u holda (14)–(15) “Qutulish chizig‘i” o‘yinida $\boldsymbol{Y}$ o‘yinchi uchun yutuqli bo‘lgan $v_{\boldsymbol{L}}(\cdot) \in \boldsymbol{V}_{\sigma}$ boshqaruv funksiya mavjud.

XULOSA

Dissertatsiyada boshqaruvlari geometrik chegaralanishlarga bo‘ysinadigan quvlovchi va qochuvchi obyektlarning harakatlari ikkinchi tartibli tenglamalar orqali ifodalangan differensial o‘yinlarda quvish, qochish va “Qutulish chizig‘i” masalalari ustida tadqiqotlar bajarildi.

Tadqiqotda quyidagi asosiy natijalarga erishildi:

1. Obyektlarning bir xil boshlang‘ich tezliklari holidagi inersion harakatli differensial o‘yinda quvish, qochish va “Qutulish chizig‘i” masalalarining yechilish shartlari olingan va obyektlarning uchrashish sohasi uchun oshkor ko‘rinishdagi formula hosil qilingan;

2. Obyektlar boshlang‘ich vaziyatlari ayirmasini ularning boshlang‘ich tezliklari ayirmasiga chiziqli bog‘liqligi holida inersion harakatli differensial o‘yinda quvish masalasining yechilish shartlariga erishilgan va obyektlarning uchrashish sohasi uchun oshkor ko‘rinishdagi formula aniqlangan;

3. Obyektlar boshlang‘ich vaziyatlari ayirmasini ularning boshlang‘ich tezliklari ayirmasiga chiziqli bog‘liqsizligi holida inersion harakatli differensial o‘yinda quvish masalasining yechilish shartlari olingan va obyektlarning uchrashish sohasi uchun oshkor ko‘rinishdagi formula hosil qilingan;

4. Bir xil boshlang‘ich tezliklar va ishqalanish koeffitisyentlari bilan berilgan obyektarning bir xil tipdagi harakat tenglamalari orqali ifodalangan Pontryagin misoli uchun quvish, qochish va “Qutulish chizig‘i” masalalarining yechilish shartlariga erishilgan va obyektlarning uchrashish sohasi uchun oshkor ko‘rinishdagi formula aniqlangan.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED AFTER V.I.ROMANOVSKIY

NAMANGAN STATE UNIVERSITY

SOYIBBOEV ULMASJON BOYKUZI UGLI

PURSUIT-EVASION PROBLEMS FOR SECOND-ORDER DIFFERENTIAL GAMES WITH VARIOUS CONSTRAINT ON CONTROLS

01.01.02 – Differential equations and mathematical physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT – 2024

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Dissertation is possible to review in Information-resource center at Institute of Mathematics named after V.I.Romanovsky (registered for № 181). (Address: University str. 9, Almazar district, Tashkent, 100174, Uzbekistan, Ph.: (+998 78) 207 91 40).

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INTRODUCTION (abstract of the PhD thesis)

Actuality and demand of the theme of dissertation. On account of the rapid expansion of research on the application of the results achieved in science to practical fields worldwide, formulation and direct implementation of mathematical models of numerous processes in connection with controlled systems given by differential equations take one of the leading places. Solving such problems on a global scale entails directly referring to the theory of differential games. In that regard, it is essential to implement mathematical models formulating for second-order differential equations using the methods of differential games to many fields such as technology, engineering, economics, and biology.

In order to increase the variety of differential games and to expand the fields related to them, scientific and research works are being conducted in the world aimed at studying game problems for high-order differential equations with various restrictions on controls. In this regard, when objects in the game have dynamics expressed by second-order differential equations and their controls are of structural constraint (acceleration) or resource constraint (energy, fuel), scientific researches on finding new solvable conditions for solving pursuit, escape, and R.Isaacs' quality problem with "Life-Line" and on building the meeting area of objects are receiving special attention scientific research on finding new conditions for solving quality issues and building the meeting area of the objects.

In our republic, comprehensive measures are being taken to pay attention to current directions of theoretical investigations aimed at scientific and practical implementation, and certain results are being achieved. In particular, important tasks[1] in the priority directions such as "the theory of differential equations, algebra and its applications, functional analysis and its applications, the theory of dynamical systems, mathematical statistics, mathematical modeling" have been defined. In the implementation of these tasks, it is momentous to build control models, which provide optimal solutions for differential game problems with various restrictions, in the progress of dynamical systems, differential equations, the theories of mathematical control and dynamic game.

The subject and object of the research of this dissertation are in line with tasks identified in the Decrees of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 "On the strategy of action for the further development of the Republic of Uzbekistan", PP-60 of January 28, 2022, "On the Development Strategy of New Uzbekistan for 2022-2026", UP-2789 dated February 17, 2017 "On measures to further improvement of the activities of the Academy of Sciences, organization, management and financing of research activities", PP-4387 from July 9, 2019 "On measures to further development of mathematical education and science, and also root improvement of the activity of the Uzbekistan Academy of

[1] Decree of the President of the Republic of Uzbekistan dated July 9, 2019 № PQ-4387 "On state support for the further development of mathematics education and subjects, as well as measures to fundamentally improve the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan".

Sciences V.I.Romanovsky Institute of Mathematics", UP-4708 dated May 7, 2020 "Quality of education in the field of mathematics", and this dissertation research serves to a certain extent in the implementation of the tasks specified in other normative-law documents related to above activities.

Connection of research to priority directions of development of science and technologies of the Republic. This research was carried out in connection with the priority areas of science and technology of the Republic of Uzbekistan IV, "Mathematics, Mechanics and Computer Science".

The degree of scrutiny of the problem. The theory of differential games went about advancing in the beginning of fiftieth years. Some of the first serious studies are the works of the American mathematician R.Isaacs, who coined the term "differential game". Since the early of sixtieth years of the last century, the conflict-controlled dynamic problems have been considered and are associated with the names of the Russian mathematicians L.S.Pontryagin, N.N.Krasovskii, E.F.Mishchenko, Yu.S.Osipov, L.A.Petrosyan, B.N.Pshenichnii, A.I.Subbotin. Among the American mathematical scientists of the late sixtieths-early seventieth years of the past century, the works by S.Baron, L.D.Bercovitz, J.V.Breakwell, O.Hajek are notable. They considered existence theorems for a cost function in a suitable class of strategies and evolved R.Isaacs' method for solving differential games by constructing singular surfaces. A great contribution to the follow-up advancement of the theory of differential games was made by A.A.Azamov, F.L. Chernousko, A.A.Chikrii, A.V.Kryajimskiy, A.A.Melikyan, N.N.Petrov, N.Yu. Satimov, N.N.Subbotina, V.I.Ukhobotov, V.N.Ushakov, and a lot of other mathematicians.

L.S.Pontryagin proffered and evolved the idea of considering a differential game from two positions of view. A.A.Azamov, A.A.Chikrii, A.N.Dar'in, N.L. Grigorenko, G.I.Ibragimov, D.V.Kornev, A.Sh.Kuchkarov, N.Yu.Lukoyanov, N.A.Mamadaliev, M.Sh.Mamatov, Yu.N.Onopchuk, A.G.Pashkov, S.V.Pashko, N.Nikandr.Petrov, B.N.Pshenichnii, I.J.Rappoport, B.B.Rikhsiev, N.Yu.Satimov, B.T.Samatov, J.Shiyuan, S.D.Terekhov, M.Tukhtasinov, L.P.Yugay, Q.Zhihua, and others carried out surveys on the problems of pursuit in discrete and differential games of many persons for geometric and integral restrictions and enlarged the techniques for solving such problems.

L.A.Petrosyan shown his own solutions to the problem of group pursuit with phase constraints and to the "Life-Line" problem of R.Isaacs by the strategy of parallel pursuit (Π-strategy) coined by him. A.A.Azamov first suggested his analytical solution of the "Life-Line" game with multi-pursuer and one escaper by applying a support function of the multi-valued mapping. B.T.Samatov individually formulated analogies of the Π-strategy for simple motion differential games of pursuit with integral, linear, non-stationary, nonlinear, Grönwall, Langenhop type restrictions, and their mixed forms. G.I.Ibragimov pioneered to consider differential games in Hilbert space, and solved pursuit and escape problems for both simple and inertial motion games represented by an infinite system of differential equations of multiple pursuers and one escaper with integral

restrictions on the controls. N.A.Mamadaliev conducted extensive surveys and achieved significant results on pursuit and escape problems of the theory of differential games with regard to the system given by a differential equation with delay argument for cases when geometric, integral and different restrictions are enforced on the players' controls. A generalization of the simple pursuit problem is the control example of L.S.Pontryagin. The problems of pursuit-escape by pursuers group and one escaper in Pontryagin's example were dissected in a number of research works by A.S.Bannikov, A.I.Blagodatskikh, N.N.Petrov, N.A.Solov'eva, D.A.Vagin.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. Dissertation research was conducted within the framework of the program "Differential Equations and central problems of mathematical fields related to them" in view of the scientific-research plan of Namangan State University.

The aim of the research consists in solving the pursuit, escape and "Life-Line" problems for second-order differential games when being selected the control parameters of objects from the family of measurable functions subjecting to geometric restrictions.

Problems of the research:

determine solvable conditions of the pursuit, escape and "Life-Line" problems in an inertial motion differential game with the same initial velocities of objects, and to build an analytical formula of the meeting area of the objects;

detect solvable conditions of the problem of pursuit in an inertial motion differential game in the case of a linear dependence of the difference of the initial positions of objects on the difference of their initial velocities, and to find an analytical formula for the meeting area of the objects;

determine solvable conditions of the problem of pursuit in an inertial motion differential game in the case of non-linear dependence of the difference of the initial positions of objects on the difference of their initial velocities, and to build an analytical formula of the meeting area of the objects;

detect solvable conditions of the pursuit, escape and "Life-Line" problems for the Pontryagin example governed by the same type of motion equations of objects with the same initial velocities and friction coefficients, and to find an analytical formula for the meeting area of the objects.

The object of the research is made up of elements of the linear theory of optimal control, linear differential equations, and the problems of pursuit, escape, and "Life-Line" of second-order differential games when the control parameters belong to the family of measurable functions subjecting to geometric restrictions.

The subject of the research is to detect solutions to the problems of pursuit, escape and "Life-Line" in conflict-controlled second-order dynamical systems with control parameters subjecting to geometric restrictions.

Methods of the research. In the research work, the strategy of parallel pursuit is implemented using the solvable function method for the problems of pursuit, and the opposite directed control method is employed for the problems of

escape, and additionally, methods of linear control theory, differential games and variational calculus are utilized.

Scientific novelty of the research is composed of the following:

solvable conditions of the pursuit, escape and "Life-Line" problems in an inertial motion differential game with identical initial velocities of objects are detected, and it is proved that a parallel pursuit strategy of pursuer is optimal;

it is proved that the problem of pursuit in an inertial motion differential game in the case of a linear dependence of the difference of the initial positions of objects on the difference of their initial velocities is solvable and the meeting area of the objects is monotone with respect to inclusion;

solvable conditions of the problem of pursuit in an inertial motion differential game in the case of non-linear dependence of the difference of the initial positions of objects on the difference of their initial velocities are detected, and an analytical formula of the meeting area of the objects are formulated;

solvable conditions of the pursuit, escape and "Life-Line" problems for the Pontryagin example governed by the same type of motion equations of objects with equal initial velocities and friction coefficients are detected, and an analytical formula for the meeting area of the objects are determined.

Practical results of the research. The results shown in the dissertation and the methods applied in elucidating their proofs are directly applied to the control problems of conflict processes governed by differential equations of two or higher order in particular. Most of the results are analyzed through numerical examples and their geometric representations are presented as well. And these help to the application of the problems solved in the research work in mathematical modeling of life processes.

The reliability of the results of the research. The linear theory of optimal control, the theory of linear differential equations, functional and mathematical analysis, calculus of variations, theorems and lemmas related to the pursuit-escape problems of the theory of differential games are used. The results presented in the research work are interpreted by the fact that their proofs are based on rigorous approaches in mathematics.

The scientific and practical significance of the research results. The scientific significance of the research results lies in formulating and implementing strategies that provide optimal solutions to conflict-controlled problems governed by dynamical systems with inertia. The given results are elucidated by the improvement of classical methods in the theories of control and differential games and by their use in determining solutions to the problems related to their practical applications.

The practical significance of the research results is glossed by the possibility of defining the optimal strategy and building computational programs for conflict-controlled problems in the fields of physics, technology, medicine, engineering and ecology as the second-order differential equations are the mathematical models of numerous complex real processes, including the connection between atoms or molecules, the motions of satellite vehicles, the laws of electric circuits.

Implementation of the research results. Relying on the achieved results in terms of second-order differential games with the controls of various restrictions:

solvability conditions of the pursuit, escape and Isaacs “Life-Line” problems in an inertial motion differential game under geometric restrictions on the controls of objects were exercised in control issues of conflict processes characterized by dynamic equations with phase restrictions on controls in the fundamental project entitled “Formulation of novel techniques of controlling conflict circumstances represented by differential equations, and their numerical executions” of number OT-F4-33 (the reference of number 04/11-1004 of the National University of Uzbekistan dated February 6, 2024). Implementation of the obtained results was feasible to demonstrate complete solutions of the problems of controlling conflict situations governed by second-order dynamic equations and subjected to geometric restrictions;

solvability conditions of the pursuit, escape and Isaacs “Life-Line” problems for Pontryagin’s control example expressed by the identical type of friction motion dynamics with geometric restrictions on objects’ controls were exercised in solving problems on optimally controlling conflict processes characterized by the identical type of second-order partial differential equations with various restrictions on controls in the fundamental project titled “Construction of adequate computational models for hyperbolic systems” of number OT-F4-28 (the reference of number 04/11-1578 of the National University of Uzbekistan dated February 28, 2024). Implementation of the obtained results was feasible to construct adequate computational models for second-order hyperbolic systems.

Approbation of the research results. The main results of the research were discussed at 16 scientific-practical conferences, including 8 international and 8 national scientific conferences.

Publications of the research results. On the topic of the dissertation, 23 scientific papers were published, 7 of which are included in the list of scientific publications proposed by the Higher Attestation Commission of the Republic of Uzbekistan for the defense of theses of the Doctor of Philosophy, including 3 of them in foreign journals, 4 of them in national scientific journals and 16 abstracts.

The structure and volume of the dissertation. The dissertation is composed of the introduction, three chapters, conclusion, and bibliography. The total volume of the dissertation is 116 pages.

THE MAIN CONTENT OF THE DISSERTATION

In introduction the motivation of the research and correspondence to the priority research areas of science and technology of the Republic are given, we present a review of international research on the topic of the dissertation and degree of scrutiny of the problem, formulate our goals and objectives, identify the object and subject of study, and state scientific novelty and practical results of the research. Moreover, we give the theoretical and practical importance of the obtained results, and also give information on the implementation of the research results, the published works and the structure of the dissertation.

The first Chapter is entitled "**Differential game with a** "**Life-Line**" **for the inertial players with equal initial velocities**", and it is researched on the pursuit, evasion, and "Life-Line" problems in an inertial motion differential game under geometric restrictions on control functions of objects with equal initial velocities.

Suppose, a controllable object $\boldsymbol{P}$, which is termed as the *Pursuer*, runs after another controllable object $\boldsymbol{E}$, which is termed as the *Evader*, in space $\mathbb{R}^n$. Let x be the state vector of the Pursuer, and let y be that of the Evader in $\mathbb{R}^n$, and let the motion dynamics of the objects are shown in the forms of the following differential equations and initial conditions:

$$\boldsymbol{P}:\ \ddot{x} = u,\ x(0) = x_{10},\ \dot{x}(0) = x_{11};\ \boldsymbol{E}:\ \ddot{y} = v,\ y(0) = y_{10},\ \dot{y}(0) = y_{11}, \tag{1}$$

where $x, y, x_{10}, y_{10}, x_{11}, y_{11}, u, v \in \mathbb{R}^n,\ n \geq 2;\ x_{10}$ and y_{10} (x_{11} and y_{11}) convey the initial state vectors (the initial velocity vectors) of the objects, respectively. Here and subsequently, we presume the vectors $x_{10}, y_{10}, x_{11}, y_{11}$ to meet the relations $x_{10} \neq y_{10}$ and $x_{11} = y_{11}$. From the physical point of view, u and v demonstrate the acceleration vectors of the objects, and maenwhile, these vectors act as the control parameters of the objects.

The temporal changes in the vectors u and v are required to be measurable functions $u(\cdot):[0,+\infty) \to \mathbb{R}^n$, $v(\cdot):[0,+\infty) \to \mathbb{R}^n$, and these vector-functions are subject to *geometric constraints* (***G*-constraints** for short) of the forms

$$|u(t)| \leq \alpha,\ |v(t)| \leq \beta \text{ almost everywhere } t \geq 0, \tag{2}$$

where α and β are given positive numbers, and they also designate the maximum values of accelerations of the objects $\boldsymbol{P}$ and $\boldsymbol{E}$.

It is worth noting that, here and later, as the norms of the control vectors $u(t)$ and $v(t)$ in (2) we mean the usual Euclidean norm.

From now on, we will use the symbol $\boldsymbol{U}_\alpha$ (respectively, $\boldsymbol{V}_\beta$) to describe the family of all control functions $u(\cdot)$ (respectively, $v(\cdot)$) conforming to the first one (respectively, the second one) of $\boldsymbol{G}$-constraints (2).

From equations (1), the triads $(x_{10}, x_{11}, u(\cdot))$ and $(y_{10}, y_{11}, v(\cdot))$, where $u(\cdot) \in \boldsymbol{U}_\alpha$ and $v(\cdot) \in \boldsymbol{V}_\beta$, produce the trajectories of the objects $\boldsymbol{P}$ and $\boldsymbol{E}$:

$$x(t) = x_{10} + x_{11}t + \int_0^t (t-s)u(s)ds,\ y(t) = y_{10} + y_{11}t + \int_0^t (t-s)v(s)ds. \qquad (3)$$

Now, presume that a closed subset $\boldsymbol{M}$, which is termed as a "*Life-Line*" is specified in space $\mathbb{R}^n$. The primary aim of the object $\boldsymbol{P}$ is to capture the object $\boldsymbol{E}$, to be exact, to accomplish the equality $x(\tilde{t}) = y(\tilde{t})$ at a finite time $\tilde{t}$, $\tilde{t} > 0$, while the object $\boldsymbol{E}$ doesn't leave the area $\mathbb{R}^n \setminus \boldsymbol{M}$. The aim of the object $\boldsymbol{E}$ is to roll up the area $\boldsymbol{M}$ earlier than bumping into the object $\boldsymbol{P}$, or to perpetuate the relation $x(t) \neq y(t)$ for all $t \in [0, +\infty)$. It is needed to be remarked that the zone $\boldsymbol{M}$ doesn't limit the movement of the object $\boldsymbol{P}$. Furthermore, it is found that the initial state vectors x_{10} and y_{10} are given under the conditions: $x_{10} \neq y_{10}$ and $y_{10} \notin \boldsymbol{M}$.

$$z(t) = x(t) - y(t),\ z(0) = z_{10} = x_{10} - y_{10},\ \dot{z}(0) = z_{11} = x_{11} - y_{11}. \qquad (4)$$

Then according to (1), (3), (4), we get the initial value problem

$$\ddot{z} = u - v,\ z(0) = z_{10},\ \dot{z}(0) = 0. \qquad (5)$$

As an alternative to the game (1)–(2), we have generated the game (2), (5), or for brevity, the game $(\boldsymbol{U}_\alpha, \boldsymbol{V}_\beta)$.

Let the symbol $\boldsymbol{S}_r$ be the ball of a radius r centered at the origin in $\mathbb{R}^n$.

Definition 1. Assume that $\alpha \geq \beta$. Then we call the function

$$\boldsymbol{u}(z_{10}, v) = v - \lambda(z_{10}, v)\xi_{10} \qquad (6)$$

the $\boldsymbol{\Pi}$-*strategy* of the object $\boldsymbol{P}$ in the game $(\boldsymbol{U}_\alpha, \boldsymbol{V}_\beta)$, where

$$\lambda(z_{10}, v) = \langle v, \xi_{10} \rangle + \sqrt{\langle v, \xi_{10} \rangle^2 + \alpha^2 - |v|^2},\ \xi_{10} = \frac{z_{10}}{|z_{10}|},$$

and $\langle v, \xi_{10} \rangle$ is the scalar product of vectors v and ξ_{10} in space $\mathbb{R}^n$. Here, we term $\lambda(z_{10}, v)$ as the *solvable function* in general.

Definition 2. We say that the $\boldsymbol{\Pi}$-strategy *guarantees to catch the object* $\boldsymbol{E}$ on the time interval $[0, t_g]$ if, for optional $v(\cdot) \in \boldsymbol{V}_\beta$: *i*) there exists such an instant $\tilde{t}$, $\tilde{t} \in [0, t_g]$ that generates the equality $z(\tilde{t}) = 0$; *ii*) the inclusion $\boldsymbol{u}(z_{10}, v(\cdot)) \in \boldsymbol{U}_\alpha$ is fulfilled on the time interval $[0, \tilde{t}]$. For this case, the number t_g is termed as the *guaranteed pursuit* (or *capture*) *time*.

Theorem 1. Let $\alpha > \beta$ be true in the game $(\boldsymbol{U}_\alpha, \boldsymbol{V}_\beta)$. Then $\boldsymbol{\Pi}$-strategy (6) guarantees to catch the object $\boldsymbol{E}$ on the time interval $[0, t_g]$, where $t_g = \sqrt{\dfrac{2|z_{10}|}{\alpha - \beta}}$.

Definition 3. We call the control function

$$\boldsymbol{v}_{\boldsymbol{E}}(t) = -\beta\xi_{10} \qquad (7)$$

the *strategy of the object* $\boldsymbol{E}$ in the game $(\boldsymbol{U}_\alpha, \boldsymbol{V}_\beta)$.

Definition 4. It is said that a strategy $\boldsymbol{v_E}(\cdot) \in \boldsymbol{V}_\beta$ *guarantees to escape* if, for any $u(\cdot) \in \boldsymbol{U}_\alpha$, the solution $z(t)$ of the Cauchy problem $\ddot{z} = u(t) - \boldsymbol{v_E}(t), z(0) = z_{10}$, $\dot{z}(0) = 0$ is nonzero, viz, $z(t) \neq 0$ for all $t \in [0, +\infty)$.

Theorem 2. *i*) Let $\alpha > \beta$. Then the strategy (7) guarantees to escape on the time interval $[0, t_g)$ in the game $(\boldsymbol{U}_\alpha, \boldsymbol{V}_\beta)$, where t_g is the guaranteed capture time (see Theorem 1); *ii*) Let $\alpha \leq \beta$. Then the strategy (7) guarantees to escape on the time interval $[0, +\infty)$ in the game $(\boldsymbol{U}_\alpha, \boldsymbol{V}_\beta)$, and a distance between the objects is estimated as follows: $|z(t)| \geq \begin{cases} |z_{10}| & \text{if } \alpha = \beta, \\ |z_{10}| + \dfrac{(\beta - \alpha)t^2}{2} & \text{if } \alpha < \beta. \end{cases}$

By any control $v(\cdot) \in \boldsymbol{V}_\beta$ and by the strategy $\boldsymbol{u}(z_{10}, v(\cdot)) \in \boldsymbol{U}_\alpha$ on the time interval $[0, \tilde{t}]$, where $0 \leq \tilde{t} < t_g$ and $x(\tilde{t}) = y(\tilde{t})$ holds, for the pair $(x(t), y(t))$ the multi-valued mapping $\boldsymbol{W}(x(t), y(t)) = \left\{ w : \ |w - x(t)| \geq \dfrac{\alpha}{\beta} |w - y(t)| \right\}$ is defined, and from this we take $\boldsymbol{W}(x_{10}, y_{10}) = \left\{ w : \ |w - x_{10}| \geq \dfrac{\alpha}{\beta} |w - y_{10}| \right\}$ at time $t = 0$.

Theorem 3. The multi-valued mapping $\boldsymbol{W}(x(t), y(t)) - x_{11}t$ is monotonically decreasing in relation to inclusion with respect to $t \in [0, \tilde{t}]$, i.e. if $t_1, t_2 \in [0, \tilde{t}]$ and $t_1 < t_2$, then $\boldsymbol{W}(x(t_2), y(t_2)) - x_{11}t_2 \subset \boldsymbol{W}(x(t_1), y(t_1)) - x_{11}t_1$.

To solve the "Life-Line" Game in favour of the object $\boldsymbol{P}$, one can define the set $\boldsymbol{W}_{\boldsymbol{P}}(x_{10}, y_{10}, x_{11}, t_g) = \bigcup_{t=0}^{t_g} \{ \boldsymbol{W}(x_{10}, y_{10}) + x_{11}t \}$, which is convex and closed.

The set $\boldsymbol{W}_{\boldsymbol{P}}(x_{10}, y_{10}, x_{11}, t_g)$ is said to be the *attainability domain of the object* $\boldsymbol{P}$ in the Problem of Pursuit.

Definition 5. It is said that $\boldsymbol{\Pi}$-strategy (6) *provides winning for the object* $\boldsymbol{P}$ on the time interval $[0, t_g]$ in the "Life-Line" Game (1)–(2), if there exists such a time instant $t_* \in [0, t_g]$ that the following conditions are valid: *i*) $x(t_*) = y(t_*)$; *ii*) $y(t) \notin \boldsymbol{M}$ for all $t \in [0, t_*]$.

Theorem 4. If: *i*) $\alpha > \beta$; *ii*) the attainability domain of the object $\boldsymbol{P}$ doesn't intersect with the zone $\boldsymbol{M}$, i.e. $\boldsymbol{W}_{\boldsymbol{P}}(x_{10}, y_{10}, x_{11}, t_g) \cap \boldsymbol{M} = \varnothing$, then $\boldsymbol{\Pi}$-strategy (6) provides win- ning for the object $\boldsymbol{P}$ in the "Life-Line" Game (1)–(2).

Definition 6. It is said that a control $v_{\boldsymbol{M}}(\cdot) \in \boldsymbol{V}_\beta$ *provides winning for the object* $\boldsymbol{E}$ in the "Life-Line" Game (1)–(2), if, for any control $u(\cdot) \in \boldsymbol{U}_\alpha$, at least

one of the following conditions holds, i.e.: *i*) there exists such a finite moment $t_{\boldsymbol{M}}$ that $y(t_{\boldsymbol{M}}) \in \boldsymbol{M}$ and $x(t) \neq y(t)$ for all $t \in [0, t_{\boldsymbol{M}})$; *ii*) $x(t) \neq y(t)$ for all $t \in [0, +\infty)$.

Consider the set $\boldsymbol{W}_{\boldsymbol{E}}(x_{11}, y_{10}, w) = \bigcup\limits_{w \in \boldsymbol{W}(x_{10}, y_{10})} \left\{ w_{\boldsymbol{E}} : \; w_{\boldsymbol{E}} = x_{11}\sqrt{\dfrac{2|w - y_{10}|}{\beta}} + w \right\}$.

The set $\boldsymbol{W}_{\boldsymbol{E}}(x_{11}, y_{10}, w)$ is said to be the *attainability domain of the object* $\boldsymbol{E}$ in the "Life-Line" Game (1)−(2).

Theorem 5. If: *i*) $\alpha > \beta$; *ii*) the attainability domain of the object $\boldsymbol{E}$ intersects with the zone $\boldsymbol{M}$, i.e. $\boldsymbol{W}_{\boldsymbol{E}}(x_{11}, y_{10}, w) \cap \boldsymbol{M} \neq \varnothing$, then there is such a control $v_{\boldsymbol{M}}(\cdot) \in \boldsymbol{V}_{\beta}$ providing winning for the object $\boldsymbol{E}$ in the "Life-Line" Game (1)−(2).

Theorem 6. Let $\alpha \leq \beta$. Then there exists such a control $v_{\boldsymbol{M}}(\cdot) \in \boldsymbol{V}_{\beta}$ that provides winning for the object $\boldsymbol{E}$ in the "Life-Line" Game (1)–(2).

The second Chapter is titled "**Applications of the Π-strategy in differential game of the inertial players with arbitrary initial velocities**", and it is surveyed on the problem of pursuit based on a novel idea in an inertial motion differential game for objects with non-equal initial velocities.

As the previous Chapter, study an inertial motion differential game with two controlled objects $\boldsymbol{P}$ and $\boldsymbol{E}$, which the first one moves based on the first equation of the dynamics (1) by the control $u(\cdot)$ adhering to the first one of $\boldsymbol{G}$-constraints (2), and the other moves based on the second equation of the dynamics (1) by the control $v(\cdot)$ complying with the second one of $\boldsymbol{G}$-constraints (2), in space $\mathbb{R}^n$.

In contrast to the above Chapter, in the present, it is presumed to be $x_{11} \neq y_{11}$.

Needless to say, by reason of denotations (4), it is directly taken from (1) that the initial value problem

$$\ddot{z} = u - v, z(0) = z_{10}, \dot{z}(0) = z_{11}. \tag{8}$$

For arbitrarily chosen admissible controls $u(\cdot) \in \boldsymbol{U}_{\alpha}$ and $v(\cdot) \in \boldsymbol{V}_{\beta}$, a solution to the problem (8) is written in the form

$$z(t) = z_{10} + z_{11}t + \int_0^t (t - s)\big(u(s) - v(s)\big)ds. \tag{9}$$

Depending on (9), let the aiming of the object $\boldsymbol{P}$ is to gain the equality $z(t) = 0$ from the given initial state vectors z_{10} and z_{11} in the shortest time t, and concurrently, the aiming of the object $\boldsymbol{E}$ is to maintain $z(t) \neq 0$ for all $t \geq 0$.

We are about to demonstrate a solution to the pursuit problem depending on z_{10} and z_{11}, for which the following two cases are possible:

Case 1. The vectors z_{10} and z_{11} are collinear, that is, there exists a finite number k such that $z_{11} = kz_{10}$.

Case 2. The vectors z_{10} and z_{11} are non-collinear, viz, they are linearly independent.

As in Chapter 1, let the object $\boldsymbol{P}$ take Π-strategy (6) in Case 1.

Theorem 7. Let at least one of the following conditions be satisfied: *i*) $\alpha > \beta$ and $k \in \mathbb{R}$; *ii*) $\alpha = \beta$ and $k < 0$. Then $\boldsymbol{\Pi}$-strategy (6) is winning in the time interval $\left[0, T_{10}\right]$ in Case 1, where

$$T_{10} = \begin{cases} \left(k|z_{10}| + \sqrt{k^2|z_{10}|^2 + 2(\alpha-\beta)|z_{10}|}\right) \Big/ (\alpha-\beta) & \text{if } \alpha > \beta,\ k \in \mathbb{R}, \\ -1/k & \text{if } \alpha = \beta,\ k < 0. \end{cases}$$

As in Chapter 1, for a pair $\big(x(t), y(t)\big)$ on the time interval $\left[0, t_{10}^*\right]$, we define the set $\boldsymbol{A}\big(x(t), y(t)\big) = \left\{ w_{\boldsymbol{A}} : \left|w_{\boldsymbol{A}} - x(t)\right| \geq \frac{\alpha}{\beta}\left|w_{\boldsymbol{A}} - y(t)\right| \right\}$.

Now, take the multi-valued mapping $\boldsymbol{A}^*\big(x(t), y(t), t\big) = \boldsymbol{A}\big(x(t), y(t)\big) - t\boldsymbol{A}_{11}$, where $\boldsymbol{A}_{11} = \boldsymbol{A}\big(x_{11}, y_{11}\big) = \frac{\alpha^2 y_{11} - \beta^2 x_{11}}{\alpha^2 - \beta^2} + \frac{\alpha\beta|z_{11}|}{\alpha^2 - \beta^2}\boldsymbol{S}$.

Property 1. In Case 1, the attainability domain of the object $\boldsymbol{E}$ is the set $\boldsymbol{A}_{\boldsymbol{E}} = \boldsymbol{A}_{10} + \bigcup_{t=0}^{T_{10}} t\boldsymbol{A}_{11}$, viz, the trajectory $y_{T_{10}}(\cdot) = \left\{ y(s) : 0 \leq s \leq T_{10} \right\}$ of the object $\boldsymbol{E}$ doesn't leave the closed convex set $\boldsymbol{A}_{\boldsymbol{E}}$, where $\boldsymbol{A}_{10} = \frac{\alpha^2 y_{10} - \beta^2 x_{10}}{\alpha^2 - \beta^2} + \frac{\alpha\beta|z_{10}|}{\alpha^2 - \beta^2}\boldsymbol{S}$.

Now, let the original non-zero vectors z_{10} and z_{11} be non-collinear, and let $\alpha > \beta$, i.e. Case 2 will be explored.

First, we are going to work out the following auxiliary problem.

Problem 1. For arbitrary control $v(\cdot) \in \boldsymbol{V}_{\beta}$ of the object $\boldsymbol{E}$, it is demanded to construct a $\boldsymbol{\Pi}$-strategy of the object $\boldsymbol{P}$, with the help of which, the equality of the velocities

$$\dot{x}(\theta) = \dot{y}(\theta) \tag{10}$$

is achieved, where θ is some finite time.

In order to solve the Problem 1, the second-order differential game (8) is reduced to a first-order differential game. To this end, we introduce new variables $x_1 = \dot{x}$ and $y_1 = \dot{y}$. Then from (1) we have

$$\dot{x}_1 = u,\ x_1(0) = x_{11};\ \ \dot{y}_1 = v,\ y_1(0) = y_{11},$$

correspondingly, where u (corresp., v) acts as the rate of change of the vector x_1 (corresp., y_1) depending on time t. It's obvious that $x_{11} \neq y_{11}$, otherwise we return to Case 1, where x_{11}, y_{11} are the initial states of the vectors x_1, y_1.

Definition 7. For the Problem 1, the function

$$\boldsymbol{u}_1\big(z_{11}, v\big) = v - \lambda_1\big(z_{11}, v\big)\xi_{11} \tag{11}$$

is called the $\boldsymbol{\Pi}_1$-*strategy* of the object $\boldsymbol{P}$, where

$$\lambda_1\big(z_{11}, v\big) = \big\langle v, \xi_{11} \big\rangle + \sqrt{\big\langle v, \xi_{11} \big\rangle^2 + \alpha^2 - |v|^2},\ \xi_{11} = \frac{z_{11}}{|z_{11}|},\ z_{11} = x_{11} - y_{11},$$

and $\boldsymbol{u}_1\big(z_{11}, v(t)\big)$, $t \geq 0$, is the *realization* of $\boldsymbol{u}_1\big(z_{11}, v\big)$ for arbitrary $v(\cdot) \in \boldsymbol{V}_\beta$.

Theorem 8. Let $\alpha > \beta$, and let the object $\boldsymbol{P}$ apply $\boldsymbol{\Pi}_1$-strategy (11). Then, for any control $v(\cdot) \in \boldsymbol{V}_\beta$ of the object $\boldsymbol{E}$, the equality (10) can be realized in some time $\theta \in \big[\theta_{10}, \theta_{11}\big]$, where $\theta_{10} = \dfrac{|z_{11}|}{\alpha + \beta}$, $\theta_{11} = \dfrac{|z_{11}|}{\alpha - \beta}$.

Theorem 9. Let $\alpha > \beta$, and let the object $\boldsymbol{P}$ utilize $\boldsymbol{\Pi}_1$-strategy (11), then for arbitrary control $v(\cdot) \in \boldsymbol{V}_\beta$ of the object $\boldsymbol{E}$, the estimate $L_{10} \leq |z(t)| \leq L_{11}$ is valid on the time interval $\big[0, \theta\big]$ for the distance $|z(t)|$ between the objects, where

$$L_{10} = \begin{cases} |z_{10}| & \text{if } \langle z_{10}, z_{11} \rangle > 0, \\ \left| z_{10} - \dfrac{\langle z_{10}, z_{11} \rangle}{|z_{11}|^2} z_{11} \right| & \text{if } \langle z_{10}, z_{11} \rangle < 0, \end{cases}$$

$$L_{11} = \begin{cases} \left| z_{10} + \dfrac{|z_{11}|}{2(\alpha - \beta)} z_{11} \right| & \text{if } \langle z_{10}, z_{11} \rangle > 0, \\ \max\left\{ |z_{10}|, \left| z_{10} + \dfrac{|z_{11}|}{2(\alpha - \beta)} z_{11} \right| \right\} & \text{if } \langle z_{10}, z_{11} \rangle < 0. \end{cases}$$

Thus, as confirmed in Theorem 9, the objects $\boldsymbol{P}$ and $\boldsymbol{E}$ can't coincide in the time interval $\big[0, \theta\big]$. However, their velocity trajectories coincide at the time instant θ. Now, starting from this time θ, we are going to deal with the differential game

$$\ddot{x} = u,\ x(\theta) = x_*,\ \dot{x}(\theta) = x_{**};\ \ \ddot{y} = v,\ y(\theta) = y_*,\ \dot{y}(\theta) = y_{**}, \tag{12}$$

where x_*, y_* are the positions of the objects, and x_{**}, y_{**} are the positions of their velocities at the time θ. Note that the equality $x_{**} = y_{**}$ has been realized, namely, the general game (8) is reduced to Case 1 again, when $k = 0$.

Definition 8. Let $\alpha > \beta$ and $t \geq \theta$. Then in the game (12), the function

$$\boldsymbol{u}_2\big(z_*, v\big) = v - \lambda_2\big(z_*, v\big)\xi_* \tag{13}$$

is called the $\boldsymbol{\Pi}_2$-*strategy* of the object $\boldsymbol{P}$, where

$$\lambda_2\big(z_*, v\big) = \langle v, \xi_* \rangle + \sqrt{\langle v, \xi_* \rangle^2 + \alpha^2 - |v|^2},\ \xi_* = \frac{z_*}{|z_*|},\ z_* = x_* - y_*,$$

and $\boldsymbol{u}_2\big(z_*, v(t)\big)$, $t \geq \theta$, is the *realization* of $\boldsymbol{u}_2\big(z_*, v\big)$ for arbitrary $v(\cdot) \in \boldsymbol{V}_\beta$.

Theorem 10. Let $\alpha > \beta$, and let the vectors z_{10} and z_{11} be non-collinear. Then, for arbitrary control $v(\cdot) \in \boldsymbol{V}_\beta$ of the object $\boldsymbol{E}$, in the game (8), the object $\boldsymbol{P}$, using the $\boldsymbol{\Pi}_1$ and $\boldsymbol{\Pi}_2$-strategies (11) and (13) successively, catches the object $\boldsymbol{E}$ on the time interval $\big[T_{21}, T_{22}\big]$, where $T_{21} = \dfrac{|z_{11}|}{\alpha + \beta} + \sqrt{\dfrac{2L_{10}}{\alpha + \beta}}$, $T_{22} = \dfrac{|z_{11}|}{\alpha - \beta} + \sqrt{\dfrac{2L_{11}}{\alpha - \beta}}$.

Theorem 11. If $\alpha > \beta$, then the inclusion $y(t) \in y_{10} + t\boldsymbol{A}_{11} + \boldsymbol{B}_{11}$ is carried out for all $t \in [0, T_{22}]$ and for arbitrary $v(\cdot) \in \boldsymbol{V}_{\beta}$ in Case 2 of the differential game (8).

Definition 9. In Case 2 of the game (1), the set $\boldsymbol{Q}_{\boldsymbol{E}} = y_{10} + \boldsymbol{B}_{11} + \bigcup_{t=0}^{T_{22}} t\boldsymbol{A}_{11}$ is called the *attainability domain of the object* $\boldsymbol{E}$.

The third Chapter is entitled "**Applications of the Π-strategy in differential games with the friction motions of players**", and it is explored on the pursuit, evasion, and "Life-Line" problems for Pontryagin's control example governed by the same type of dynamics of movements of two inertial objects with identical initial velocities and influences of frictions.

Let a controllable player $\boldsymbol{X}$, viz, the *Pursuer*, come after other controllable player $\boldsymbol{Y}$, viz, the *Evader* in $\mathbb{R}^n$. Suppose that positions of the players $\boldsymbol{X}$ and $\boldsymbol{Y}$ are described by x and y, respectively, and their movements are governed by

$$\boldsymbol{X}: \ddot{x} + a\dot{x} = u, x(0) = x_{10}, \dot{x}(0) = x_{20}; \ \boldsymbol{Y}: \ddot{y} + a\dot{y} = v, y(0) = y_{10}, \dot{y}(0) = y_{20}, \quad (14)$$

where $x, y, x_{10}, y_{10}, x_{20}, y_{20}, u, v \in \mathbb{R}^n$, $n \geq 2$, $a > 0$; the vectors x_{10} and y_{10} are the initial positions of the players $\boldsymbol{X}$ and $\boldsymbol{Y}$, whereas the vectors x_{20} and y_{20} are their initial velocity vectors. Here, it is required that $x_{10} \neq y_{10}$ and $x_{20} = y_{20}$.

In common with the first Chapter, the control vectors u and v of the players are thought of as measurable functions $u(\cdot): [0, +\infty) \to \mathbb{R}^n$ and $v(\cdot): [0, +\infty) \to \mathbb{R}^n$, and on these vector-functions, we enforce the $\boldsymbol{G}$-constraints

$$|u(t)| \leq \rho, \ |v(t)| \leq \sigma \text{ almost everywhere } t \geq 0, \quad (15)$$

where ρ and σ are positive parametric numbers designating the maximum values of accelerations of the players $\boldsymbol{X}$ and $\boldsymbol{Y}$.

First off, let us write the notations

$$z(t) = x(t) - y(t), z(0) = z_{10} = x_{10} - y_{10}, \dot{z}(0) = z_{20} = x_{20} - y_{20}. \quad (16)$$

Then according to (14), (16), we get the initial value problem

$$\ddot{z} + a\dot{z} = u - v, z(0) = z_{10}, \dot{z}(0) = z_{20} = 0. \quad (17)$$

As in Chapter 1, come to the game (15), (17), or briefly, the game $(\boldsymbol{U}_{\rho}, \boldsymbol{V}_{\sigma})$.

Consider the function $p(t) = e^{-at} + at - b$, where $b = 1 + \dfrac{a^2 |z_{10}|}{\rho - \sigma}$.

Proposition 1. Suppose $\rho > \sigma$. Then the equation $p(t) = 0, t \geq 0$, has one and only one positive root that will be denoted by T.

Definition 10. We say that the vector-function

$$\boldsymbol{u}_{\Pi}(v, z_{10}) = v - \gamma(v, z_{10})\xi_{10} \quad (18)$$

is the Π-*strategy* in the game $(\boldsymbol{U}_{\rho}, \boldsymbol{V}_{\sigma})$, where

$$\gamma(v, z_{10}) = \langle v, \xi_{10} \rangle + \sqrt{\langle v, \xi_{10} \rangle^2 + \rho^2 - |v|^2},$$

$\xi_{10} = \dfrac{z_{10}}{|z_{10}|}$, and $\langle v, \xi_{10} \rangle$ is the scalar product of vectors v and ξ_{10} in $\mathbb{R}^n$.

Theorem 12. If $\rho > \sigma$, then $\boldsymbol{\Pi}$-strategy (18) is winning for the player $\boldsymbol{X}$ on the time interval $t \in \left[0, T\right]$.

Definition 11. In the game $\left(\boldsymbol{U}_\rho, \boldsymbol{V}_\sigma\right)$, the control function

$$\boldsymbol{v}_{\boldsymbol{Y}}(t) = -\sigma \xi_{10} \tag{19}$$

is called the *strategy of the player* $\boldsymbol{Y}$.

Theorem 13. *i*) Let $\rho > \sigma$. Then the strategy (19) is winning for the player $\boldsymbol{Y}$ in the time interval $\left[0, T\right)$; *ii*) Let $\rho \le \sigma$. Then the strategy (19) is winning for the player $\boldsymbol{Y}$ in the time interval $\left[0, +\infty\right)$, and the distance between the players $\boldsymbol{X}$ and $\boldsymbol{Y}$ is evaluated as: $|z(t)| \ge \begin{cases} |z_{10}| & \text{if } \rho = \sigma, \\ |z_{10}| + \dfrac{\sigma - \rho}{a^2}\left(e^{-at} + at - 1\right) & \text{if } \rho < \sigma. \end{cases}$

By any control $v(\cdot) \in \boldsymbol{V}_\sigma$ and by the strategy $\boldsymbol{u}_{\boldsymbol{\Pi}}\left(v(\cdot), z_{10}\right) \in \boldsymbol{U}_\rho$ on the time interval $\left[0, T_*\right]$, where $0 < T_* \le T$ and $x\left(T_*\right) = y\left(T_*\right)$ holds, for the pair $\left(x(t), y(t)\right)$, the multi-valued mapping $\boldsymbol{\Delta}\left(x(t), y(t)\right) = \left\{\delta :\ |\delta - x(t)| \ge \dfrac{\rho}{\sigma}|\delta - y(t)|\right\}$ is defined, and at $t = 0$, we have $\boldsymbol{\Delta}\left(x_{10}, y_{10}\right) = \left\{\delta :\ |\delta - x_{10}| \ge \dfrac{\rho}{\sigma}|\delta - y_{10}|\right\}$

Lemma 1. Let $\boldsymbol{\Psi}\left(x(t), y(t), x_{20}\right) = \boldsymbol{\Delta}\left(x(t), y(t)\right) - \dfrac{x_{20}}{a}\left(1 - e^{-at}\right)$. Then $\boldsymbol{\Psi}\left(x(t_2), y(t_2), x_{20}\right) \subset \boldsymbol{\Psi}\left(x(t_1), y(t_1), x_{20}\right)$ holds when $t_1 < t_2$ for any $t_1, t_2 \in \left[0, T_*\right]$.

Definition 12. The set $\boldsymbol{\Delta}_{\boldsymbol{X}}\left(x_{10}, x_{20}, y_{10}, T\right) = \bigcup_{t=0}^{T} \left\{\boldsymbol{\Delta}\left(x_{10}, y_{10}\right) + \dfrac{x_{20}}{a}\left(1 - e^{-at}\right)\right\}$ is called the *attainability domain of the player* $\boldsymbol{X}$ in the game $\left(\boldsymbol{U}_\rho, \boldsymbol{V}_\sigma\right)$.

Assume that a closed subset $\boldsymbol{L}$, called the "*Life-Line*", is specified in $\mathbb{R}^n$. It is imagined that the initial position of the player $\boldsymbol{Y}$ conforms to the relation $y_{10} \notin \boldsymbol{L}$.

Let's take the sets $\boldsymbol{T}_{\boldsymbol{L}}\left(\delta, y_{10}\right) = \bigcup_{\delta \in \boldsymbol{\Delta}\left(x_{10}, y_{10}\right)} \left\{t_{\boldsymbol{L}} :\ e^{-at_{\boldsymbol{L}}} + at_{\boldsymbol{L}} = 1 + \dfrac{a^2|\delta - y_{10}|}{\sigma}\right\}$,

$$\boldsymbol{\Delta}_{\boldsymbol{Y}}\left(\delta, x_{20}, t_{\boldsymbol{L}}\right) = \bigcup_{\delta \in \boldsymbol{\Delta}\left(x_{10}, y_{10}\right)} \left\{\delta_{\boldsymbol{Y}} :\ \delta_{\boldsymbol{Y}} = \delta + \frac{x_{20}}{a}\left(1 - e^{-at_{\boldsymbol{L}}}\right)\right\}. \tag{20}$$

The set (20) is said to be the *attainability domain of the player* $\boldsymbol{Y}$ in the "Life-Line" Game (14)–(15).

Theorem 14. If $\rho > \sigma$ and $\Delta_{\boldsymbol{Y}}(\delta, x_{20}, t_{\boldsymbol{L}}) \cap \boldsymbol{L} \neq \varnothing$, then there exists such a control function $v_{\boldsymbol{L}}(\cdot) \in \boldsymbol{V}_{\sigma}$ that is winning for the player $\boldsymbol{Y}$ in the "Life-Line" Game (14)–(15).

CONCLUSION

In the dissertation, it has been performed surveys on the pursuit, evasion and "Life-Line" problems in differential games such that movements of the pursuing and escaping objects, whose controls are subject to geometric restrictions, are governed by second-order equations.

The research produced the following core results:

1. Solvable conditions of the pursuit, evasion and "Life-Line" problems in an inertial motion differential game in the case of equal initial velocities of the objects were gleaned, and the formula for the meeting domain of the objects was explicitly produced;

2. Solvable conditions of the problem of pursuit in an inertial motion differential game in the case of linear dependence of subtraction of initial positions of the objects to that of their initial velocities were achieved, and the formula was explicitly created for the meeting domain of the objects;

3. Solvable conditions of the problem of pursuit in an inertial motion differential game in the case of linear independence of subtraction of initial positions of the objects to that of their initial velocities were attained, and the explicit formula was generated for the meeting domain of the objects;

4. Solvable conditions of the pursuit, evasion and "Life-Line" problems for Pontryagin's example governed by the same type of motion equations of the objects given with equal initial velocities and friction coefficients were gained, and the explicit formula for the meeting domain of the objects was turned up.

НАУЧНЫЙ СОВЕТ DSc.02/30.12.2019.FM.86.01 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО

НАМАНГАНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

СОЙИББОЕВ УЛМАСЖОН БОЙКУЗИ УГЛИ

ЗАДАЧИ ПРЕСЛЕДОВАНИЯ-УБЕГАНИЯ ДЛЯ ДИФФЕРЕНЦИАЛЬНЫХ ИГР ВТОРОГО ПОРЯДКА С РАЗЛИЧНЫМИ ОГРАНИЧЕНИЯМИ НА УПРАВЛЕНИЯ

01.01.02 – Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

ТАШКЕНТ – 2024

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования состоит в решении задачи преследования, убегания и «Линий жизни» для дифференциальных игр второго порядка при выборе управляющих параметров объектов из семейства измеримых функций, подчиняющихся геометрическим ограничениям.

Объектом исследования составляют элементы линейной теории оптимального управления, линейные дифференциальные уравнения, а также задачи преследования, убегания и «Линий жизни» дифференциальных игр второго порядка, когда параметры управления принадлежат семейству измеримых функций с геометрическими ограничениями.

Научная новизна исследования состоит в следующем:

определена разрешимые условия задачи преследования, убегания и «Линий жизни» в дифференциальной игре инерционного движения с одинаковыми начальными скоростями объектов и доказано что стратегия параллельного преследования преследователя оптимальна;

доказано что задача преследования в дифференциальной игре по инерционному движению в случае линейной зависимости разности начальных состояний объектов от разности их начальных скоростей разрешима и область встречи объектов монотонна по отношению к включению;

определена разрешимые условия задача преследования в дифференциальной игре по инерционному движению в случае нелинейной зависимости разности начальных состояний объектов от разности их начальных скоростей, а также сформулирована аналитическая формула области встречи объекты;

найдены разрешимые условия задачи преследования, убегания и «Линий жизни» для примера Понтрягина, подчиняющиеся однотипным уравнениям движения объектов с равными начальными скоростями и коэффициентами трения, а также получена аналитическая формула для области встречи объектов.

Внедрение результатов исследования. На основе полученных результатов о дифференциальных играх второго порядка с управлениями геометрическими ограничениями:

условия разрешимости задачи преследования, убегания и «Линий жизни» Айзекса в дифференциальной игре с инерциального движения при геометрических ограничениях на управления объектами были использованы в задачах управления конфликтными ситуации, характеризуемыми динамическими уравнениями с фазовыми ограничениями на управления в фундаментальный проект под названием «Формулирование новых методов управления конфликтными обстоятельствами, представленными дифференциальными уравнениями и их численные реализации», под номером ОТ-Ф4-33 (справка № 04/11-1004 от 6 февралья 2024 года Национального университета Узбекистана). Применение полученных результатов стало возможным показывать полные решения задач управления

конфликтными ситуациями, подчиняющиеся динамическим уравнениям второго порядка и налагаемые геометрическими ограничениями;

условия разрешимости задачи преследования, убегания и «Линий жизни» Айзекса для контрольный пример Понтрягина, выражаемые однотипной динамикой тренного движения с геометрическими ограничениями на управление объектами были использованы в решении задач по оптимальному управлению конфликтными процессами характеризующихся однотипными уравнениями в частных производных второго порядка с различными ограничениями на управления в фундаментальный под названием «Построение адекватных вычислительных моделей для гиперболических систем», под номером ОТ-Ф4-28 (справка № 04/11-1578 от 28 февралья 2024 года Национального университета Узбекистана). Применение полученных результатов стало возможным построить адекватные вычислительные модели для гиперболических систем второго порядка.

Структура и объем диссертации. Диссертация состоит из введения, трех глав, разбитых на одинадцать параграфов, заключения и списка использованной литературы. Объем диссертации составляет 116 страниц.

E’LON QILINGAN ISHLAR RO‘YXATI
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1-bo‘lim (part 1; часть 1)

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