V.I. ROMONOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

MATEMATIKA INSTITUTI

QILICHOV OYBEK SHARAFIDDINOVICH

JUFT TARTIBLI XUSUSIY HOSILALI DIFFERENSIAL TENGLAMALAR UCHUN CHEGARAVIY MASALALAR

01.01.02 – Differensial tenglamalar va matematik fizika

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

TOSHKENT – 2023

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

V.I. ROMONOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

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TOSHKENT – 2023

Fizika-matematika fanlari bo‘yicha falsafa doktori (Doctor of Philosophy) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Oliy ta’lim, Fan va Innovatsiyalar Vazirligi huzuridagi Oliy attestatsiya komissiyasida № B2022.4.PhD/FM789 raqam bilan ro‘yxatga olingan.

Dissertatsiya V. I. Romanovskiy nomidagi matematika institutida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o‘zbek, ingiliz, rus (резюме)) Ilmiy kengash veb-sahifasi (http:ik-fizmat.nuu.uz) va «Ziyonet» Axborot ta’lim portalida (www.ziyonet.uz) joylashtirilgan.

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zaruriyati. Jahon miqyosida olib borilgan ko'plab ilmiy va amaliy tadqiqotlar juft tartibli xususiy hosilali differensial tenglamalar uchun boshlang'ich-chegaraviy masalalarni o'rganish muhimligini ko'rsatadi. Amaliy xarakterdagi masalalarni o'rganish jarayonida juft tartibli xususiy hosilali differentsial tenglamalar uchun chegaraviy masalalar ustuvor yo'nalishlardan biridir. Boshlang'ich-chegaraviy masalalarni yechishning ahamiyati ularni gaz dinamikasi va magnit gidrodinamika sohalaridagi ko'plab jarayonlar masalalariga qo'llanilishi bilan izohlanadi. Amaliyot ehtiyojlaridan kelib chiqib, juft tartibli tenglamalar nazariyasiga e'tibor kuchaydi. Juft tartibli tenglamalar bilan bog'liq jarayonlarning murakkabligi va analitik usullarning yo'qligi sababli, juft tartibli xususiy hosilali differentsial tenglamalarni o'rganish ustuvor yo'nalishlardan biri hisoblanadi.

Hozirgi vaqtda matematik fizikaning dolzarb muammolaridan biri chegaraviy masalasi hisoblanadi. So‘nggi yillarda juft tartibli xususiy hosilali differensial tenglamalar uchun boshlang'ich va chegaraviy masalalari jadal rivojlanmoqda, chunki ular amaliyotga tatbiqi bilan chambarchas bog‘liq. Bunday masalalar ixtiyoriy chegaralangan sohada parabolik tipdagi tenglamalarga qo‘yilgan masalalarini yechishga keltiriladi. Hozirgi kunda global miqyosda yuqori juft tartibli xususiy hosilali differentsial tenglamalar uchun boshlang'ich va chegaraviy masalalarni o'rganish muhim rol o'ynaydi. Yuqori juft tartibli xususiy hosilali bo'lgan differensial tenglamalar uchun boshlang'ich va chegaraviy masalalarni to'g'ri qo'yish masalaning yagona yechilishini o'rnatishni talab qiladi. Yuqoridagilarni hisobga olib, maqsadli ilmiy tadqiqotlarni amalga oshirish muhim vazifaga aylanib borayotganini ta’kidlaymiz.

Mamlakatimizdagi fundamental fanlarning ilmiy va amaliy tatbiqiga ega bo‘lgan matematika, fizika va optimal boshqruv yo‘nalishlariga e’tibor kuchaytirildi. Jumladan, o‘yinlar nazaryasi va boshlang'ich-chegaraviy masalalarini tadqiq qilishga alohida e’tibor qaratildi. Oddiy differensial tenglamalar, xususiy hosilali differensial tenglamalar va ularning sistemalari, oddiy differensial tenglamalar va ularning sistemalari orqali tavsiflanadigan jarayonlarni boshqarish nazariyasiga doir ajoyib natijalarga erishildi. “Funksional analiz, differensial tenglamalar va matematik fizika fanlarining ustuvor yo‘nalishlari bo‘yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish asosiy vazifalar va faoliyat yo‘nalishlari” etib belgilandi[1]. Qaror ijrosini ta’minlashda differensial tenglamalar va matematik fizika masalalarini hal qilishda chegaraviy va chegaraviy masalalarini rivojlantirish muhim ahamiyatga ega.

O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha harakatlar strategiyasi to‘g‘risida”gi PF-4947-son Farmoni, 2019 yil 9 iyuldagi “Matematika ta’limi va fanlarni yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston

[1] O‘zbekiston Respublikasi Vazirlar Maxkamasining 2017-yil 18-maydagi “O‘zbekiston Respublikasi Fanlar Akademiyasining yangidan tashkil etilgan ilmiy tadqiqot muassasalari faoliyatini tashkil etish to‘g‘risida”gi 292-sonli qarori.

Respublikasi Fanlar akademiyasining V.I. Romonovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi PQ-4387-son Qarori va 2020-yil 7-maydagi “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora tadbirlari to‘g‘risida”gi PQ-4708-sonli Qarori hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnalogiyalar rivojlanishining ustuvor yo‘nalishlariga bog‘liqligi. Ushbu tadqiqot O‘zbekiston Respublikasi fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarildi.

Muammoning o‘rganilganlik darajasi. Chegaraviy masalalarining matematik nazariyasi o‘tgan asrda rivojlana boshlagan. Ilmiy markazlar va oliy o‘quv yurtlarida ikkinchi va yuqori juft tartibli xususiy hosilali differensial tenglamalar, shuningdek, ikkinchi va yuqori juft tartibli aralash turdagi turli xil tenglamalar uchun turli chegaraviy masalalarni o‘rganish bo‘yicha tadqiqotlar olib borilmoqda. Xususiy hosilali differensial tenglamalar fan va texnikada bir nechta parametrlarni, shuningdek, o'zgaruvchilarning boshqalariga nisbatan o'zgarishini o'z ichiga olgan real dunyoni qo'llash masalalarini modellashtirish uchun ishlatiladi. Ushbu masalalarning aksariyati chegaraviy masalalarni yechishni talab qiladi, ya'ni xususiy hosilali differensial tenglamalarni yechish muayyan shartlar va ma'lumotlarni qanoatlantirishga majbur bo'ladi. Biroq, real dunyo masalalarining ko'pchiligini modellashtirish juda murakkab vazifa va ko'p shakllarga ega. Aniq yechimni topish ham qiyin. Dunyo miqyosida olib borilayotgan ko’pgina ilimiy-amaliy tadqiqotlar juft tartibli, xususiy hosilali tenglamalar uchun boshlang’ich-chegaraviy masalalarning muhim ekanligini ko’rsatmoqda. Amaliy tadbiqga doir masalalarni tadqiq qilish jarayonida, juft tartibli xususiy hosilali differensial tenglamalar uchun boshlang’ich masalalar eng ustuvor yo’nalish hisoblanadi. Chegaraviy masalalar yechimlarining muhimligi, uning tovush tezligigacha va tovush tezligi atrofidagi gaz dinamikasi, magnit gidrodinamika sohalaridagi jarayonlarning masalariga tadbig’i bilan tushuntiriladi. Juft tartibli tenglamalar bilan bog’liq murakkab jarayonlar va analitik usullarning mavjud emasligi, juft tartibli xususiy hosilali differensial tenglamalarni tadqiqini dolzarb yo’nalishlardan biriga olib chiqdi.

Dunyo miqyosida yuqori juft tartibli xususiy hosilai tenglamalar uchun boshlang’ich va chegaraviy masalalar muhim rol o’ynaydi. Yuqori juft tartibli, xususuy hosilali tenglamalar uchun chegaraviy masalalarni to’gri qo’yish qo’yilgan masalaning bir qiymatli yechilishini aniqlashni talab qiladi. Yuqori tartibli hosilalarni o'z ichiga olgan chegara shartlariga ega Laplas, Puasson va Helmgolts tenglamalari uchun chegaraviy masalalari Bavrin, Karachik, Sokolovskiy asarlarida o'rganilgan. Birinchi bo’lib soha chegarasining qismiga qo’yilgan yuqori tartibli masalani A.N. Tixonov o’rgangan. U ishda bir jinsli issiqlik tarqalish tenglamasini

$$\sum_{k=0}^{m} a_k \frac{\partial^k u}{\partial x^k}(0,t) = f(t),\ u(x,0) = 0,$$

shart bilan $(0 < x < \infty, t > 0)$ sohada o'rgangan. A.V. Bitsadze Laplas tenglamasini $\frac{d^m u}{dv^m} = f(x), x \in \partial D$ sharti bilan $n-$o'lchovli chegaralangan D sohada tadqiq qilgan va uning fredgolm bo'lishini isbotlagan. Yuqori tartibli hosilaga ega cheraviy shart bilan berilgan Laplas Puasson va Gelmgolts cheraviy masalalari I.I. Bavrin, V.V. Karachik va B.X. Turmetov, V.V. Karachik, V.B. Sokolovskiy va bir nechta ishlarda o'rganilgan. Quyida biz keltirilgan masalalar uchun Fury'e usulini qo'llaymiz. Normal sohalarda, umumiy bo'lgan chiziqli giperbolik va parabolik tenglamalar uchun Fury'e usulining asosli bo'lishi V.A. Ilin tomonidan ko'rsatilgan.

Hozirgi vaqtda geofizika, okeanlogiya, atmosfera fizikasi muammolari va boshqa bir qator muammolardan kelib chiqgan holda turli birjinsli bo'lmagan suyuqliklar dinamikasiga qiziqish oshgan bo'lib, ushbu tenglamalar to'rtinchi tartibli xususiy hosilali tenglamalar uchun boshlang'ich-chegaraviy masalalarga kelib qoladi. To'sinning ko'ndalang tebranishi, to'rtinchi tartibli tenglamani o'rganishga olib keladi. Fizika va texnikaning ko'pgina muammolarini yechishga harakat qilinganda to'rtinchi va yuqori tartibli xususiy xosilali tenglamalar uchun chegaraviy masalarni tadqiq qilish o'ta ahamiyatli ekani yuzaga chiqadi. Shunga o'xshash tadqiqotlarni aralash tipdagi tenglamalar uchun muhim ekanligini A.V. Bitsadze qayd etgan. U tomonidan model tenglama sifatida to'rtinchi tartibli bir jinsli tenglamani taklif qilgan. M.M. Smironov, M.M. Meredov boshqa bir qator olimlar tomonidan to'rtinchi tartibli tenglamalar o'rganilgan. K. B. Sabitov., Zhao H., Zhu C., Yu Z mualliflar tomonidan to'rtinchi tartibli chiziqli va nochiziqli tenglamalar uchun aralash masalalar o'rganilgan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta'lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog'liqligi.

Dissertatsiya tadqiqoti V.I. Romanovskiy nomidagi matematika institutida O'zbekiston Respublikasi Innovatsion rivojlanish vazirligining F-FA-2021-424– sonli ilmiy tadqiqot grantining rejalashtirilgan mavzusiga muvofiq amalga oshirildi.

Tadqiqotning maqsadi berilgan sohada juft tartibli xususiy hosilali differensial tenglama uchun qo'yilgan, boshlang'ich shartida yuqori tartibli hosilalar qatnashgan chegaraviy masalaning yechimi mavjudligi va yagonaligini isbotlashdan iborat.

Tadqiqotning vazifalari:

issiqlik tarqalish va to'sin tebranishi tenglamalari uchun boshlangich shartlarida yuqori tartibli hosilalar bo'lgan chegaraviy masalaning korrektligini isbot qilish;

to'g'ri to'rtburchak sohada, parabolik va elliptik tenglamalar uchun qo'yilgan boshlang'ich shartida vaqt bo'yicha yuqori tartibli hosilalar qatnashgan chegaraviy masalalarning yechimini topish;

juft tartibli xususiy hosilali differensial tenglama uchun chegaraviy masalalarning regulyar yechimlarini baholash;

berilgan sohada parabolik va elliptik tenglamalarning yechimi mavjudligi Furye usuli bilan yagonaligi esa spektral usul yordamida isbotlash.

Tadqiqotning ob'yekti. Boshlang'ich shartida yuqori tartibli hosilalar qatnashgan to'rtinchi tartibli parabolik va elliptik tipdagi tenglamalar.

Tadqiqotning predmeti. Parabolik va gipoelliptik tipdagi tenglamalar uchun chegaraviy va boshlang'ich shartida yuqori tartibli hosila qatnashgan masalalari.

Tadqiqotning usullari. Dissertatsiya ishida matematik fizikaning zamonaviy usullari, Furye usuli va Furye qatorlari nazariyalaridan keng foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

boshlang'ich shartida k-tartibli hosila qatnashgan parabolik va elliptik tenglamalar uchun chegaraviy masalalarning yechimi, k sonining juft yoki toqligiga bog'liqligi ko'rsatilgan;

to'g'ri to'rtburchak sohada, boshlang'ich shartida yuqori tartibli hosilalar bo'lgan to'rtinchi tartibli tenglamalar uchun qo'yilgan chegaraviy masalalarning regulyar yechimi mavjudligi va yagonaligi isbotlangan;

boshlang'ich shartida yuqori tartibli hosilalar qatnashgan to'rtinchi tartibli xususiy hosilali differensial tenglamalarga qo'yilgan chegaraviy masalalar uchun rekurent formulalar tuzilgan;

berilgan sohada parabolik va elliptik tenglamalarning yechimi mavjudligi Furye usuli bilan yagonaligi esa spektral usul yordamida isbotlangan.

Tadqiqotning amaliy natijalari. Dissertatsiya ishida olingan natijalar va qo'llanilgan usullar oliy o'quv yurtlari magistrantlari va doktorantlari uchun magistratura bosqichi sifatida o'qitilishi mumkin. Bundan tashqari, parabolik, gipoelliptik va parabolik tipdagi tenglamalar bo'yicha dissertatsiya natijalari boshqa turdagi tenglamalar uchun qo'llash imkonini beradi.

Tadqiqot natijalarining ishonchliligi. Natijalar matematik fizikaning zamonaviy usullari, matematik tahlil usullaridan foydalanilgan holda olingan. Olingan natijalarning isbotlari matematik jihatdan to'g'ri.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati shundan iboratki, berilgan to'g'ri to'rtburchak sohada Dirixli yoki boshlang'ich shartlari bilan berilgan juft tartibli xususiy hosilali differensial tenglamalar uchun chagaraviy masalalarni yechimini topish usulini va boshlang'ich shartida yuqori tartibli hosilalar bilan berilgan masalalarda foydalanilganligi bilan izohlanadi.

Tadqiqotning amaliy ahamiyati esa magistrlarni tayyorlashda hamda boshlang'ich shartida yuqori tartibli hosila qatnashgan, ikkinchi va to'rtinchi tartibli tenglamalarga qo'yilgan lokal va nolokal chegaraviy masalalarni matematik modellashtirishda qo'llash imkoni bilan belgilaniladi.

Tadqiqot natijalarining joriy qilinishi. Juft tartibli xususiy hosilali differensial tenglamalar uchun boshlang'ich va chegaraviy masalalar bo'yicha olingan natijalar asosida:

boshlang'ich shartida yuqori tartibli hosilalar qatnashgan juft tartibli xususiy hosilali differensial tenglamalar uchun chegaraviy masalaning regulyar yechimning mavjudligidan С-ОКП-17 raqamli "Takomillashtirilgan 138 arrali jin ishchi kamerasini ishlab chiqish" mavzusidagi amaliy grant loyihasida juft tartibli differensial operatorlar uchun chegaraviy masalalarning korrektligini tekshirishda

foydalanilgan (Mexanika va inshootlar seysmik mustahkamligi institutining 2023 yil 5-iyundagi №422-3-sonli ma'lumotnomasi). Ilmiy natijalarning qo'llanishi differensial operatorlar uchun chegaraviy masalalarning yagona yechimi mavjudligini isbotlash imkonini bergan;

to'rtinchi tartibli xususiy hosilali parabolik va elliptik tipdagi tenglamalar uchun boshlang'ich shartida k-va k+1- tartibli hosilalar qatnashgan chegaraviy masalalarning yechimlaridan Rossiya Federatsiyasi Prezidentining № МД-758.2022.1.1 raqamli loyiha doirasida dinamik jarayonlarni o'rganishda foydalanilgan (Kamchatka davlat universitetining 2023 yil 24-yanvardagi №02-12-sonli ma'lumotnomasi). Ilmiy natijalarning loyiha ishida qo'llanilishi matematik fizika tenglamalari uchun qo'yilgan chegaraviy masalalarning regulyar yechimini topish, yechimdagi qatorlarning absolyut va tekis yaqinlashishini isbotlash imkonini bergan.

Tadqiqot natijalarini aprobatsiyasi. Mazkur tadqiqot natijalari 2 ta xalqaro va 4 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o'tkazilgan.

Tadqiqot natijalarining e'lon qilinganligi. Dissirtatsiya mavzusi bo'yicha jami 8 ta ilmiy ish chop etilgan, shulardan, O'zbekiston Respublikasi Oliy attestatsiya komissiyasining falsafa doktori dissertatsiyalarini asosiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 6 ta maqola, jumladan, 2 tasi xorijiy va 4 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzulishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan tashkil topgan. Dissertatsiyaning hajmi 93 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurligi asoslangan, tadqiqotning respublika fan va texnologiyasini rivojlantirishning ustuvor yo'nalishlariga mosligi ko'rsatilgan, xorijiy va mahalliy ilmiy tadqiqotlar haqida umumiy ma'lumot berilgan, muammoning o'rganilganlik darajasi, maqsad va vazifalari belgilangan, tadqiqotning ob'yekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari ochib berilgan. Olingan natijalarning nazariy va amaliy ahamiyati bayon etilgan, ilmiy-tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzulishi bo'yicha ma'lumotlar keltirilgan.

Dissertatsiyaning "$u_t + u_{xxxx} = f(x,t)$ **tenglama uchun boshlang'ich-chegaraviy masalalar**" deb nomlangan birinchi bobida $u_t + u_{xxxx} = f(x,t)$ tenglama uchun boshlang'ich-chegaraviy masala qaralgan.

Birinchi paragrafda chegaraviy masalalarni o'rganishda foydalaniladigan asosiy tushunchalar va ba'zi yordamchi formulalar keltirilgan.

Ikkinchi paragrafda boshlang'ich shartida yuqori tartibli hosila qatnashgan, parabolik tenglama uchun lokal chegaraviy masalasi ko'rib chiqiladi.

Uchinchi paragrafda $u_t + u_{xxxx} = f(x,t)$ tenglama uchun nolokal chegaraviy masalasi o'rganildi.

Dissertatsiyaning ikkinchi bobi **“Boshlang’ich shartlarida vaqt bo’yicha yuqori tartibli hosilalar bo’lgan nostasionar tenglamalar uchun lokal va nolokal chegaraviy masalalar”** deb nomlangan.

Birinchi paragrafda to‘g‘ri to‘rtburchak sohada issiqlik tarqalish tenglamasi uchun boshlang’ich shartida yuqori tartibli hosila qatnashgan nolokal chegaraviy masala tadqiq qilingan.

Ikkinchi paragrafda to’sin tebranish tenglamasi uchun boshlang'ich-chegaraviy masalasi o'rganilgan. Bu to’rtinchi tartibli tenglama mexanik masalalarni yechishda zarur tenglamalardan biri hisoblanadi. Bu paragrafda qaralgan masala boshlang’ich shartida tenglama tartibidan oshib ketadigan t ga nisbatan yuqori tartibli hosilalar berilgan.

Berilgan $\Omega = \{(x,t): 0 < x < p,\ 0 < t < T\}$ to’g’ri to’rtburchakda

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial x^4} = f(x,t) \tag{1}$$

to’sin tebranish tenglamasini

$$u(0,t) = 0,\ \ u(p,t) = 0,\quad 0 \le t \le T, \tag{2}$$

$$u_{xx}(0,t) = 0,\ u_{xx}(p,t) = 0,\ \ 0 \le t \le T, \tag{3}$$

chegaraviy shartlar va vaqt bo’yicha yuqori tartibli hosila qatnashgan

$$\frac{\partial^k u}{\partial t^k}(x,0) = \varphi_k(x),\quad 0 \le x \le p, \tag{4}$$

$$\frac{\partial^{k+1} u}{\partial t^{k+1}}(x,0) = \psi_k(x),\quad 0 \le x \le p, \tag{5}$$

boshlang’ich shartlar bilan ko’rib chiqamiz.

Bu yerda $f(x,t)-$funksiya $\overline{\Omega}$ da uzluksiz funksiya va $1 \le k -$tayinlangan natural son.

1-Masala. (1) tenglamaning Ω sohada, (2)-(5) shartlarni qanoatlantiruvchi $u(x,t)$ yechimini toping. Masala yechimining yagonaligi spektral usul bilan isbotlangan.

1-teorema. (1)-(5) masalaning yechimi yagona bo’ladi, agar u mavjud bo’lsa.

Furye usulidan foydalanib, 1-masalaning yechimini ushbu

$$u(x,t) = \sum_{n=1}^{\infty} X_n(x)\left\{ \frac{\varphi_{kn}}{\left(\lambda_n^2\right)^k}\cos\left(\frac{\pi k}{2} - \lambda_n^2 t\right) - \frac{\psi_{kn}}{\left(\lambda_n^2\right)^{k+1}}\sin\left(\frac{\pi k}{2} - \lambda_n^2 t\right) - \right.$$

$$- \sum_{s=0}^{\left[\frac{k}{2}\right]-1} (-1)^s \frac{f_n^{(k-2-2s)}(0)}{\left(\lambda_n^2\right)^{k-2s}}\cos\left(\frac{\pi k}{2} - \lambda_n^2 t\right) +$$

$$\left. + \sum_{s=0}^{\left[\frac{k+1}{2}\right]-1} (-1)^s \frac{f_n^{(k-1-2s)}(0)}{\left(\lambda_n^2\right)^{k+1-2s}}\sin\left(\frac{\pi k}{2} - \lambda_n^2 t\right) + \frac{1}{\lambda_n^2}\int_0^t f_n(\tau)\sin\lambda_n^2(t-\tau)\,d\tau \right\} \tag{6}$$

ko’rinishda olamiz.

2-teorema. Agar

$$f(x,t)\in W_2^{(2k+1,k-1)}(\Omega),\ \varphi_k(x)\in W_2^3(0,p),\ \psi_k(x)\in W_2^1(0,p),$$

$$\varphi_k(0)=\varphi_k(p)=0,\ \varphi_k''(0)=\varphi_k''(p)=0,\ \psi_k(0)=\psi_k(p)=0,$$

$$\frac{\partial^{2l}f(0,t)}{\partial x^{2l}}=\frac{\partial^{2l}f(p,t)}{\partial x^{2l}}=0,\ l=0,1,...,(k),$$

$$\frac{\partial^{2l+1}f(0,t)}{\partial t\partial x^{2l}}=\frac{\partial^{2l+1}f(p,t)}{\partial t\partial x^{2l}}=0,\ l=0,1,...,(k-1),$$

$$\frac{\partial^{2l+2}f(0,t)}{\partial t^2\partial x^{2l}}=\frac{\partial^{2l+2}f(p,t)}{\partial t^2\partial x^{2l}}=0,\ l=0,1,...,(k-3)$$

shartlar bajarilsa, (6) qator $\overline{\Omega}$ sohada absolyut va tekis yaqinlashuvchi bo'ladi. (6) funksional qator bilan aniqlangan yechim, (1) tenglamani qanoatlantiradi va (2)-(5) shartlar bajariladi.

Dissertatsiyaning **"To'rtinchi tartibli tenglama uchun lokal va nolokal chegaraviy masalalar"** deb nomlangan uchinchi bobida chekli sohada to'rtinchi tartibli tenglama uchun lokal va nolokal masalalar o'rganilgan. Bu masalalarning boshlang'ich shartlarida tenglama tartibidan oshib ketadigan bitta o'zgaruvchiga nisbatan yuqori tartibli hosilalar berilgan. Har bir masala yechimining mavjudligi va yagonaligi haqidagi teoremalar isbotlangan.

Yuqori tartibli xususiy hosilali differensial tenglamalarning ko'p turlari uchun chegaraviy masalalar ko'plab mualliflarning ishlarida o'rganilgan.

Uchinchi bobning birinchi paragrafida to'rtinchi tartibli tenglama uchun chegaraviy masala o'rganilgan.

2-masala. Berilgan chekli $\Omega=\{(x;\ y):0\ <x<\ p,\ 0<\ y<T\}$ to'g'ri to'rtburchak sohada

$$\frac{\partial^2 u}{\partial y^2}=\frac{\partial^4 u}{\partial x^4}+f(x,y) \tag{7}$$

to'rtinchi tartibli tenglamaning

$$u(0,y)=u(p,y)=0,\quad 0\le y\le T, \tag{8}$$

$$u_{xx}(0,y)=u_{xx}(p,y)=0,\quad 0\le y\le T, \tag{9}$$

chegaraviy shartlar va

$$\frac{\partial^k u}{\partial y^k}(x,0)=\varphi(x),\ 0\le x\le p, \tag{10}$$

$$\frac{\partial^k u}{\partial y^k}(x,T)=\psi(x),\ 0\le x\le p, \tag{11}$$

boshlang'ich shartlarni qanoatlantiruvchi $u(x,y)$ yechimi topilsin.

Bu yerda $f(x,y),\ \varphi(x),\ \psi(x)-$funksiyalar $\overline{\Omega}$ sohada uzluksiz, $2\le k-$ tayinlangan natural son.

3-teorema. Agar (7)-(11) masalaning yechimi mavjud bo'lsa, u yechim yagona bo'ladi. Teorema spektral usul yordamida isbot qilinadi.

Furye usulidan foydalanib, 2-masalaning yechimini

$$u(x,y)=\sum_{n=1}^{\infty}X_n(x)\left[-\frac{e^{-\lambda_n^2(T-y)}-(-1)^k e^{\lambda_n^2(T-y)}}{\left(\lambda_n^2\right)^k\left(e^{\lambda_n^2T}-e^{-\lambda_n^2T}\right)}\cdot\varphi_n+\frac{e^{\lambda_n^2y}-(-1)^k e^{-\lambda_n^2y}}{\left(\lambda_n^2\right)^k\left(e^{\lambda_n^2T}-e^{-\lambda_n^2T}\right)}\cdot\psi_n-\right.$$

$$-\frac{e^{\lambda_n^2y}-(-1)^k e^{-\lambda_n^2y}}{\left(\lambda_n^2\right)^k\left(e^{\lambda_n^2T}-e^{-\lambda_n^2T}\right)}\sum_{s=0}^{\left[\frac{k+(-1)^k}{2}\right]-1}\lambda_n^{4s}f_n^{(k-2-2s)}(T)+$$

$$\left.+\frac{e^{-\lambda_n^2(T-y)}-(-1)^k e^{\lambda_n^2(T-y)}}{\left(\lambda_n^2\right)^k\left(e^{\lambda_n^2T}-e^{-\lambda_n^2T}\right)}\sum_{s=0}^{\left[\frac{k+(-1)^k}{2}\right]-1}\lambda_n^{4s}f_n^{(k-2-2s)}(0)-\frac{1}{\lambda_n^2}\int_0^T K_{1n}(y,\tau)f_n(\tau)d\tau\right] \quad (12)$$

ko'rinishda topamiz. Bu yerda,
agar k-juft son bo'lsa,

$$K_{1n}(y,\tau)=\begin{cases}\dfrac{sh\lambda_n^2\tau\cdot sh\lambda_n^2(T-y)}{sh\lambda_n^2T},\ 0\le\tau\le y,\\ \dfrac{sh\lambda_n^2y\cdot sh\lambda_n^2(T-\tau)}{sh\lambda_n^2T},\ y\le\tau\le T,\end{cases}$$

va k-toq son bo'lsa,

$$K_{1n}(y,\tau)=\begin{cases}\dfrac{ch\lambda_n^2\tau\cdot ch\lambda_n^2(T-y)}{sh\lambda_n^2T},\ 0\le\tau\le y,\\ \dfrac{ch\lambda_n^2y\cdot ch\lambda_n^2(T-\tau)}{sh\lambda_n^2T},\ y\le\tau\le T.\end{cases}$$

4-teorema. Agar $f(x,y)\in W_2^{(2k-1,k-2)}(\overline{\Omega})$, $\varphi(x)\in W_2^1(0,p)$, $\psi(x)\in W_2^1(0,p)$,

$$\varphi(0)=\varphi(p)=0,\ \psi(0)=\psi(p)=0,$$

$$\frac{\partial^{2l}f(0,y)}{\partial x^{2l}}=\frac{\partial^{2l}f(p,y)}{\partial x^{2l}}=0,\ l=0,1,...,(k-1),$$

$$\frac{\partial^{2l+1}f(0,y)}{\partial y\,\partial x^{2l}}=\frac{\partial^{2l+1}f(p,y)}{\partial y\,\partial x^{2l}}=0,\ l=0,1,...,(k-3),$$

shartlar bajarilsa, u holda (12) qatorlar $\overline{\Omega}$ sohada tekis va absolyut yaqinlashadi, (12) qator bilan aniqlangan $u(x,y)$ funksiya (7) tenglamani va (8)-(11) shartlarni qanoatlantiradi.

Ikkinchi paragrafda berilgan chekli $\Omega=\{(x,y):0<x<p,\ 0<y<q\}$ sohada biz

$$\frac{\partial^2 u}{\partial y^2} - \frac{\partial^4 u}{\partial x^4} = f(x,y) \tag{13}$$

differensial tenglamani qaraymiz, bu yerda $f(x,y)-$funksiya $\overline{\Omega}$ sohada berilgan uzluksiz funksiya.

3-masala. (13) tenglamaning

$$u(0,y)=0,\ \ u(p,y)=0,\quad 0\le y\le q, \tag{14}$$

$$u_{xx}(0,y)=0,\ u_{xx}(p,y)=0,\ \ 0\le y\le q, \tag{15}$$

chegaraviy va

$$\left.\frac{\partial^k u}{\partial y^k}\right|_{y=0} = \left.\frac{\partial^k u}{\partial y^k}\right|_{y=q}, \qquad 0\le x\le p, \tag{16}$$

$$\left.\frac{\partial^{k+1} u}{\partial y^{k+1}}\right|_{y=0} = \left.\frac{\partial^{k+1} u}{\partial y^{k+1}}\right|_{y=q}, \qquad 0\le x\le p, \tag{17}$$

boshlang'ich shartlarni qanoatlantiruvchi $u(x,y)$ yechimini toping. Bu yerda $1\le k-$ tayinlangan natural son.

5-teorema. Agar (13)-(17) masalaning yechimi mavjud bo'lsa, u holda bu yechim yagona bo'ladi. Teorema spektral usuli qo'llanilib isbotlangan.

Uchinchi masalaning yechimini, o'zgaruvchilarni ajratish usuli yordamida ushbu

$$u(x,y)=\sum_{n=1}^{\infty} X_n(x)\Bigg[K_{0n}(y)\sum_{s=0}^{\left[\frac{k-2}{2}\right]}\lambda_n^{4s-2k}\left(f_n^{(k-2-2s)}(q)-f_n^{(k-2-2s)}(0)\right)+$$

$$+M_{0n}(y)\sum_{s=0}^{\left[\frac{k-2}{2}\right]}\lambda_n^{4s-2k-2}\left(f_n^{(k-1-2s)}(q)-f_n^{(k-1-2s)}(0)\right)+$$

$$+M_{0n}(y)\frac{H_k}{\lambda_n^4}\left(f_n(q)-f_n(0)\right)+\frac{1}{2\lambda_n^2}\int_0^q K_{1n}(y,\tau)f_n(\tau)d\tau\Bigg] \tag{18}$$

ko'rinishda olamiz.

Bu yerda, agar $k-$ juft son bo'lsa,

$$K_{0n}(y)=\frac{ch\lambda_n^2 y - ch\lambda_n^2(y-q)}{2(1-ch\lambda_n^2 q)},\ \ M_{0n}(y)=\frac{sh\lambda_n^2 y - sh\lambda_n^2(y-q)}{2(1-ch\lambda_n^2 q)},$$

$k-$ toq son bo'lsa,

$$K_{0n}(y)=\frac{sh\lambda_n^2 y - sh\lambda_n^2(y-q)}{2(1-ch\lambda_n^2 q)},\ \ M_{0n}(y)=\frac{ch\lambda_n^2 y - ch\lambda_n^2(y-q)}{2(1-ch\lambda_n^2 q)}.$$

$$K_{1n}(y,\tau)=\begin{cases}\dfrac{sh\lambda_n^2(q-y+\tau)+sh\lambda_n^2(y-\tau)}{(1-ch\lambda_n^2 q)}, & 0<\tau<y,\\ \dfrac{sh\lambda_n^2(q+y-\tau)-sh\lambda_n^2(y-\tau)}{(1-ch\lambda_n^2 q)}, & y<\tau<q.\end{cases}$$

6-teorema. Agar $f(x,y)\in W_2^{(2k+1,k-1)}(\Omega)$,

$$\frac{\partial^{2l}f(0,y)}{\partial x^{2l}}=\frac{\partial^{2l}f(p,y)}{\partial x^{2l}}=0,\ l=0,1,...,k,$$

$$\frac{\partial^{2l+1}f(0,y)}{\partial y\partial x^{2l}}=\frac{\partial^{2l+1}f(p,y)}{\partial y\partial x^{2l}}=0,\ l=0,1,...,k-1,$$

$$\frac{\partial^{2l+2}f(0,y)}{\partial y^2\partial x^{2l}}=\frac{\partial^{2l+2}f(p,y)}{\partial y^2\partial x^{2l}}=0,\ l=0,1,...,k-3,$$

shartlar o'rinli bo'lsa, u holda (18) qator $\overline{\Omega}$ sohada absolyut va tekis yaqinlashuvchi bo'ladi. (18) yechim (13) tenglamani va (14)-(17) shartlarni qanoatlantiradi.

XULOSA

- Birinchi bobning birinchi paragrafida asosiy tushunchalar va ba'zi formulalar keltirilgan. Ikkinchi paragrafda parabolik tenglama uchun chegaraviy lokal masala yechilgan. Masala yechimi mavjud va yagona ekanligi isbot qilingan. Uchinchi paragrafda to'rtinchi tartibli xususiy hosilali differensial tenglama uchun boshlang'ich shartida k-tartibli hosila qatnashgan chegaraviy masala o'rganilgan. Masala yechimining mavjudligi va yagonaligi haqidagi teoremalar isbotlagan.
- Ikkinchi bobning birinchi paragrafida issiqlik tarqalish tenglamasi uchun nolokal chegaraviy masala yechilgan, masala yechimining mavjudligi va yagonaligi isbot qilingan. Ikkinchi paragrafda mexanianing asosiy tenglamalaridan biri bo'lgan, to'rtinchi tartibli to'sin tebranish tenglamasi uchun boshlang'ich-chegaraviy masala yechilgan, masala yechimining mavjudligi va yagonaligi isbotlangan.
- Uchinchi bobda, to'rtinchi tartibli xususiy hosilali differensial tenglama uchun boshlang'ich shartida yuqori tartibli hosila qatnashgan lokal va nolokal chegaraviy masalalar o'rganilgan. O'rganilgan masalalarning yechimi mavjudligi haqidagi teoremalar Furye usuli bilan, yagonaligi haqidagi teoremalar esa spektral usul bilan isbot qilingan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED AFTER V.I. ROMANOVSKIY

INSTITUTE OF MATHEMATICS

KILICHOV OYBEK SHARAFIDDINOVICH

BOUNDARY VALUE PROBLEMS FOR PARTIAL DERIVATIVE EQUATIONS OF EVEN ORDER

01.01.02-Differential equations and mathematical physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT-2023

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INTRODUCTION

Actuality and demand of the theme of the thesis. Many scientific and practical studies carried out on a global scale show the importance of studying initial-boundary problems for differential equations with partial derivative of even order. In the process of studying problems of a practical nature, boundary value problems for even-order partial differential equations are one of the priority directions. The importance of solving initial-boundary problems is explained by their application to problems of many processes in the fields of gas dynamics and magnetohydrodynamics. Due to the needs of practice, the attention to the theory of even-order equations has increased. Due to the complexity of processes related to even-order equations and the lack of analytical methods, the study of even-order partial differential equations is one of the priority directions.

Currently, one of the urgent problems of mathematical physics is the boundary problem. In recent years, initial and boundary value problems for even-order partial differential equations have been rapidly developing, as they are closely related to practical applications. Such problems are brought to the solution of parabolic-type equations in an arbitrary bounded area. Nowadays, the study of initial and boundary value problems for high-order partial differential equations on a global scale plays an important role. For differential equations with higher even-order partial derivatives, the correct formulation of the initial and boundary value problems requires the establishment of a unique solution to the problem. Taking into account the above, we emphasize that the implementation of targeted scientific research is becoming an important task.

In our country much attention has been paid to mathematics, physics and optimal management, which have scientific and practical application of fundamental sciences. In particular, special attention was paid to the research of game theory and initial-boundary problems. Excellent results have been achieved on the theory of control of processes characterized by ordinary differential equations, differential equations with particular derivatives and their systems, ordinary differential equations and their systems. "The main tasks and areas of activity are to conduct scientific research at the level of international standards in functional analysis, differential equations and the priority areas of mathematical physics". It is important to develop boundary and boundary problems in solving problems of differential equations and mathematical physics in ensuring the execution of decisions.

The dissertation research serves to a certain extent in the implementation of tasks defined in.Decree No. PF-4947 of the President of the Republic of Uzbekistan dated February 7, 2017 "On the strategy of actions for the further development of the Republic of Uzbekistan", July 9, 2019 "State support for the further development of mathematics education and sciences, as well as the Republic of Uzbekistan Academy of Sciences Decision No. PQ-4387 "On measures to fundamentally improve the activities of the Institute of Mathematics named after V.I. Romonovsky" and Decision No. PQ-4708 of May 7, 2020 "On measures to improve the quality of education and development of scientific research in the field of mathematics" and other normative legal documents related to this activity.

Dependence of the research on the priority directions of the republic's science and technology development. This study is part of IV. of Science and Technology Development of the Republic of Uzbekistan. It was carried out within the priority direction "Mathematics, mechanics and informatics".

Level of study of the problem. Mathematical theory of boundary value problems began to develop in the last century. In scientific centers and institutions of higher education, research is being conducted on the study of various boundary-value problems for differential equations with second and higher even order partial derivatives, as well as on various equations of mixed types of second and higher even order. Partial differential equations are used in science and engineering to model real-world application problems that involve multiple parameters as well as the change of variables with respect to others. Most of these problems require the solution of boundary value problems, that is, the solution of partial differential equations is forced to satisfy certain conditions and data. However, modeling most real-world problems is a complex task and takes many forms. It's hard to find an exact solution. Many scientific-practical studies show the importance of initial-boundary problems for even-order partial differential equations. In the process of researching the problems of practical application, elementary problems for partial differential equations of even order are considered the most priority. The importance of solutions of boundary problems is explained by its application to problems of gas dynamics up to and around the speed of sound, processes in the fields of magnetohydrodynamics. The non-availability of complex processes and analytical methods related to even-order equations has led the study of even-order partial differential equations to one of the current directions.

Initial and boundary problems play an important role for higher even-order partial differential equations. The correct setting of the boundary value problems for partial differential equations of higher even order requires the determination of the single-valued solution of the given problem. Boundary problems for Laplace, Poisson and Helmholtz equations with boundary conditions containing higher-order derivatives are studied in the works of Bavrin, Karachik, Sokolovsky. Firstly A.N. Tikhonov studied a high-order problem on the part of the domain boundary. In his works he studied equations of homogeneous heat distribution

$$\sum_{k=0}^{m} a_k \frac{\partial^k u}{\partial x^k}(0,t) = f(t),$$

with the condition $u(x,0)=0$ in the domain $(0<x<\infty, t>0)$. Bitsadze studied Laplace's equation $\frac{d^m u}{dv^m} = f(x)$ with the condition $x \in \partial D$ in $n-$dimensional bounded domain D and proved that it is Fredholm. Laplace-Poisson and Helmholtz boundary problems with a boundary condition containing higher-order derivative are studied in the works of I.I. Bavrin, V.V. Karachik and B.Kh. Turmetov, V.V. Karachik, V.B. Sokolovsky and others. Below we use the Fourier method for the

given problems. In normal domains, V.A. Ilin showed that Foure's method for general linear hyperbolic and parabolic equations is valid.

Currently, due to the problems of geophysics, oceanology, atmospheric physics and a number of other problems, interest in the dynamics of various inhomogeneous fluids has increased and these kind of equations lead to the initial-boundary value problem of the fourth-order partial differential equations. When trying to solve many problems of physics and technology, it turns out that the research of boundary value problems for equations with fourth and higher order partial derivatives is extremely important. A.V. Bitsadze noted that similar studies are important for mixed-type equations. He proposed a homogeneous equation of the fourth order as a model equation. Fourth-order equations were studied by M.M. Smironov, M.M. Meredov and a number of other scientists. The authors K. B. Sabitov, Zhao H., Zhu C., Yu Z studied mixed problems for linear and nonlinear equations of the fourth order.

The connection of the dissertation research with the research plans of the higher educational institution where the dissertation was completed.

Dissertation research was carried out within the framework of the project (no. F-FA-2021-424) on the topic "Solution of boundary value problems for partial differential equations of integer and fractional order" of the Mathematical Institute named after V.I. Romanovsky.

The purpose of the research is to prove the existence and uniqueness of the solution of the boundary value problem, which is set for the differential equation with double-order particular derivatives, in the initial condition of which higher-order derivatives are involved.

Tasks of the research:

proving the correctness of the boundary value problem with high-order derivatives in the initial conditions for the equations of heat distribution and beam vibration;

finding the solution of boundary value problems involving high-order derivatives in time in the initial condition set for parabolic and elliptic equations in the rectangular area;

evaluation of regular solutions of boundary value problems for differential equations with even-order differential equations;

in the given domain proving the existence of a solution of parabolic and elliptic equations using the Fourier method and the uniqueness of it using the spectral method.

The object of the research. Equations of parabolic and elliptic type of the fourth order with higher derivatives in the initial conditions.

The subject of research. Problems involving higher-order derivatives in boundary and initial conditions for parabolic and hypoelliptic type equations.

Research methods. Modern methods of mathematical physics, Fourier method and theories of Fourier series were widely used in the dissertation work.

The scientific novelty of the research is as follows:

it is shown that the solution of boundary value problems for parabolic and elliptic equations with the k-order derivative in the initial condition depends on whether the number k is even or odd;

theorems about the existence and uniqueness of the regular solution of boundary value problems for fourth-order equations with higher-order derivatives in the initial condition in the rectangular domain are proved;

recurrent formulas for boundary value problems on fourth-order partial differential equations with higher-order derivatives in the initial condition were created;

existence of solutions of parabolic and elliptic equations in the given domain was proved by Fourier method and the uniqueness of it was proved by spectral method.

Practical results of the research. The results obtained in the dissertation work and the methods used can be taught as a master's degree for master's and doctoral students of higher educational institutions. In addition, the results of the dissertation on parabolic, hypoelliptic and parabolic type equations can be applied to other types of equations.

Reliability of research results. The results were obtained using modern methods of mathematical physics, methods of mathematical analysis. The proofs of the obtained results are mathematically correct.

Scientific and practical significance of research results. The scientific significance of the research results lies in the fact that the method of finding the solution of boundary value problems for differential equations with double order differential equations given by Dirichlet or initial conditions in a given rectangular area and by the fact that they are used in problems given by higher order derivatives in the initial condition explained.

The practical importance of the research is determined by the possibility of applying it in mathematical modeling of local and non-local boundary value problems in second and fourth order equations, in which high-order derivatives are involved in the initial conditions.

Implementation of research results. Based on the results obtained for the initial and boundary value problems for even-order partial differential equations:

the existence of a regular solution of the boundary value problem for even-order differential equations with high-order derivatives in the initial condition was used in the research of the correctness of boundary-value problems for even-order differential operators in S-OKP-17 "Development of an improved working chamber of saw gin". (Reference No. 422-3 dated 06/05/2023 of the Institute of Seismic Resistance of Mechanics and Structures). Application of scientific results made it possible to prove the existence of a unique solution of boundary value problems for differential operators;

the solution of the boundary value problems with k- and k+1-order derivatives in the initial conditions for parabolic and elliptic type equations with fourth-order partial derivatives was used in the study of dynamic processes within the framework of the project No. MD-758.2022.1.1 of the President of the Russian Federation.

(Certificate of the Kamchatka State University No.02-12 dated January 24, 2023). The use of scientific results in the project made it possible to find a regular solution of boundary problems for mathematical physics equations, to prove the absolute and smooth convergence of the series in the solution.

Approval of research results. The results of this research were discussed at 2 international and 4 national scientific-practical conferences.

Publication of research results. A total of 8 scientific works were published on the topic of the dissertation, of which 6 articles were published in scientific publications, including 2 foreign and 4 in national journals, in which the main results of Doctor of Philosophy dissertations of the Higher Attestation Commission of the Republic of Uzbekistan were recommended for publication.

Dissertation structure and volume. The dissertation consists of an introduction, three chapters, a conclusion and a list of references. The length of the dissertation is 93 pages.

THE MAIN CONTENT OF THE DISSERTATION

In the introduction, the relevance and necessity of the dissertation topic is based, the compatibility of the research with the priority directions of the development of science and technology of the republic is shown, general information is given about foreign and local scientific research, the level of research of the problem, goals and tasks are determined, the object and subject of the research are described, the research scientific innovation and practical results are disclosed. The theoretical and practical significance of the obtained results is described, information on the implementation of scientific research results, published works and the composition of the dissertation is provided.

The first chapter of the dissertation, entitled "initial-boundary problems for the equation $u_t + u_{xxxx} = f(x,t)$" deals with the initial-boundary problem for the equation $u_t + u_{xxxx} = f(x,t)$.

The first paragraph presents the main concepts and some auxiliary formulas used in the study of boundary problems.

In the second paragraph, the local boundary problem for the parabolic equation, whose initial condition includes a higher-order derivative, is considered.
In the third paragraph, the nonlocal boundary value problem for the equation $u_t + u_{xxxx} = f(x,t)$ was studied.

The second chapter of the thesis is called "Local and nonlocal boundary value problems for non-stationary equations with high-order time derivatives in the initial conditions".

In the first paragraph, a nonlocal boundary value problem with a high-order derivative was studied in the initial condition for the equation of heat diffusion in a rectangular area.

In the second paragraph, the initial-boundary problem for the vibration equation of the beam is studied. This fourth-order equation is one of the necessary

equations for solving mechanical problems. In the initial condition of the problem considered in this paragraph, higher-order derivatives are given compared to which exceed the order of the equation.

In the given rectangle $\Omega = \{(x,t): 0 < x < p,\ 0 < t < T\}$ we will consider the vibration equation of the beam

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial x^4} = f(x,t) \tag{1}$$

with boundary conditions

$$u(0,t) = 0, \quad u(p,t) = 0, \quad 0 \le t \le T, \tag{2}$$

$$u_{xx}(0,t) = 0, \ u_{xx}(p,t) = 0, \quad 0 \le t \le T, \tag{3}$$

and with initial conditions with higher-order derivatives in time

$$\frac{\partial^k u}{\partial t^k}(x,0) = \varphi_k(x), \quad 0 \le x \le p, \tag{4}$$

$$\frac{\partial^{k+1} u}{\partial t^{k+1}}(x,0) = \psi_k(x), \quad 0 \le x \le p. \tag{5}$$

Here, the function $f(x,t)-$ is a continuous function in $\overline{\Omega}$ and $1 \le k -$is an assigned natural number.

Problem 1. Find a solution of equation (1) that $u(x,t)$ in the domain Ω and satisfies conditions (2)-(5). The uniqueness of the regular solution of the problem is proved by the spectral method.

Theorem 1. The solution to problem (1)-(5) is unique, if it exists.

We obtain the solution of the problem 1 using the Fourier method:

$$u(x,t) = \sum_{n=1}^{\infty} X_n(x) \left\{ \frac{\varphi_{kn}}{\left(\lambda_n^2\right)^k} \cos\left(\frac{\pi k}{2} - \lambda_n^2 t\right) - \frac{\psi_{kn}}{\left(\lambda_n^2\right)^{k+1}} \sin\left(\frac{\pi k}{2} - \lambda_n^2 t\right) - \right.$$

$$- \sum_{s=0}^{\left[\frac{k}{2}\right]-1} (-1)^s \frac{f_n^{(k-2-2s)}(0)}{\left(\lambda_n^2\right)^{k-2s}} \cos\left(\frac{\pi k}{2} - \lambda_n^2 t\right) +$$

$$\left. + \sum_{s=0}^{\left[\frac{k+1}{2}\right]-1} (-1)^s \frac{f_n^{(k-1-2s)}(0)}{\left(\lambda_n^2\right)^{k+1-2s}} \sin\left(\frac{\pi k}{2} - \lambda_n^2 t\right) + \frac{1}{\lambda_n^2} \int_0^t f_n(\tau) \sin \lambda_n^2 (t-\tau) d\tau \right\} \tag{6}$$

Theorem 2. If the conditions

$$f(x,t) \in W_2^{(2k+1,k-1)}(\Omega), \ \varphi_k(x) \in W_2^3(0,p), \ \psi_k(x) \in W_2^1(0,p),$$

$$\varphi_k(0)=\varphi_k(p)=0,\ \varphi_k''(0)=\varphi_k''(p)=0,\ \psi_k(0)=\psi_k(p)=0,$$

$$\frac{\partial^{2l}f(0,t)}{\partial x^{2l}}=\frac{\partial^{2l}f(p,t)}{\partial x^{2l}}=0,\ l=0,1,...,(k),$$
$$\frac{\partial^{2l+1}f(0,t)}{\partial t\partial x^{2l}}=\frac{\partial^{2l+1}f(p,t)}{\partial t\partial x^{2l}}=0,\ l=0,1,...,(k-1),$$
$$\frac{\partial^{2l+2}f(0,t)}{\partial t^2\partial x^{2l}}=\frac{\partial^{2l+2}f(p,t)}{\partial t^2\partial x^{2l}}=0,\ l=0,1,...,(k-3)$$

are met, series (6) is absolutely and uniformly convergent in the domain $\overline{\Omega}$. (6) is the solution defined by the functional series, satisfies equation (1), and conditions (2)-(5) are satisfied.

In the third chapter of the dissertation entitled "Local and non-local boundary value problems for the fourth-order equation" local and non-local problems for the fourth-order equation in the finite domain are studied. In the initial conditions of these problems, high-order derivatives with respect to one variable exceeding the order of the equation are given. Theorems about the existence and uniqueness of the solution to each problem are proved.

Boundary value problems for many types of higher-order partial differential equations have been studied in the works of many authors.

In the first paragraph of the third chapter, the boundary problem for the fourth-order equation is studied.

Problem 2. In the given finite rectangular area $\Omega=\{(x;\ y):0\ <x<\ p,\ 0<\ y<T\}$ find the solution $u(x,y)$ of the fourth-order equation

$$\frac{\partial^2 u}{\partial y^2}=\frac{\partial^4 u}{\partial x^4}+f(x,y) \tag{7}$$

with boundary conditions

$$u(0,y)=u(p,y)=0,\quad 0\le y\le T, \tag{8}$$
$$u_{xx}(0,y)=u_{xx}(p,y)=0,\quad 0\le y\le T, \tag{9}$$

and that satisfies the initial conditions

$$\frac{\partial^k u}{\partial y^k}(x,0)=\varphi(x),\ 0\le x\le p, \tag{10}$$
$$\frac{\partial^k u}{\partial y^k}(x,T)=\psi(x),\ 0\le x\le p. \tag{11}$$

Here the functions $f(x,y),\ \varphi(x),\ \psi(x)-$are continuous in the domain $\overline{\Omega}$, and $2\le k-$is assigned natural numbers.

Theorem 3. If there is a solution to problem (7)-(11), then that solution is unique. The theorem is proved using the spectral method.

Solution of problem 2 can be found using the Fourier method

$$u(x,y)=\sum_{n=1}^{\infty}X_n(x)\left[-\frac{e^{-\lambda_n^2(T-y)}-(-1)^k e^{\lambda_n^2(T-y)}}{\left(\lambda_n^2\right)^k\left(e^{\lambda_n^2T}-e^{-\lambda_n^2T}\right)}\cdot\varphi_n+\frac{e^{\lambda_n^2y}-(-1)^k e^{-\lambda_n^2y}}{\left(\lambda_n^2\right)^k\left(e^{\lambda_n^2T}-e^{-\lambda_n^2T}\right)}\cdot\psi_n-\right.$$

$$-\frac{e^{\lambda_n^2y}-(-1)^k e^{-\lambda_n^2y}}{\left(\lambda_n^2\right)^k\left(e^{\lambda_n^2T}-e^{-\lambda_n^2T}\right)}\sum_{s=0}^{\left[\frac{k+(-1)^k}{2}\right]-1}\lambda_n^{4s}f_n^{(k-2-2s)}(T)+$$

$$\left.+\frac{e^{-\lambda_n^2(T-y)}-(-1)^k e^{\lambda_n^2(T-y)}}{\left(\lambda_n^2\right)^k\left(e^{\lambda_n^2T}-e^{-\lambda_n^2T}\right)}\sum_{s=0}^{\left[\frac{k+(-1)^k}{2}\right]-1}\lambda_n^{4s}f_n^{(k-2-2s)}(0)-\frac{1}{\lambda_n^2}\int_0^T K_{1n}(y,\tau)f_n(\tau)d\tau\right] \quad (12)$$

Here if k – is even number

$$K_{1n}(y,\tau)=\begin{cases}\dfrac{sh\lambda_n^2\tau\cdot sh\lambda_n^2(T-y)}{sh\lambda_n^2T},\ 0\le\tau\le y,\\ \dfrac{sh\lambda_n^2y\cdot sh\lambda_n^2(T-\tau)}{sh\lambda_n^2T},\ y\le\tau\le T,\end{cases}$$

and if k is odd number

$$K_{1n}(y,\tau)=\begin{cases}\dfrac{ch\lambda_n^2\tau\cdot ch\lambda_n^2(T-y)}{sh\lambda_n^2T},\ 0\le\tau\le y,\\ \dfrac{ch\lambda_n^2y\cdot ch\lambda_n^2(T-\tau)}{sh\lambda_n^2T},\ y\le\tau\le T.\end{cases}$$

Theorem 4. If the conditions

$$f(x,y)\in W_2^{(2k-1,k-2)}(\overline{\Omega}),\ \varphi(x)\in W_2^1(0,p),\ \psi(x)\in W_2^1(0,p),$$
$$\varphi(0)=\varphi(p)=0,\ \psi(0)=\psi(p)=0,$$
$$\frac{\partial^{2l}f(0,y)}{\partial x^{2l}}=\frac{\partial^{2l}f(p,y)}{\partial x^{2l}}=0,\ l=0,1,...,(k-1),$$
$$\frac{\partial^{2l+1}f(0,y)}{\partial y\partial x^{2l}}=\frac{\partial^{2l+1}f(p,y)}{\partial y\partial x^{2l}}=0,\ l=0,1,...,(k-3),$$

are met, then the series (12) are smooth and absolutely convergent in the domain $\overline{\Omega}$, the function $u(x,y)$ defined by the series (12) satisfies equation (7) and conditions (8)-(11).

In the finite domain $\Omega=\{(x,y):0<x<p,\ 0<y<q\}$ given in the second paragraph, we consider the differential equation

$$\frac{\partial^2u}{\partial y^2}-\frac{\partial^4u}{\partial x^4}=f(x,y) \quad (13)$$

where the function $f(x,y)-$is a continuous function given on the domain $\overline{\Omega}$.

Problem 3. Find the solution $u(x,y)$ of the equation (13) satisfies the boundary conditions

$$u(0,y)=0,\quad u(p,y)=0,\quad 0\le y\le q, \tag{14}$$

$$u_{xx}(0,y)=0,\ u_{xx}(p,y)=0,\quad 0\le y\le q, \tag{15}$$

and initial conditions

$$\left.\frac{\partial^k u}{\partial y^k}\right|_{y=0}=\left.\frac{\partial^k u}{\partial y^k}\right|_{y=q},\qquad 0\le x\le p, \tag{16}$$

$$\left.\frac{\partial^{k+1} u}{\partial y^{k+1}}\right|_{y=0}=\left.\frac{\partial^{k+1} u}{\partial y^{k+1}}\right|_{y=q},\qquad 0\le x\le p. \tag{17}$$

Here $1\le k$ – assigned natural number.

Theorem 5. If there is a solution to problem (13)-(17), then this solution is unique. The theorem is proved using the spectral method.

We obtain the solution to the third problem using the method of separation of variables

$$u(x,y)=\sum_{n=1}^{\infty}X_n(x)\Bigg[K_{0n}(y)\sum_{s=0}^{\left[\frac{k-2}{2}\right]}\lambda_n^{4s-2k}\left(f_n^{(k-2-2s)}(q)-f_n^{(k-2-2s)}(0)\right)+$$

$$+M_{0n}(y)\sum_{s=0}^{\left[\frac{k-2}{2}\right]}\lambda_n^{4s-2k-2}\left(f_n^{(k-1-2s)}(q)-f_n^{(k-1-2s)}(0)\right)+$$

$$+M_{0n}(y)\frac{H_k}{\lambda_n^4}\left(f_n(q)-f_n(0)\right)+\frac{1}{2\lambda_n^2}\int_0^q K_{1n}(y,\tau)f_n(\tau)d\tau\Bigg] \tag{18}$$

Here, if k – is an even number, then

$$K_{0n}(y)=\frac{ch\lambda_n^2 y-ch\lambda_n^2(y-q)}{2(1-ch\lambda_n^2 q)},\quad M_{0n}(y)=\frac{sh\lambda_n^2 y-sh\lambda_n^2(y-q)}{2(1-ch\lambda_n^2 q)},$$

and if k – is an odd number, then

$$K_{0n}(y)=\frac{sh\lambda_n^2 y-sh\lambda_n^2(y-q)}{2(1-ch\lambda_n^2 q)},\quad M_{0n}(y)=\frac{ch\lambda_n^2 y-ch\lambda_n^2(y-q)}{2(1-ch\lambda_n^2 q)}.$$

$$K_{1n}(y,\tau)=\begin{cases}\dfrac{sh\lambda_n^2(q-y+\tau)+sh\lambda_n^2(y-\tau)}{(1-ch\lambda_n^2 q)}, & 0<\tau<y,\\[2ex] \dfrac{sh\lambda_n^2(q+y-\tau)-sh\lambda_n^2(y-\tau)}{(1-ch\lambda_n^2 q)}, & y<\tau<q.\end{cases}$$

Theorem 6. If the conditions $f(x,y)\in W_2^{(2k+1,k-1)}(\Omega)$,

$$\frac{\partial^{2l}f(0,y)}{\partial x^{2l}}=\frac{\partial^{2l}f(p,y)}{\partial x^{2l}}=0,\ l=0,1,...,k,$$

$$\frac{\partial^{2l+1} f(0,y)}{\partial y \partial x^{2l}} = \frac{\partial^{2l+1} f(p,y)}{\partial y \partial x^{2l}} = 0, \quad l = 0,1,...,k-1,$$

$$\frac{\partial^{2l+2} f(0,y)}{\partial y^2 \partial x^{2l}} = \frac{\partial^{2l+2} f(p,y)}{\partial y^2 \partial x^{2l}} = 0, \quad l = 0,1,...,k-3,$$

are valid, then the series (18) is absolutely and smoothly convergent in the domain $\overline{\Omega}$. The solution (18) satisfies equation (13) and conditions (14)-(17).

CONCLUSION

• The first paragraph of the first chapter presents the main concepts and some formulas. In the second paragraph, the boundary local problem for the parabolic equation is solved. It is proved that the solution of the problem exists and is unique. In the third paragraph, the boundary value problem with the k-order derivative is studied in the initial condition for the fourth-order partial differential equation. Theorems about the existence and uniqueness of the solution to the problem are proved.

• In the first paragraph of the second chapter, the nonlocal boundary value problem for the heat diffusion equation is solved, and the existence and uniqueness of the solution of the problem is proved. In the second paragraph, the initial-boundary problem for the vibration equation of the fourth order beam, which is one of the main equations of mechanics, is solved, and the existence and uniqueness of the solution of the problem is proved.

• In the third chapter, local and non-local boundary value problems with a high-order derivative are studied in the initial condition for a differential equation with a fourth-order particular derivative. Theorems about the existence of a solution to the studied problems were proved by the Fourier method, and the theorems about the uniqueness were proved by the spectral method.

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ИНСТИТУТЕ МАТЕМАТИКИ

КИЛИЧОВ ОЙБЕК ШАРАФИДДИНОВИЧ

КРАЕВЫЕ ЗАДАЧИ ДЛЯ УРАВНЕНИЙ В ЧАСТНЫХ ПРОИЗВОДНЫХ ЧЕТНОГО ПОРЯДКА

01.01.02 – Дифференциальные уравнения и математическая физика

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)
ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

ТАШКЕНТ-2023

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министрстве Высшего образования, Науки и Инноваций Республики Узбекистан за B2022.4. PhD/FM789.

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования доказаны теоремы существования и единственности решения краевых задач для параболических и эллиптических уравнений со высокими производными в начальных условиях.

Объектом исследования уравнения параболического и эллиптического типа четвертого порядка с высшими производными в начальных условиях.

Научная новизна исследования состоит в следующем:

показано решение краевых задач для параболических и эллиптических уравнений с производной k-го порядка в начальных условиях в зависит от того, четно или нечетно число k;

доказаны теоремы существования и единственности регулярного решения краевых задач в прямоугольной области для уравнений четвертого порядка со старшими производными в начальных условиях;

созданы рекуррентные формулы для краевых задач, поставленных для дифференциальных уравнений четвертого порядка со старшими производными в начальных условиях;

существование решений параболических и эллиптических уравнений в заданной области доказано спектральным методом а единственность по методу Фурье.

Внедрение результатов исследования. На основании полученных результатов для начально-краевой задачи для уравнений в частных производных четного порядка:

существование регулярного решения краевой задачи для дифференциальных уравнений двойного порядка со старшими производными в начальных условиях использовалось при исследовании корректности краевых задач для дифференциальных операторов двойного порядка в S-ОКР-№ 17 «Разработка усовершенствованной рабочей камеры 138-пильного джина». (справка № 422-3 от 05.06.2023 Института сейсмостойкости механики и сооружений). Применение научных результатов позволило доказать существование единственного решения краевых задач для дифференциальных операторов;

решение краевых задач с производными k- и k+1 порядка в начальных условиях для уравнений параболического и эллиптического типов с частными производными четвертого порядка использовалось при исследовании динамических процессов в рамках проекта № МД -758.2022.1.1 Президента Российской Федерации. (справка Камчатского государственного университета № 02-12 от 24 января 2023 г). Использование научных результатов в проектной работе позволило найти регулярное решение краевых задач для уравнений математической физики, а также доказать абсолютную и равномерную сходимость рядов в решении.

Структура и объем диссертации. Диссертация состоит из введения, трёх глав, заключения и списка использованной литературы. Объем диссертации составляет 93 страниц.

E’LON QILINGAN ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo’lim (part I, I часть)

1. Kilichov O.Sh. On a boundary value problem for a fourth-order partial differential equation. // Bulletin of the Institute of Mathematics 2022, Vol. 5, №4, P. 42-47. (01.00.00; №17).
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II bo’lim (part II, II часть)

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9. Киличов О.Ш. Нелокальная начально-краевая задача для 2-параболического уравнения. // Научной конференции «Операторные алгебры, неассоциативные структуры и смежные проблемы». 14–15 сентября 2022 года Ташкент, Узбекистан. –С. 238-239.
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