**V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH**

MATEMATIKA INSTITUTI

JUMANIYOZOV DOSTON ERKABOY O‘G‘LI

KICHIK O‘LCHAMLI NOASSOTSIATIV ALGEBRALARNING TASNIFI VA SODDA *n*-LI ALGEBRALARNI QURISH

01.01.06 – Algebra

**FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD)
DISSERTATSIYASI AVTOREFERATI**

Toshkent – 2023

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

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Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, Fan va Innovatsiyalar Vazirligi huzuridagi Oliy attestatsiya komissiyasida B2023.3.PhD/FM915 raqam bilan ro'yxatga olingan.

Dissertatsiya V.I. Romanovskiy nomidagi Matematika institutida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, ingliz, rus(rezyume)) Ilmiy kengash veb-sahifasi (www. kengash.mathinst.uz) va "ZiyoNet" Axborot ta'lim portalida (www.ziyonet.uz) joylashtirilgan.

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Dissertatsiya himoyasi V.I.Romanovskiy nomidagi Matematika Instituti huzuridagi DSc.02/30.12.2019.FM.86.01 raqamli Ilmiy kengashning 2023 yil «19» dekabr soat 16:00 dagi majlisida bo'lib o'tadi. (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko'chasi, 9-uy. Tel.: (+99871)-207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

Dissertatsiya bilan V.I. Romanovskiy nomidagi Matematika Institutining Axborot-resurs markazida tanishish mumkin (171-raqami bilan ro'yxatga olingan). (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko'chasi, 9-uy. Tel.: (+99871)-207-91-40.

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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Bugungi kunda nazariy fizikada boʻlgani kabi, matematikaning koʻplab obyektlarining xossalarini noassotsiativ algebra elementlariga bogʻlagan holda aniqlash usullari yaratilmoqda. Noassotsiativ algebralar nisbiylik va kvant maydonlar nazariyasi, kvant mexanikasi hamda torlar nazariyasi kabi koʻplab nazariyalarga tadbiq qilinadi. Noassotsiativ algebralar nazariyasi masalalari uzoq yillar davomida dolzarb muammolar hisoblanib, Li algebralariga doir masalalar eng muhim masalalar qatorida sanaladi. Muayyan koʻphadlar oilasi orqali aniqlangan biror koʻpxillikda kichik oʻlchamli algebralarni izomorfizm aniqligida tasniflash assotsiativ boʻlmagan algebralar nazariyasidagi klassik muammodir. Berilgan koʻpxillikda nilpotent algebralarning algebraik tasnifini esa shu koʻpxillikdagi kichikroq oʻlchamdagi algebralarning markaziy kengaytmalarini hisoblash orqali olish mumkin. Chekli oʻlchamli algebralar nazariyasining yana bir dolzarb masalalaridan biri ularni geometrik nuqtai nazardan oʻrganish boʻlib, bunda algebralarning degeneratsiyalarini hisoblash, ushbu koʻpxillikdagi barcha keltirilmas komponentalarni aniqlash imkonini beradi.

Noassotsiativ algebralar nazariyasining yana bir qiziqarli yoʻnalishi bu Li algebralarini umumlashmasi boʻlgan n-Li algebralari nazariyasidir. Yakobian orqali berilgan cheksiz oʻlchamli n-Li algebrasi muhim misollardan biri boʻlib, keyinchalik u boshqa cheksiz oʻlchamli n-Li algebralarning qurilishiga turtki boʻlgan. Hozirgi kunga kelib, cheksiz oʻlchamli sodda n-Li algebralarning Yakobian, Vronskiyan kabi ataluvchi operatorlar yordamida qurilgan sinflari mavjud boʻlib, ulardan boshqa cheksiz oʻlchamli sodda n-Li algebrasi mavjudmi degan savol n-ar algebralar nazariyasining dolzarb masalalaridan biri boʻlib kelmoqda.

Soʻnggi yillarda mamlakatimizda ilmiy va amaliy qoʻllanilishiga ega boʻlgan fundamental fanlar - matematika, fizika, geologiya hamda biologiya fanlariga e’tibor kuchaymoqda. Xususan, noassotsiativ algebralar boʻyicha fundamental tadqiqotlarni rivojlantirishga alohida e’tibor qaratildi. Ushbu fundamental tadqiqotlar doirasida chekli oʻlchamli assotsiativ boʻlmagan algebralarni oʻrganishda muhim natijalarga erishildi. Ilmiy ishning[1] asosiy vazifasi “Algebra va funksional analiz” ustuvor yoʻnalishlari boʻyicha xalqaro standartlar darajasida tadqiqot olib borishdan iborat. Ilmiy natijalarni tegishli fan sohasida qoʻllash uchun chekli oʻlchamli noassotsiativ algebralar nazariyasini ishlab chiqish belgilangan vazifaning bajarilishini ta’minlashda muhim ahamiyatga ega.

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[1] Oʻzbekiston Respublikasi Vazirlar mahkamasining 2017 yil 18 maydagi “Oʻzbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy tadqiqot muassasalari faoliyatini tashkil etish toʻgʻrisida”gi 292-sonli qarori.

O‘zbekiston Respublikasi Prezidentining 2017 yil 7 fevraldagi PF-4947-sonli «O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha xarakatlar strategiyasi to‘g’risida»gi Farmoni, 2017 yil 17 fevraldagi PQ-2789 sonli “Fanlar akademiyasi faoliyatini yanada takomillashtirish, ilmiy-tadqiqot faoliyatini tashkil etish, boshqarish va moliyalashtirish chora-tadbirlari to‘g‘risida” qarori, 2018 yil 27 apreldagi PQ-3682 sonli “Innovatsion g‘oyalar, texnologiyalar va loyihalarni amaliyotga tatbiq etish tizimi to‘g‘risida”gi qarori, 2019 yil 9 iyuldagi PQ-4387-son «Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek, O‘zbekiston Respublikasi Fanlar Akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida»gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning Respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi. Mazkur tadqiqot O‘zbekiston Respublika fan va texnologiyalar rivojlanishining IV «Matematika, mexanika va informatika» ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Li algebralari 19-asrning 70 yillarida M. Sofus Li tomonidan inifinitesmal transformatsiyalarni o‘rganish maqsadida fanga kiritilgan. Yordan algebralari esa 1933 yilda kvant elektrodinamikasi masalalarini tatqiq qilish natijasida nemis matematigi Paskal Jordan tomonidan aniqlangan. Keyinchalik, Li va Yordan algebralarining bir qator umumlashmalari, Maltsev algebralari, Leibniz algebralari, kvadratik Yordan algebralari, nokommutativ Yordan algebralari, kommutativ $\mathfrak{CD}$-algebralari kabi algebralar A.I.Maltsev, J.L.Loday, K.MakKrimmon, M.Osborn, I.Kaygorodov va P.Zusmanovich tomonidan o‘rganila boshlangan.

Li algebralari va ularning umumlashmalari noassosiativ algebralar nazariyasining alohida yo’nalishi sifatida shakllandi. Xususan, Leybnits algebralari so‘nggi yillarda eng dolzarb sohalardan biri hisoblanib, uning strukturaviy nazariyasi va tasniflash masalalari bo‘yicha Sh.Ayupov, S.Albeverio, K.Kudaybergenov, B.Omirov va I.Rahimov tomonidan ko‘plab natijalar olingan. A.Albert 1948 yilda alternativ va Yordan algebralarining bir qancha umumiy xossalaridan foydalangan holda tabiiy ravishda bu algebralar biror umumiyroq sinfning qismi bo‘ladi degan farazni ilgari surib, nokommutativ Yordan algebralarini aniqlagan. Keyinchalik, 1955 yilda R.Shafer nokommutativ Yordan algebralari ko‘pxilligi aynan Yordan ayniyati hamda egiluvchanlik ayniyati bilan aniqlanishini ko‘rsatgan.

Noassotsiativ algebralarning yangi sinflaridan biri bo‘lgan $\mathfrak{CD}$-algebralari esa I.Kaygorodov va P.Zusmanovichning ishida kiritilgan bo‘lib, bu algebralarning aniqlanishiga Li va Yordan algebralarida ixtiyoriy ikkita o‘ng yoki chap ko‘paytirish operatorlarining kommutatori differensiallash bo‘lish sharti asos hisoblanadi.

So‘nggi yillarda ko‘plab tadqiqotchilar nilpotent noassotsiativ algebralarning algebraik tasnifiga e’tibor qaratishmoqda. Buning asosiy sabablaridan biri bu berilgan ko‘phillikdagi algebralar haqida aniqroq ma’lumotga ega bo‘lishdir. Bundan tashqari, algebraik tasnif keyinchalik ularni geometrik nuqtai nazardan tadqiq qilish imkonini beradi. Ya’ni algebralarni geometrik tasniflash orqali muhim masalalardan biri hisoblangan biror ko‘pxillikdagi barcha keltirilmas komponentlarini tavsiflash muammosi hal qilinadi. Nilpotent noassotsiativ algebralarning algebraik va geometrik tasnifiga oid ko‘plab ilmiy ishlar chop qilingan. Jumladan, kompleks sonlar maydoni ustida nilpotent assotsiativ kommutativ algebralarning algebraik tasnifi I.Kaygorodov, I.Rahimov va Sh.K.Said Husayn tomonidan o‘rganilgan bo‘lsa, A.Hegazi hamda H.Abdelvahab besh o‘lchamli nilpotent noassotsiativ Yordan algebralarini tasnifladilar. I.Kashuba va M.Martin bu algebralarning geometrik xossalarini tadqiq qilishgan. Bundan tashqari, kompleks sonlar maydoni ustida to‘rt o‘lchamli Li algebralarining orbita yopilmalarining tasnifi D.Burde va C.Steinhoff tomonidan o‘rganilgan.

Noassotsiativ algebralar nazariyasining yana bir muhim yo‘nalishi n-Li algebralar nazariyasi hisoblanadi. Dastlab, Ω-algebra tushunchasi A.G.Kurosh tomonidan kiritilgan bo‘lib, keyinchalik V.T.Filippov tomonidan Yakobian operatori orqali n-Li algebralari aniqlangan. Bundan tashqari, A.Djumadildayev esa Vronskian operatori orqali aniqlanuvchi sodda n-Li algebrasiga misol keltirgan. N.Kantarini va V.Kats tomonidan yuqoridagi 2 ta sodda n-Li algebrasidan tashqari yana bir sodda n-Li algebrasi keltirilib, n-Li algebralar nazariyasida hozirda mavjud uchta cheksiz o‘lchamli sodda n-Li algebralaridan boshqa cheksiz o‘lchamli sodda n-Li algebrasi mavjudmi degan savol qo‘yilgan va bu muammo hozirigi kungacha hal qilinmagan.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti V.I.Romanovskiy nomidagi matematika institutining «Operator algebralarining avtomorfizmlari, cheksiz o‘lchamli noassotsiativ algebralar va superalgebralarning tasnifi» mavzusidagi ilmiy-tadqiqot loyihasi (FZ-2020093065, 2021-2025 yy.) doirasida bajarilgan.

Tadqiqotning maqsadi kichik o‘lchamli niplotent nokommutativ Yordan va kommutativ algebralar ko‘pxilliklarining algebraik tasnifini aniqlash, ushbu ko‘pxilliklarning barcha keltirilmas komponentalalarini topish hamda cheksiz o‘lchamli sodda n-Li algebralarni qurishdan iborat.

Tadqiqotning vazifalari:

kichik o‘lchamli nilpotent noassotsiativ algebralarning ayrim sinflarini tasniflash;

kichik o‘lchamli nilpotent noassotsiativ algebralarning geometrik tasnifini olish;

cheksiz o‘lchamli sodda n-Li algebralarini qurish.

Tadqiqotning obyekti: nokommutativ Yordan algebralar, Kommutativ $\mathfrak{CD}$-algebralar, n-Li algebralar.

Tadqiqotning premeti: to‘rt o‘lchamli niplotent nokommutativ Yordan algebralari, besh o‘lchamli nilpotent kommutativ $\mathfrak{CD}$-algebralar, cheksiz o‘lchamli sodda n-Li algebralar.

Tadqiqotning usullari. Dissertatsiyada markaziy kengaytmalar usuli, tasniflash usullari, keltirilmas komponentalarni aniqlash usuli, shuningdek, invariantlar nazariyasi usullari foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

to‘rt o‘lchamli niplotent nokommutativ Yordan va besh o‘lchamli nilpotent kommutativ algebralari to‘liq tasniflangan;

to‘rt o‘lchamli nilpotent nokommutativ Yordan algebralar ko‘pxilligining barcha keltirilmas komponentalari topilgan va ushbu ko‘pxillikdagi qattiq algebralar soni beshta ekanligi isbotlangan;

o‘ng yoki chapdan ko‘paytirish operatorlarining kommutatori differensiallash bo‘ladigan algebralar ko‘pxilligi o‘nta keltirilmas komponentadan iboratligi isbotlangan;

Yakobian va Vronskian operatorlari bilan aniqlangan n-Li algebralarni umumlashtirish orqali yangi n-Li algebralar qurilgan hamda ushbu algebralarning ba’zilari sodda bo‘lishi isbotlangan.

Tadqiqotning amaliy natijalari. Olingan natijalar va dissertatsiyada qo‘llanilgan usullar oliy o‘quv yurtlari magistr talabalari va doktorantlari uchun maxsus kurslarda o‘qitilishi mumkin. Bundan tashqari, cheksiz o‘lchamli sodda n-Li algebralari bo‘yicha dissertatsiya natijalari sodda n-Li algebralari haqidagi bir qancha ilmiy farazlarni tekshirish imkonini beradi.

Tadqiqot natijalarining ishonchliligi. Matematik mulohazalarning qat’iyligi, algebralarning boshqa sinflaridagi ma’lum usullaridan, kogomologiyalar nazariyasi, algebralarining strukturaviy nazariyasidagi fundamental natijalardan foydalanilganligi bilan asoslanadi. Mathematica 13 dasturlash tilida yaratilgan maxsus dasturlar yordamida kichik o‘lchamli algebralarning differensiallashlari va degeneratsiyalariga doir natijalar tekshirilgan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati boshqa ko‘pxillikdagi algebralarni tadqiq qilish uchun ishlatilishi mumkin. Xususan, ushbu dissertatsiyada ishlab chiqilgan texnika va usullar cheksiz o‘lchamli n-Li algebralari haqidagi ilmiy farazni hal qilishda qo‘llanilishi mumkin.

Bitiruv malakaviy ishining amaliy ahamiyati shundaki, olingan natijalardan algebraik strukturalar nazariyasida foydalanish mumkin. Dissertatsiyadagi barcha natijalar va qo‘llanilgan usullardan oliy ta’lim muassasalarining magistratura va doktorantura talabalari uchun maxsus kurslarda foydalanish mumkin.

Tadqiqot natijalarining joriy qilinishi. Kichik o‘lchamli noassotsiativ algebralarning tasnifi va sodda n-Li algebralarni qurish bo‘yicha olingan natijalar asosida:

o‘ng yoki chapdan ko‘paytirish operatorlarining kommutatori differensiallash bo‘ladigan algebralar ko‘pxilligini tasniflashda olingan natijalardan AP08051987 raqamli “Dinamik tizimlardagi noregulyar to‘plamlar” mavzusidagi xorijiy grant loyihasida besh o‘lchamli yagona hosil qiluvchiga ega nilpotent assosimmetrik algebralarning tasnifini olish uchun foydalanilgan (Suleyman Demirel Universitetining 2023 yil 1-noyabrdagi №1.1-11/1141-sonli ma’lumotnomasi, Qozog‘iston). Ilmiy natijaning qo‘llanilishi besh o‘lchamli yagona hosil qiluvchiga ega nilpotent assosimmetrik algebralar ko‘pxilligining keltirilmas komponentlarini aniqlash imkonini bergan;

to‘rt o‘lchamli niplotent nokommutativ Yordan algebralarning to‘liq tasnifidan xorijiy ilmiy jurnallardagi maqolalarda (Journal of Algebra and its Applications, 20, 2021, 2150198; Communications in Mathematics, 29, 2021, 215-226; Journal of Pure and Applied Algebra, 226, 2022, 106850; Journal of Algebra and its Applications, 21, 2022, 2250031) nilpotent algebralarning algebraik va geometrik tasniflarini olishda foydalanilgan. Ilmiy natijaning qo‘llanilishi besh va olti o‘lchamli bitta hosil qiluvchili nilpotent kompleks bikommutativ algebralarning to‘liq tasnifini olish imkonini berdi.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 3 ta xalqaro va 2 ta respublika ilmiy-amaliy anjumanlarda muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya mavzusi bo‘yicha jami 11 ta ilmiy ish chop etilgan, shulardan, O‘zbekiston Respublikasi Oliy Attestatsiya komissiyasining falsafa doktori dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 6 ta maqola, jumladan, 5 tasi xorijiy, 1 tasi respublika jurnallarida, shuningdek 5 ta ma’ruza tezislari ilmiy konferensiya materiallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qism, o‘nta bo‘limga bo‘lingan to‘rtta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning hajmi 121 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiya kirish qism, to‘rtta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat.

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustivor yo‘nalishlariga mosligi ko‘rsatilgan. Shuningdek, bu qismda dissertatsiya mavzusi bo‘yicha muammoning o‘rganilganlik darajasi keltirilgan, tadqiqotning maqsadi, vazifalari, obyekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning “**Asosiy tushunchalar**” nomli birinchi bobida noassotsiativ algebralar nazariyasining boshlang‘ich tushunchalari qatorida bir qator noassotsiativ algebralar ta’riflari keltirilgan. Bundan tashqari, bu bob noassotsiativ algebralarni tasniflash va ularning keltirilmas komponentalarini topish usullari bilan bir qatorda n-Li algebralar nazariyasining boshlang‘ich tushunchalarini o‘z ichiga oladi.

Birinchi bobning birinchi bo‘limida boshlang‘ich tushunchalar, Yordan, nokommutativ Yordan va $\mathfrak{CD}$-algebralarining ta’riflari keltirilgan.

Ma’lumki, chiziqli fazoda ko‘paytirish amali aniqlangan bo‘lsa, u holda bu chiziqli fazo ushbu ko‘paytirish amali bilan birgalikda algebra deb ataladi. Bizga **A** va **B** algebralar berilgan bo‘lsin. Quyidagi

$$\varphi(xy) = \varphi(x)\varphi(y)$$

shartni qanoatlantiruvchi $\varphi : \mathbf{A} \to \mathbf{B}$ chiziqli akslantirish esa gomomorfizm deyiladi. Biyektiv gomomorfizmga esa izomorfizm deb ataladi. Algebrani o‘zini o‘ziga akslantirvchi izomorfizm esa avtomorfizm deyiladi.

1-ta’rif. *Agar* **A** *algebraning ixtiyoriy* $x, y \in \mathbf{A}$ *elementlari uchun quyidagi ayniyatlar o‘rinli bo‘lsa*:

$$xy = yx, \qquad x^2(yx) = (x^2y)x,$$

u holda **A** *Yordan algebrasi deyiladi.*

2-ta’rif. *Agar* **A** *algebraning ixtiyoriy* $x, y \in \mathbf{A}$ *elementlari uchun quyidagi ayniyatlar o‘rinli bo‘lsa*:

$$x(yx) = (xy)x, \qquad x^2(yx) = (x^2y)x,$$

u holda **A** *nokommutativ Yordan algebrasi deyiladi.*

Takidlash kerakki, har qanday Yordan algebra nokommutativ Yordan algebra bo‘ladi.

Berilgan **A** algebra uchun quyidagi *quyi markaziy qator* deb ataluvchi ketma-ketlikni aniqlaylik:

$$\mathbf{A}^1 = \mathbf{A}, \ \mathbf{A}^{k+1} = \mathbf{A}^k\mathbf{A} + \mathbf{A}^{k-1}\mathbf{A}^2 + \ldots + \mathbf{A}\mathbf{A}^k, \ k \geq 1.$$

3-ta’rif. *Agar biror* $s \in \mathbb{N}$ *natural son topilib* $\mathbf{A}^s = 0$ *tenglik o‘rinli bo‘lsa, u holda* **A** *algebra nilpotent deyiladi. Bu shartni qanoatlantiruvchi eng kichik s son nilpotentlik indeksi deyiladi. Agar algebraning nilpotentlik indeksi* 3 *ga teng bo‘lsa bunday algebraga* 2*-qadam algebra deyiladi*.

Agar $d : \mathbf{A} \to \mathbf{A}$ chiziqli akslantirish barcha $x, y \in \mathbf{A}$ elementlar uchun quyidagi shartni qanoatlantirsa:

$$d(xy) = d(x)y + xd(y),$$

u holda d differensiallash deyiladi. Der(**A**) orqali algebraning barcha differensiallashlari to‘plamini belgilaymiz.

4-ta’rif. *Agar* **A** *algebra uchun* $[R_a, R_b], [R_a, L_b], [L_a, L_b]$ *operatorlar differensiallash bo‘lsa, u holda bu algebra* $\mathfrak{CD}$*-algebrasi deyiladi.*

Boshqacha aytganda, agar **A** algebra uchun quyidagi ayniyatlar o‘rinli bo‘lsa:

$$\big((xy)a\big)b + \big((xb)a\big)y + x\big((yb)a\big) = \big((xy)b\big)a + \big((xa)b\big)y + x\big((ya)b\big),$$
$$b\big((xy)a\big) + \big((bx)a\big)y + x\big((by)a\big) = \big(b(xy)\big)a + \big(b(xa)\big)y + x\big(b(ya)\big),$$
$$b\big(a(xy)\big) + \big(a(bx)\big)y + x\big(a(by)\big) = a\big(b(xy)\big) + \big(b(ax)\big)y + x\big(b(ay)\big).$$

U holda bu algebra $\mathfrak{CD}$-algebra deb ataladi.

Birinchi bobning ikkinchi bo‘limida markaziy kengaytma usuli keltirilgan.

Aytaylik $(\mathbf{A},\cdot)$ biror $F(x_1,\ldots,x_n)=0$ ayniyat orqali aniqlangan biror algebra va $\mathbb{V}$ chiziqli fazo bo‘lsin. $\theta:\mathbf{A}\times\mathbf{A}\to\mathbb{V}$ bichiziqli akslantirishni qaraylik. Bu akslantirish yordamida $\mathbf{A}_\theta=\mathbf{A}\oplus\mathbb{V}$ chiziqli fazoda quyidagi shakldda ko‘paytirish amalini aniqlaymiz:

$$[x+x_0,y+y_0]=x\cdot y+\theta(x,y) \text{ for all } x,y\in\mathbf{A},x_0,y_0\in\mathbb{V}.$$

Yuqoridagi kabi aniqlangan $\mathbf{A}_\theta$ algebra $\mathbf{A}$ algebraning $\mathbb{V}$ vektor fazo orqali k-o‘lchamli markaziy kengaytmasi deyiladi.

Ravshanki, $\mathbf{A}_\theta$ algebrada $F(x_1,\ldots,x_n)=0$ ayniyat o‘rinli bo‘lishi uchun θ akslantirish $\theta(F(x_1,\ldots,x_n))=0$ ayniyat orqali aniqlanuvchi trivial 2-kosikllik shartini qanoatlantirishi zarur va yetarlidir.

Masalan, ayniyatning ko‘rinishi $F(x,y,z)=x(yz)+z(yx)-(xy)z-(zy)x$ kabi bo‘lsa, u holda $\theta(F(x,y,z))$ ifoda quyidagi ko‘rinishda bo‘ladi:

$$\theta(F(x,y,z))=\theta(x,yz)+\theta(z,yx)-\theta(xy,z)-\theta(zy,x). \qquad (1)$$

Ushbu $\theta(F(x_1,\ldots,x_n))=0$ ayniyatni qanoatlantiruvchi barcha $Z^2(\mathbf{A},\mathbb{V})$ bichiziqli akslantirishlar to‘plamini $Z^2(\mathbf{A},\mathbb{V})$ kabi belgilaymiz. Bu elementlar trivial 2-kosikllar deyiladi. Berilgan $f:\mathbf{A}\to\mathbb{V}$ chiziqli akslantirish uchun $\delta f:\mathbf{A}\times\mathbf{A}\to\mathbb{V}$ bichiziqli akslantirishni $\delta f(x,y)=f(xy)$ kabi aniqlasak, u holda $\delta f\in Z^2(\mathbf{A},\mathbb{V})$ bo‘ladi. Demak, $B^2(\mathbf{A},\mathbb{V})=\{\theta=\delta f : f$ chiziqli akslantirish$\}$ kabi aniqlangan chiziqli fazo $Z^2(\mathbf{A},\mathbb{V})$ fazoning qismfazosi bo‘ladi. U holda, $\mathbf{A}$ algebraning ikkinchi kogomologik fazosi $H^2(\mathbf{A},\mathbb{V})$ esa $Z^2(\mathbf{A},\mathbb{V})/B^2(\mathbf{A},\mathbb{V})$ faktor fazo orqali aniqlanadi.

Aytaylik, $\mathbf{A}$ algebraning avtomorfizmlar gruppasi $Aut(\mathbf{A})$ berilgan bo‘lsin. Ushbu $Aut(\mathbf{A})$ gruppaning $Z^2(\mathbf{A},\mathbb{V})$ fazoga ta’sirini

$$\phi.\theta(x,y)=\theta(\phi(x),\phi(y))$$

kabi aniqlaymiz, bu yerda $\phi\in Aut(\mathbf{A})$ va $\theta\in Z^2(\mathbf{A},\mathbb{V})$. U holda, $B^2(\mathbf{A},\mathbb{V})$ qismfazo bu ta’sirga nisbatan invariant bo‘ladi. Demak, biz $Aut(\mathbf{A})$ gruppaning $H^2(\mathbf{A},\mathbb{V})$ fazoga ta’sirini ham qarashimiz mumkin.

$Ann(\theta)=\{x\in\mathbf{A}:\theta(x,\mathbf{A})=\theta(\mathbf{A},x)=0\}$ to‘plam θ elementning annulyatori va $Ann(\mathbf{A})=\{x\in\mathbf{A}:x\mathbf{A}=\mathbf{A}x=0\}$ to‘plam esa $\mathbf{A}$ algebraning annulyatori deb ataladi. Qayd etish joizki, $Ann(\mathbf{A}_\theta)=(Ann(\theta)\cap Ann(\mathbf{A}))\oplus\mathbb{V}$ tenglik o‘rinli bo‘ladi.

Quyidagi natija A.Hegazi va H.Abdelvahab tomonidan Yordan algebralari uchun isbotlangan. Aslida bu natija ixtiyoriy noassotsiativ algebra uchun o‘rinlidir. Unga ko‘ra har qanday annulyatori noldan farqli algebra kichikroq o‘lchamli algebraning markaziy kengaytmasidir.

1-lemma. *Aytaylik*, $\mathbf{A}$ *algebra n-o‘lchamli muayyan tipdagi algebra va* $dim(Ann(\mathbf{A}))=m\neq 0$ bo‘lsin. *U holda*, *ayni tipdagi yagona* $(n\text{-}m)$*-o‘lchamli* $\mathbf{A}'$ *algebra va* $Ann(\theta)\cap Ann(\mathbf{A})=0$ *shartni qanoatlantiruvchi* $\theta\in Z^2(\mathbf{A},\mathbb{V})$ *mavjud bo‘lib,* $\mathbf{A}\cong \mathbf{A}_\theta'$, $\mathbf{A}/Ann(\mathbf{A})\cong\mathbf{A}'$ *munosabatlar o‘rinli bo‘ladi*, *bu yerda*, $\mathbb{V}$ *m-o‘lchamli chiziqli fazo*.

5-ta’rif. *Aytaylik* $\mathbf{A}$ *biror algebra va* $\boldsymbol{I}$ *esa* $Ann(\mathbf{A})$ *ning qism fazosi bo‘lsin*. *Agar* $\mathbf{A}=\mathbf{A_0}\oplus \boldsymbol{I}$ *bo‘lsa, u holda* $\boldsymbol{I}$ *fazo* $\mathbf{A}$ *algebraning annulyator komponenti deyiladi*. $\mathbf{A}$ *algebraning annulyator komponentasiz markaziy kengaytmasi ajralmaydigan markaziy kengaytma deyiladi.*

Aytaylik $e_1,e_2,\ldots,e_s$ va θ_1, $\theta_2,\ldots$, θ_s lar mos ravishda $\mathbb{V}$ hamda $Z^2(\mathbf{A},\mathbb{V})$ fazolarning bazislari bo‘lsin. U holda, ixtiyoriy θ element yagona ravishda $\theta(x,y)=\sum_{i=1}^{s}\theta_i(x,y)e_i$ kabi ifodalanadi, bu yerda $\theta_i\in Z^2(\mathbf{A},\mathbb{C})$. Bundan tashqari, $Ann(\theta)=Ann(\theta_1)\cap Ann(\theta_2)\cap\ldots\cap Ann(\theta_s)$ tenglik o‘rinli bo‘lib, $\theta\in B^2(\mathbf{A},\mathbb{V})$ munosabat faqat va faqat $\theta_i\in B^2(\mathbf{A}, \mathbb{C})$ bo‘lganda o‘rinli bo‘ladi.

Quyidagi natija A.Hagazi va H.Abdelvahab tomonidan Yordan algebralari uchun isbotlangan. Aslida bu natija ixtiyoriy noassotsiativ algebra uchun o‘rinlidir.

2-lemma. *Aytaylik*, $\theta(x,y)=\sum_{i=1}^{s}\theta_i(x,y)e_i\in Z^2(\mathbf{A},\mathbb{V})$ *va* $Ann(\theta)\cap Ann(\mathbf{A})=0$ *bo‘lsin*. *U holda*, $\mathbf{A}_\theta$ *annulyator componentaga ega bo‘lishi uchun* $[\theta_1],[\theta_2],\ldots,[\theta_s]$ *elementlar* $H^2(\mathbf{A},\mathbb{C})$ *fazoda chiziqli bog‘liq bo‘lishi zarur va yetarlidir*.

Yuqoridagi lemmadan foydalangan holda $(n\text{-}s)$-o‘lchamli $\mathbf{A}'$ algebraning barcha ajralmaydigan markaziy kengaytmalarini topish uchun quyidagicha tartibdagi ishlarni amalga oshirish zarurligiga ega bo‘lamiz:

1. Berilgan $(n\text{-}s)$-o‘lchamli $\mathbf{A}'$ algebra uchun $H^2(\mathbf{A},\mathbb{C})$, $Ann(\mathbf{A}')$ va $Aut(\mathbf{A}')$ larni aniqlaymiz.
2. $Aut(\mathbf{A})$ gruppaning $H^2(\mathbf{A},\mathbb{C})$ fazoga ta’sirini topib, $H^2(\mathbf{A},\mathbb{C})$ ning elementlaridan tuzilgan qism fazoning orbitasini aniqlaymiz.
3. Har bir orbita uchun unga mos algebrani quramiz.

Endi, $\mathbf{A}$ algebra va uning $e_1,e_2,\ldots,e_n$ bazisi uchun Δ_{ij} bichiziqli formani quyidagicha kiritamiz:

$$\Delta_{ij}:\mathbf{A}\times\mathbf{A}\to\mathbb{C},\quad \Delta_{ij}(e_l,e_m)=\delta_{il}\delta_{jm}.$$

U holda, $\{\Delta_{ij} : 1 \le i, j \le n\}$ to‘plam **A** algebradagi bichiziqli formalar uchun bazis bo‘ladi, ya’ni har qanday $\theta \in Z^2(\mathbf{A},\mathbb{V})$ element $\theta = \sum_{1\le i\le j\le n} c_{ij}\Delta_{ij}$ kabi yagona usulda ifodalanadi, bu yerda $c_{ij} \in \mathbb{C}$.

Dissertatsiyaning “**Kichik o‘lchamli noassotsiativ algebralarning algebraik tasnifi**” nomli ikkinchi bobi 4-o‘lchamli nilpotent nokommutativ Yordan algebralar, 5-o‘lchamli nilpotent kommutativ $\mathfrak{CD}$-algebralar va 5-o‘lchamli nilpotent kommutativ algebralar ko‘pxilliklarini kompleks sonlar maydoni ustida algebraik tasniflashga bag‘ishlangan.

Aytaylik, $(\mathbf{J},\cdot)$ kompleks sonlar maydoni ustida berilgan Yordan algebra va $\mathbb{V}$ shu maydon ustida chiziqli fazo bo‘lsin. $Z^2(\mathbf{J}, \mathbb{V})$ orqali quyidagi ayniyatlarni qanoatlantiruvchi barcha bichiziqli $\theta:\mathbf{J}\times\mathbf{J}\to\mathbb{V}$ akslantirishlar to‘plamini belgilaymiz:

$$\theta(x, yz) + \theta(z, yx) = \theta(xy, z) + \theta(zy, x), \qquad (2)$$

$$\theta(xt, yz) + \theta(tx, yz) + \theta(tz, yx) + \theta(xz, yt) + \theta(zt, yx) + \theta(zx, yt) =$$
$$\theta((xt)y, z) + \theta((tx)y, z) + \theta((xz)y, t) + \theta((tz)y, x) + \theta((zx)y, t) + \theta((zt)y, x). \qquad (3)$$

Bu bichiziqli akslantirishlar **J** nokommutativ Yordan algebraning trivial 2-kosikllari deb ataladi.

Ravshanki, bir o‘lchamli notrivial nilpotent nokommutativ Yordan algebrasi mavjud emas. O‘lchami ikkiga teng bo‘lgan notrivial nilpotent nokommutativ Yordan algebralar soni bitta va u quyidagicha:

$$\mathbf{J}^2_{01} : e_1e_1 = e_2.$$

A.Martin Kalderon, A.Ouaridi Fernandez va I.Kaygorodov tomonidan olingan natijalarga asosan uch o‘lchamli nilpotent nokommutativ Yordan algebralari quyidaglardir:

$\mathbf{J}^3_{02}$: $e_1e_2 = e_3 \quad e_2e_1 = e_3$,

$\mathbf{J}^3_{03}$: $e_1e_2 = e_3 \quad e_2e_1 = -e_3$,

$\mathbf{J}^3_{04}(\lambda)$: $e_1e_1 = \lambda e_3 \quad e_1e_2 = e_3 \quad e_2e_1 = e_3$,

$\mathbf{J}^3_{05}$: $e_1e_1 = e_2 \quad e_1e_2 = e_3 \quad e_2e_1 = e_3$.

Biz quyidagi tasdiqda 3-o‘lchamli nilpotent nokommutativ Yordan algebralarining ikkinchi kogomologik fazolarini tasnifini keltiramiz:

1-tasdiq. *Quyidagi elementlar* 3*-o‘lchamli nilpotent nokommutativ Yordan algebralarining ikkinchi kogomologik fazolari uchun bazis bo‘ladi*:

A	$Z^2(\mathbf{A},\mathbb{C})$	$B^2(\mathbf{A},\mathbb{C})$	$H^2(\mathbf{A},\mathbb{C})$
$\mathbf{J}^3_{01}$	$\left\langle \begin{array}{l}\Delta_{11}, \Delta_{12}+\Delta_{21}, \Delta_{13}, \\ \Delta_{23}+\Delta_{32}, \Delta_{31}, \Delta_{33}\end{array}\right\rangle$	$\langle \Delta_{11} \rangle$	$\left\langle \begin{array}{l}[\Delta_{12}+\Delta_{21}], [\Delta_{13}], \\ [\Delta_{23}+\Delta_{32}], [\Delta_{31}], [\Delta_{33}]\end{array}\right\rangle$

$\mathbf{J}_{02}^{3}$	$\left\langle \begin{matrix} \Delta_{11}, \Delta_{12}, \Delta_{13} + \Delta_{31}, \\ \Delta_{21}, \Delta_{22}, \Delta_{23} + \Delta_{32} \end{matrix} \right\rangle$	$\langle \Delta_{12} + \Delta_{21} \rangle$	$\left\langle \begin{matrix} [\Delta_{11}], [\Delta_{13} + \Delta_{31}], \\ [\Delta_{21}], [\Delta_{22}], [\Delta_{23} + \Delta_{32}] \end{matrix} \right\rangle$
$\mathbf{J}_{03}^{3}$	$\left\langle \begin{matrix} \Delta_{11}, \Delta_{12}, \Delta_{13} - \Delta_{31}, \\ \Delta_{21}, \Delta_{22}, \Delta_{23} - \Delta_{32} \end{matrix} \right\rangle$	$\langle \Delta_{12} - \Delta_{21} \rangle$	$\left\langle \begin{matrix} [\Delta_{11}], [\Delta_{13} - \Delta_{31}], \\ [\Delta_{21}], [\Delta_{22}], [\Delta_{23} - \Delta_{32}] \end{matrix} \right\rangle$
$\mathbf{J}_{04}^{3}(\lambda)$	$\langle \Delta_{11}, \Delta_{12}, \Delta_{21}, \Delta_{22} \rangle$	$\langle \lambda\Delta_{11} + \Delta_{21} + \Delta_{22} \rangle$	$\langle [\Delta_{11}], [\Delta_{12}], [\Delta_{22}] \rangle$
$\mathbf{J}_{05}^{3}$	$\left\langle \begin{matrix} \Delta_{11}, \Delta_{12} + \Delta_{21}, \\ \Delta_{13} + \Delta_{22} + \Delta_{31} \end{matrix} \right\rangle$	$\langle \Delta_{11}, \Delta_{21} + \Delta_{21} \rangle$	$\langle [\Delta_{13} + \Delta_{22} + \Delta_{31}] \rangle$

bu yerda $\mathbf{J}_{01}^{3} = \mathbf{J}_{01}^{2} \oplus \mathbb{C}e_3$.

Quyidagi teorema bu bo‘limning asosiy natijasi hisoblanadi:

1-teorema. *Har qanday* 4-*o‘lchamli nilpotent nokommutativ Yordan algebra* 2-*qadam nilpotent yoki quyidagi o‘zaro izomorf bo‘lmagan algebralarning birortasiga izomorf bo‘ladi*:

$\mathbf{J}_{01}^{4}:\quad e_1e_1 = e_2 \quad e_1e_2 = e_3 \quad e_2e_1 = e_3$

$\mathbf{J}_{02}^{4}:\quad e_1e_1 = e_2 \quad e_2e_3 = e_4 \quad e_3e_2 = e_4 \quad e_3e_1 = e_4$

$\mathbf{J}_{03}^{4}:\quad e_1e_1 = e_2 \quad e_1e_2 = e_4 \quad e_2e_1 = e_4 \quad e_3e_1 = e_4 \quad e_3e_3 = e_4$

$\mathbf{J}_{04}^{4}:\quad e_1e_1 = e_2 \quad e_1e_2 = e_4 \quad e_2e_1 = e_4 \quad e_3e_1 = e_4$

$\mathbf{J}_{05}^{4}:\quad e_1e_1 = e_2 \quad e_2e_3 = e_4 \quad e_3e_2 = e_4$

$\mathbf{J}_{06}^{4}:\quad e_1e_1 = e_2 \quad e_1e_2 = e_4 \quad e_2e_1 = e_4 \quad e_3e_3 = e_4$

$\mathbf{J}_{07}^{4}:\quad e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = e_3 + e_4 \quad e_2e_3 = e_4 \quad e_3e_1 = e_4 \quad e_3e_2 = e_4$

$\mathbf{J}_{08}^{4}:\quad e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = e_3 + e_4 \quad e_2e_2 = e_4 \quad e_3e_1 = e_4$

$\mathbf{J}_{09}^{4}:\quad e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = e_3 + e_4 \quad e_3e_1 = e_4$

$\mathbf{J}_{10}^{4}:\quad e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = e_3 \quad e_2e_3 = e_4 \quad e_3e_1 = e_4 \quad e_3e_2 = e_4$

$\mathbf{J}_{11}^{4}:\quad e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = e_3 \quad e_3e_1 = e_4$

$\mathbf{J}_{12}^{4}:\quad e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = e_3 \quad e_2e_2 = e_4 \quad e_3e_1 = e_4$

$\mathbf{J}_{13}^{4}:\quad e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = -e_3 \quad e_3e_1 = -e_4$

$\mathbf{J}_{14}^{4}:\quad e_1e_1 = e_4 \quad e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = -e_3 \quad e_3e_1 = -e_4$

$\mathbf{J}_{15}^{4}:\quad e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = -e_3 + e_4 \quad e_3e_1 = -e_4$

$\mathbf{J}_{16}^{4}:\quad e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = -e_3 \quad e_2e_2 = e_4 \quad e_3e_1 = -e_4$

$\mathbf{J}_{17}^{4}:\quad e_1e_1 = e_4 \quad e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = -e_3 \quad e_2e_2 = e_4 \quad e_3e_1 = -e_4$

$\mathbf{J}_{18}^{4}:\quad e_1e_1 = e_2 \quad e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = e_3 \quad e_2e_2 = e_4 \quad e_3e_1 = e_4$

Ikkinchi bobning ikkinchi bo‘limida kommutativ $\mathfrak{CD}$-algebralarning algebraik tasnifi keltirilgan.

Aytaylik, $(\mathbf{A},\cdot)$ kompleks sonlar maydoni ustida berilgan kommutativ $\mathfrak{CD}$-algebra va $\mathbb{V}$ shu maydon ustida chiziqli fazo bo‘lsin. $\mathrm{Z}^2(\mathbf{A},\mathbb{V})$ orqali quyidagi ayniyatlarni qanoatlantiruvchi barcha bichiziqli θ:**A**×**A**→$\mathbb{V}$ akslantirishlar to‘plamini belgilaymiz:

$$\theta(x,y)=\theta(y,x), \tag{4}$$

$$\theta((xy)a,b)+\theta((xb)a,y)+\theta(x,(yb)a)=\theta((xy)b,a)+\theta((xa)b,y)+\theta(x,(ya)b). \tag{5}$$

Bu bichiziqli akslantirishlar kommutativ $\mathfrak{CD}$-algebraning trivial 2-kosikllari deb ataladi. Qayd etish joizki, Yordan algebralari ko‘pxilligi kommutativ $\mathfrak{CD}$-algebralari ko‘pxilligining qism ko‘pxilligi bo‘ladi. Shuning uchun, (3) va (4) ayniyatlarni qanoatlantiruvchi trivial ikkinchi kosikl kommutativ $\mathfrak{CD}$-algebrasi uchun ham trivial ikkinchi kosikl bo‘ladi. Shunga ko‘ra, $\mathrm{H}^2_{\mathfrak{J}}(\mathbf{A})$ va $\mathrm{H}^2_{\mathfrak{CD}}(\mathbf{A})$ orqali **A** algebraning mos ravisgda Yordan va kommutativ $\mathfrak{CD}$-algebra algebraga nisbatan ikkinchi kogomologik fazolarini belgilaymiz.

Quyidagicha beilgilashlarni kiritaylik:

C^{i}_{j} – *j-sondagi i-o‘lchamli kommutative* $\mathfrak{CD}$ (*non-Jordan*) *algebra.*

C^{i*}_{j} – *j-sondagi i-o‘lchamli kommutative* $\mathfrak{CD}$ (*Jordan*) *algebra.*

A.Kalderon Martin, A.Fernandez Ouaridi hamda I.Kaygorodov ilmiy ishida quyidagi kompleks 4-o‘lchamli kommutativ $\mathfrak{CD}$-algebralar ro‘yxati keltirilgan:

C^{2*}_{01} : $e_1e_1=e_2$

C^{3*}_{02} : $e_1e_1=e_2$ $e_1e_2=e_3$

C^{3*}_{03} : $e_1e_2=e_3$

C^{3}_{01} : $e_1e_1=e_2$ $e_2e_2=e_3$

C^{4*}_{04} : $e_1e_1=e_3$ $e_2e_2=e_4$

C^{4*}_{05} : $e_1e_1=e_3$ $e_1e_2=e_4$

C^{4*}_{06} : $e_1e_1=e_4$ $e_2e_3=e_4$

$\mathrm{C}^{4}_{02}(0)$: $e_1e_1=e_2$ $e_1e_2=e_4$ $e_2e_2=e_3$

Bundan tashqari, A.Fernandez Ouaridi, I.Kaygorodov, M.Khrypchenko va Y.Volkov ishlarida 3-o‘lchamli kommutativ $\mathfrak{CD}$-algebralarning markaziy kengaymalari orqali hosil qilingan quyidagi 4-o‘lchamli kommutativ $\mathfrak{CD}$-algebralar keltirilgan:

C^{4*}_{07} : $e_1e_1=e_2$ $e_2e_3=e_4$

C^{4*}_{08} : $e_1e_1=e_2$ $e_1e_2=e_4$ $e_3e_3=e_4$

C^{4}_{03} : $e_1e_1=e_2$ $e_1e_3=e_4$ $e_2e_2=e_4$

C^{4}_{04} : $e_1e_1=e_2$ $e_2e_2=e_4$ $e_3e_3=e_4$

C^{4*}_{09} : $e_1e_1=e_2$ $e_1e_2=e_3$ $e_1e_3=e_4$ $e_2e_2=e_4$

$\mathrm{C}^{4}_{02}(\alpha)$: $e_1e_1=e_2$ $e_1e_2=e_3$ $e_1e_3=\alpha e_4$ $e_2e_2=(\alpha+1)e_4$

C^{4*}_{10} : $e_1e_2=e_3$ $e_1e_3=e_4$ $e_2e_2=e_4$

C_{11}^{4*} : $e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_3 = e_4$

C_{12}^{4*} : $e_1e_2 = e_3 \quad e_1e_3 = e_4$

C_{05}^{4} : $e_1e_2 = e_3 \quad e_3e_3 = e_4$

C_{06}^{4} : $e_1e_1 = e_4 \quad e_1e_2 = e_3 \quad e_2e_2 = e_4 \quad e_3e_3 = e_4$

C_{07}^{4} : $e_1e_1 = e_4 \quad e_1e_2 = e_3 \quad e_3e_3 = e_4$

Ma'lumki, C_{01}^{2*} algebraning har qanday 3-o'lchamli markaziy kengaytmasi ajraluvchi algebra bo'ladi. Shuning uchun, 5-o'lchamli ajralmaydigan algebralarning to'liq tasnifini hosil qilish uchun $\mathrm{C}_{01}^{3*} = \mathrm{C}_{01}^{2*} \oplus \mathbb{C}e_3$, C_{02}^{3*}, C_{03}^{3*}, C_{01}^{3} algebralarning 2-o'lchamli markaziy kengaytmalarini va 4-o'lchamli nilpotent kommutativ $\mathfrak{CD}$-algebralarning 1-o'lchamli markaziy kengaytmalarini qarash yetarli.

Quyida, biz 3- va 4-o'lchamli kommutativ $\mathfrak{CD}$-algebralarning ikkinchi kogomologik fazolarining tasnifini keltriamiz. Biz Yordan bo'lmagan kommutativ $\mathfrak{CD}$-algebralarni topishni maqsad qilganimiz uchun uchbu natijalarda algebralarning Yordan kogomologiyalari ham keltirilgan.

2-tasdiq. *Quyidagi elementlar 3-o'lchamli nilpotent kommutativ $\mathfrak{CD}$-algebraining ikkinchi kogomologik fazolari uchun bazis bo'ladi*:

Algebra	Yordan kogomologiya	$\mathfrak{CD}$-kogomologiya
C_{01}^{3*}	$\mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{01}^{3*}) = \langle [\Delta_{12}],[\Delta_{13}],[\Delta_{23}],[\Delta_{33}] \rangle$	$\mathrm{H}_{\mathfrak{CD}}^2(\mathrm{C}_{01}^{3*}) = \mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{01}^{3*}) \oplus \langle [\Delta_{22}]$
C_{02}^{3*}	$\mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{02}^{3*}) = \langle [\Delta_{13} + \Delta_{22}] \rangle$	$\mathrm{H}_{\mathfrak{CD}}^2(\mathrm{C}_{02}^{3*}) = \mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{02}^{3*}) \oplus \langle [\Delta_{22}] \rangle$
C_{03}^{3*}	$\mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{03}^{3*}) = \langle [\Delta_{11}],[\Delta_{22}],[\Delta_{13}],[\Delta_{23}] \rangle$	$\mathrm{H}_{\mathfrak{CD}}^2(\mathrm{C}_{03}^{3*}) = \mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{03}^{3*}) \oplus \langle [\Delta_{33}] \rangle$
C_{01}^{3}	–	$\mathrm{H}_{\mathfrak{CD}}^2(\mathrm{C}_{01}^{3}) = \langle [\Delta_{12}] \rangle$

3-tasdiq. C_{07}^{4*}, C_{11}^{4*}, C_{12}^{4*}, C_{01}^{4}, C_{03}^{4}, C_{04}^{4}, C_{05}^{4}, C_{06}^{4}, C_{07}^{4} *har qanday markaziy kengaymasi trivial bo'ladi hamda* C_{01}^{4*}, C_{02}^{4*}, ..., C_{09}^{4*}, $\mathrm{C}_{02}^{4}(\alpha)$ *algebraining ikkinchi kogomologik fazolari uchun basis elementlar quyidagicha bo'ladi.*

Algebra	Yordan kogomologiya	$\mathfrak{CD}$-kogomologiya
C_{01}^{4*}	$\mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{01}^{4*}) = \langle [\Delta_{ij}] \rangle_{1\le i\le j\le 4}^{(i,j)\notin\{(1,1),(2,2)\}}$	$\mathrm{H}_{\mathfrak{CD}}^2(\mathrm{C}_{01}^{4*}) = \mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{01}^{4*}) \oplus \langle [\Delta_{22}]$
C_{02}^{4*}	$\mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{02}^{4*}) = \left\langle \begin{matrix} [\Delta_{13} + \Delta_{22}],[\Delta_{14}], \\ [\Delta_{24}],[\Delta_{44}] \end{matrix} \right\rangle$	$\mathrm{H}_{\mathfrak{CD}}^2(\mathrm{C}_{02}^{4*}) = \mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{02}^{4*}) \oplus \langle [\Delta_{13}] \rangle$
C_{03}^{4*}	$\mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{03}^{4*}) = \langle [\Delta_{ij}] \rangle_{1\le i\le j\le 4}^{(i,j)\notin\{(1,2),(3,3)\}}$	$\mathrm{H}_{\mathfrak{CD}}^2(\mathrm{C}_{03}^{4*}) = \mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{03}^{4*}) \oplus \langle [\Delta_{33}] \rangle$
C_{04}^{4*}	$\mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{04}^{4*}) = \left\langle \begin{matrix} [\Delta_{12}],[\Delta_{13}],[\Delta_{14}], \\ [\Delta_{23}],[\Delta_{24}] \end{matrix} \right\rangle$	$\mathrm{H}_{\mathfrak{CD}}^2(\mathrm{C}_{04}^{4*}) = \mathrm{H}_{\mathfrak{J}}^2(\mathrm{C}_{04}^{4*}) \oplus \langle [\Delta_{33}],[\Delta_{34}],[\Delta_{44}] \rangle$

C_{05}^{4*}	$H_{\mathfrak{J}}^2(C_{05}^{4*}) = \left\langle \begin{matrix}[\Delta_{13}],[\Delta_{14}],[\Delta_{22}], \\ [\Delta_{23}],[\Delta_{24}]\end{matrix} \right\rangle$	$H_{\mathfrak{CD}}^2(C_{05}^{4*}) = H_{\mathfrak{J}}^2(C_{05}^{4*}) \oplus \langle [\Delta_{33}],[\Delta_{34}],[\Delta_{44}] \rangle$
C_{06}^{4*}	$H_{\mathfrak{J}}^2(C_{06}^{4*}) = \langle [\Delta_{ij}] \rangle_{1\le i\le j\le 4}^{(i,j)\notin\{(1,1),(4,4)\}}$	$H_{\mathfrak{CD}}^2(C_{06}^{4*}) = H_{\mathfrak{J}}^2(C_{06}^{4*}) \oplus \langle [\Delta_{44}] \rangle$
C_{08}^{4*}	$H_{\mathfrak{J}}^2(C_{08}^{4*}) = \left\langle \begin{matrix}[\Delta_{13}],[\Delta_{14}+\Delta_{22}], \\ [\Delta_{23}],[\Delta_{33}]\end{matrix} \right\rangle$	$H_{\mathfrak{CD}}^2(C_{08}^{4*}) = H_{\mathfrak{J}}^2(C_{08}^{4*}) \oplus \langle [\Delta_{14}] \rangle$
C_{09}^{4*}	$H_{\mathfrak{J}}^2(C_{09}^{4*}) = \langle [\Delta_{14}] + [\Delta_{23}] \rangle$	$H_{\mathfrak{CD}}^2(C_{09}^{4*}) = H_{\mathfrak{J}}^2(C_{09}^{4*}) \oplus \langle [\Delta_{22}] \rangle$
$C_{02}^{4}(\alpha)$	–	$H_{\mathfrak{CD}}^2(\mathfrak{C}_{02}^4(\alpha)_{\alpha\neq 0,1}) = \langle [\Delta_{22}],[\Delta_{14}+(3+\alpha)\Delta_{23}] \rangle$ $H_{\mathfrak{CD}}^2(\mathfrak{C}_{02}^4(1)) = \langle [\Delta_{22}],[\Delta_{14}+4\Delta_{23}],[\Delta_{24}] \rangle$ $H_{\mathfrak{CD}}^2(\mathfrak{C}_{02}^4(0)) = \langle [\Delta_{13}],[\Delta_{14}+3\Delta_{23}] \rangle$

bu yerda $C_{01}^{4*} = C_{01}^{2*} \oplus \mathbb{C}e_3 \oplus \mathbb{C}e_4$.

Bu bo‘limning asosiy natijasi quyidagi teoremadan iborat:

2-teorema. *Har qanday* 5*-o‘lchamli kommutativ* $\mathfrak{CD}$*-algebra yoki Yordan algebra yoki* **A ilova** (81 *ta algebra, ulardan* 12 *tasi bitta parametrli,* 2 *tasi ikkita parametrli oilalardan iborat*) *dagi algebralardan biriga izomorf bo‘ladi*.

Ikkinchi bobning uchinchi bo‘limida 5-o‘lchamli nilpotent kommutativ algebralar to‘liq tasniflangan. Biz oldingi bo‘limda kommutativ $\mathfrak{CD}$-algebra algebralarni tasniflaganimiz uchun bu bo‘limda $\mathfrak{CD}$ bo‘lmagan kommutativ algebralar tasnifiga e’tibor qaratamiz.

Aytaylik, $(\mathbf{A},\cdot)$ kompleks sonlar maydoni ustida berilgan kommutativ algebra va $\mathbb{V}$ shu maydon ustida chiziqli fazo bo‘lsin. $Z^2(\mathbf{A},\mathbb{V})$ orqali $\theta(x,y)=\theta(y,x)$ ayniyatni qanoatlantiruvchi barcha bichiziqli $\theta{:}\mathbf{A}\times\mathbf{A}\to\mathbb{V}$ akslantirishlar to‘plamini belgilaymiz. Bu bichiziqli akslantirishlar **A** kommutativ algebraning trivial 2-kosikllari deb ataladi.

Quyidagi teorema ushbu bo‘limning asosiy natijasi hisoblanadi:

3-teorema. *Har qanday* 5*-o‘lchamli kommutativ algebra yoki* $\mathfrak{CD}$*-algebra yoki* **B ilova***dagi* (409 *ta algebra, ulardan* 99 *tasi bitta parametrli,* 63 *tasi ikkita parametrli, 28 tasi uch parametrli 11 tasi to‘rt parametrli 6 tasi besh parametrli, bittasi olti parametrli oilalardan iborat*) *algebralardan biriga izomorf bo‘ladi*.

Dissertatsiyaning “**Kichik o‘lchamli noassotsiativ algabralarning geometrik tasnifi**” nomli uchinchi bobida 4-o‘lchamli nilpotent nokommutativ Yordan algebralari va 5-o‘lchamli kommutativ $\mathfrak{CD}$-algebralarining geometrik xossalari o‘rganilgan.

Aytaylik, $\mathbb{V}$ – n-o‘lchamli chiziqli fazo bo‘lsin, u holda barcha bichiziqli formalar to‘plami Hom($\mathbb{V}\otimes\mathbb{V},\mathbb{V}$) ni n^3-o‘lchamli chiziqli fazo sifatida qarash mumkin. Bu $\mathbb{C}^{n^3}$ affin ko‘pxilligi strukturasiga ega bo‘ladi. e_1, e_2, …, e_n elementalar $\mathbb{V}$ fazoning bazisi bo‘lsin. U holda, har qanday $\mu\in$Hom($\mathbb{V}\otimes\mathbb{V},\mathbb{V}$) element n^3 ta strukturaviy konstantalar $c_{ij}^k\in\mathbb{C}$ orqali aniqlanadi, ya’ni $\mu(e_i\otimes e_j)=\sum_{k=1}^{n}c_{ij}^k e_k$. Agar Hom($\mathbb{V}\otimes\mathbb{V},\mathbb{V}$) fazoning qism to‘plami $c_{ij}^k\ (1\le i,j,k\le n)$ o‘zgaruvchilarning biror ko‘phadlari to‘plamidan iborat bo‘lsa, u holda bu qism to‘plam yopiq (Zariskiy topologiyasida) to‘plam deyiladi.

Aytaylik, T qandaydir ayniyatlar to‘plami bo‘lsin. U holda, T to‘plamdagi ayniyatlarni qanoatlantiruvchi $\mathbb{V}$ fazodagi algebraik strukturalar Hom($\mathbb{V}\otimes\mathbb{V},\mathbb{V}$) ko‘pxillikning yopiq qism to‘plami bo‘ladi. Bu qism to‘plamni $\mathbb{L}(T)$ orqali belgilaymiz. $\mathbb{L}(T)$ to‘plamda umumiy chiziqli gruppa GL($\mathbb{V}$) orqali quyidagicha ta’sir aniqlaymiz:

$$(g*\mu)(x\otimes y)=g\mu(g^{-1}x\otimes g^{-1}y),$$

bu yerda $x,y\in\mathbb{V}$, $\mu\in\mathbb{L}(T)\subset$Hom($\mathbb{V}\otimes\mathbb{V},\mathbb{V}$) va $g\in$GL($\mathbb{V}$). Berilgan $\mu\in\mathbb{L}(T)$ elementning orbitasini $O(\mu)$ va bu orbitaning yopig‘ini $\overline{O(\mu)}$ kabi belgilaymiz.

A va **B** T to‘plamdagi ayniyatlarni qanoatlantiruvchi n-o‘lchamli algebralar va $\mu,\lambda\in\mathbb{L}(T)$ mos ravishda ulardagi ko‘paytirish amallari bo‘lsin.

6-ta’rif. *Agar* $\lambda\in\overline{O(\mu)}$ *bo‘lsa, u holda* **A** *algebra* **B** *algebraga degeneratsiyalanadi deyiladi va* $\lambda\to\mu$ *kabi yoziladi*.

Aytaylik, **A** biror algebra va $\mu\in\mathbb{L}(T)$ ko‘paytirish amali bo‘lsin. Agar $O(\mu)$ ochiq to‘plam bo‘lsa, u holda **A** algebra qattiq deyiladi. Agar ko‘pxillikning biror qism to‘plami chekli sondagi notrivial yopiq to‘plamlarning birlashmasi sifatida ifodalanmasa, u holda u keltirilmas deyiladi. Ko‘pxillikning maksimal keltirilmas yopiq qism to‘plami shu ko‘pxillikning keltirilmas komponentasi deyiladi. Ma’lumki, har qanday chekli o‘lchamli affin ko‘pxillik chekli sondagi keltirilmas komponentalarning birlashmasi ko‘rinishida yagona ravishda ifodalanadi.

Uchinchi bobning birinchi bo‘limida 4-o‘lchamli nilpotent nokommutativ Yordan algebralari ko‘pxilligining keltirilmas komponentalari tasniflangan.

4-teorema. 4-*o‘lchamli nilpotent nokommutativ Yordan algebralari ko‘pxilligining o‘lchami* 14 *ga teng. Bu ko‘pxillik quyidagi algebralar orbitalari yopilmalarining birlashmasi ko‘rinishida ifodalanadi*:

$\mathfrak{N}_2(\alpha):\quad e_1e_1=e_3\quad e_1e_2=e_4\quad e_2e_1=-\alpha e_3\quad e_2e_2=-e_4,$

$\mathfrak{N}_3(\alpha):\quad e_1e_1=e_4\quad e_1e_2=\alpha e_4\quad e_2e_1=-\alpha e_4\quad e_2e_2=e_4\quad e_3e_3=e_4,$

$\mathbf{J}_{07}^4:\quad e_1e_2=e_3\quad e_1e_3=e_4\quad e_2e_1=e_3+e_4\quad e_2e_3=e_4\quad e_3e_1=e_4\quad e_3e_2=e_4,$

$\mathbf{J}_{17}^4$: $\quad e_1e_1 = e_4 \quad e_1e_2 = e_3 \quad e_2e_1 = -e_3 \quad e_1e_3 = e_4 \quad e_2e_2 = e_4 \quad e_3e_1 = -e_4$

$\mathbf{J}_{18}^4$: $\quad e_1e_1 = e_2 \quad e_1e_2 = e_3 \quad e_2e_1 = e_3 \quad e_1e_3 = e_4 \quad e_2e_2 = e_4 \quad e_3e_1 = e_4$.

Bu yerda ($\alpha \in \mathbb{C}$).

Ikkinchi bobning ikkinchi bo‘limida 5-o‘lchamli kommutativ $\mathfrak{CD}$-algebra ko‘pxilligining keltirilmas komponentalari tasniflangan. Takidlash joizki, 5-o‘lchamli nilpotent assotsiativ kommutativ algebralar ko‘pxilligining keltirilmas komponentalari G.Mazzola tomonidan hamda 5-o‘lchamli nilpotent Yordan algebralari ko‘pxilligining keltirilmas komponentalari I.Kashuba va M.Martinlar tomonidan topilgan.

5-teorema. *5-o‘lchamli nilpotent kommutativ $\mathfrak{CD}$-algebralari ko‘pxilligining o‘lchami* 24 *ga teng va* 10 *ta keltirilmas komponentaga ega. Bundan tashqari,* $\mathfrak{C}_{69}^5, \mathfrak{C}_{72}^5, \mathfrak{C}_{76}^5, \mathfrak{C}_{77}^5, \mathfrak{C}_{81}^5, \mathfrak{J}_{21}$ *algebralar qattiq algebralar bo‘ladi.*

Dissertatsiyaning “**Cheksiz o‘lchamli sodda *n*-Li algebralar**” nomli to‘rtinchi bobi cheksiz o‘lchamli *n*-Li algebralarni o‘rganishga bag‘ishlangan. Bu bobdagi asosiy maqsad cheksiz o‘lchamli sodda *n*-Li algebralarini qurishdan iborat. Cheksiz o‘lchamli *n*-Li algebralarga bir qancha misollar keltirilib, bu algebralarning ba’zilari sodda bo‘lishi ko‘rsatilgan.

7-ta’rif. **A** *chiziqli fazoda anti-kommutativ n-ar amal* $[-,\dots,-]$ *aniqlangan bo‘lib barcha* $x_1,\dots,x_n,y_1,\dots,y_{n-1} \in \mathbf{A}$ *elementlar uchun quyidagi ayniyatlar o‘rinli bo‘lsa*:

$$[[x_1,\dots,x_n],y_1,\dots,y_{n-1}] = \sum_{s=1}^{n}[x_1,\dots,x_{s-1},[x_s,y_1,\dots,y_{n-1}],x_{s+1},\dots,x_n],$$

u holda **A** *n*-Li algebra deb ataladi. Takidlash joizki, 2-Li algebrasi Li algebrasi bo‘ladi.

Aytaylik, **A** assotsiativ kommutativ algebra bo‘lsin. Berilgan o‘zaro o‘rin almashinuvchi $d_1,\dots,d_n$ differensiallashlar orqali $x_1,\dots,x_n \in \mathbf{A}$ elementlar uchun Yakobian quyidagicha aniqlanadi:

$$Jac(x_1,\dots,x_n) = \begin{vmatrix} d_1(x_1) & \cdots & d_1(x_n) \\ \vdots & \ddots & \vdots \\ d_n(x_1) & \cdots & d_n(x_n) \end{vmatrix}.$$

Yakobian orqali aniqlangan algebrani $\mathbf{A}^*(d_1,\dots,d_n)$ kabi belgilaymiz.

Quyida V.T.Filippov tomonidan isbotlangan natijani keltiramiz.

6-teorema. *Assotsiativ, kommutativ A algebra uchun Yakobian orqali aniqlangan* $\mathbf{A}^*(d_1,\dots,d_n)$ *algebra n-Li algebra bo‘ladi.*

Shunga o‘xshash agar assotsiativ, kommutativ **A** algebra o‘zaro o‘rin almashinuvchi $d_1,\dots,d_{n-1}$ differensiallashlar orqali qaralsa, u holda A.Djumadildayev olgan natijaga ko‘ra **A** chiziqli fazo quyidagicha aniqlangan *n*-ar amal $[x_1,\dots,x_n]_W = Wr(x_1,\dots,x_n)$ ga nisbatan *n*-Li algebra bo‘ladi

$$Wr(x_1,\dots,x_n)=\begin{vmatrix} x_1 & x_2 & \cdots & x_n \\ d_1(x_1) & d_1(x_2) & \cdots & d_1(x_n) \\ \vdots & \vdots & \ddots & \cdots \\ d_{n-1}(x_1) & d_{n-1}(x_2) & \cdots & d_{n-1}(x_n) \end{vmatrix}.$$

Bu algebraga Vronskian algebrasi deyiladi.

N.Kantarini va V.Kats yuqoridagi ikkita va quyidagi misoldan boshqa cheksiz o‘lchamli sodda n-Li algebalari mavjud emas degan farazni ilgari surishgan.

1-misol. Aytaylik, **A** assotsiativ, kommutativ algebra, ***g*** esa, **A** algebraning differensiallashlari hosil qilgan Li algebra bo‘lsin. $SW(\mathbf{A},D)=\mathbf{A}^{<1>}\oplus\ldots\oplus\mathbf{A}^{<n\text{-}1>}$, bu yerda **A** algebraning n-1 ta to‘g‘ri yig‘indisi qaralgan bo‘lib, n-ar amal quyidagicha aniqlanadi: $h\in SW(\mathbf{A},\mathrm{D})$, $h^{<k>}\in\mathbf{A}^{<k}$ bo‘lsin, u holda

$$[h_1^{<1>},\dots h_{k-1}^{<k-1>},h_k^{<k>},h_{k+1}^{<k>},\dots,h_n^{<n-1>}]=$$
$$(-1)^{k+n-1}(h_1\dots h_{k-1}(D(h_k)h_{k+1}-h_kD(h_{k+1})h_{k+2}\dots h_n))^{<k>}$$

agar $\{j_1,\dots,j_n\}\supset\{1,\dots,n-1\}$ shart bajarilmasa $[h_1^{<j_1>},\dots,h_n^{<j_n>}]=0.$

Aytaylik, $\mathcal{A}$ assotsiativ kommutativ algebra va $d_1,\dots,d_{n+1}$ bu algebraning o‘zaro o‘rin almashinuvchi differsiallashlari bo‘lsin. Berilgan $\alpha_1,\alpha_2,\dots,\alpha_{n+1}\in\mathcal{A}$ va $x_1,\dots,x_n\in\mathcal{A}$ elementlar uchun $\mathcal{A}$ algebrada n-ar amalni quyidagicha aniqlaymiz:

$$[x_1,\dots,x_n]_\alpha=Jac(x)_\alpha=\begin{vmatrix} d_1(x_1) & \cdots & d_1(x_n) & \alpha_1 \\ \vdots & \ddots & \vdots & \vdots \\ d_{n+1}(x_1) & \cdots & d_{n+1}(x_n) & \alpha_{n+1} \end{vmatrix}.$$

7-teorema. *Barcha* $x_1,\dots,x_n,y_1,\dots,y_{n-1}\in\mathcal{A}$ *elementlar uchun quyidagi tenglik o‘rinli bo‘ladi:*

$$\sum_{s=1}^{n}[x_1,\dots,x_{s-1},[x_s,y_1,\dots,y_{n-1}]_\alpha,x_{s+1},\dots,x_n]_\alpha-[[x_1,\dots,x_n]_\alpha,y_1,\dots,y_{n-1}]_\alpha$$
$$=\sum_{k=1,k\neq i,j}^{n+1}\sum_{1\le i<j\le n+1}(-1)^{i+j+k}\left(\begin{vmatrix}\alpha_i & \alpha_j\\ d_k(\alpha_i) & d_k(\alpha_j)\end{vmatrix}+\begin{vmatrix}\alpha_j & \alpha_k\\ d_i(\alpha_j) & d_i(\alpha_k)\end{vmatrix}+\begin{vmatrix}\alpha_k & \alpha_k\\ d_j(\alpha_i) & d_j(\alpha_k)\end{vmatrix}\right)\times$$
$$Jac_k(x)Jac_{i,j}(y).$$

1-natija. $\langle\mathcal{A},[-,\dots,-]_\alpha\rangle$ *algebra n-Li algebra bo‘lishi uchun quyidagi shartlar o‘rinli bo‘lishi zarur va yetarli:*

$$\begin{vmatrix}\alpha_i & \alpha_j\\ d_k(\alpha_i) & d_k(\alpha_j)\end{vmatrix}+\begin{vmatrix}\alpha_j & \alpha_k\\ d_i(\alpha_j) & d_i(\alpha_k)\end{vmatrix}+\begin{vmatrix}\alpha_k & \alpha_k\\ d_j(\alpha_i) & d_j(\alpha_k)\end{vmatrix}=0,$$

bu yerda, $i,j,k\in\{1,\dots,n+1\}$ *va* $i\neq j$, $k\neq i$, $k\neq j$.

Aytaylik, $\mathcal{A}$ assotsiativ kommutativ algebra va $d_1,\dots,d_n$ bu algebraning o‘zaro o‘rin almashinuvchi differensiallashlari bo‘lsin. Berilgan $\alpha_0,\alpha_1,\dots,\alpha_n\in\mathcal{A}$

va $x_1,\dots,x_n\in\mathcal{A}$ elementlar uchun $\mathcal{A}$ algebrada n-ar amalni quyidagicha aniqlaymiz:

$$\{x_1,\dots,x_n\}_\alpha=Wr(x)_\alpha=\begin{vmatrix} x_1 & \cdots & x_n & \alpha_0\\ d_1(x_1) & \cdots & d_1(x_n) & \alpha_1\\ \vdots & \ddots & \vdots & \vdots\\ d_n(x_1) & \cdots & d_n(x_n) & \alpha_n\end{vmatrix}.$$

8-teorema. *Barcha* $x_1,\dots,x_n,y_1,\dots,y_{n-1}\in\mathcal{A}$ *elementlar uchun quyidagi tenglik o'rinli bo'ladi:*

$$\sum_{s=1}^{n}\{x_1,\dots,x_{s-1},\{x_s,y_1,\dots,y_{n-1}\}_\alpha,x_{s+1},\dots,x_n\}_\alpha-\{\{x_1,\dots,x_n\}_\alpha,y_1,\dots,y_{n-1}\}_\alpha$$

$$=\sum_{j,k=1,j\neq k}^{n+1}(-1)^{j+k+1}\left(\begin{vmatrix}\alpha_0 & \alpha_j\\ d_k(\alpha_0) & d_k(\alpha_j)\end{vmatrix}+\begin{vmatrix}\alpha_j & \alpha_0\\ d_i(\alpha_j) & d_i(\alpha_0)\end{vmatrix}\right)Wr_k(x)Jac_j(y)$$

$$+\sum_{j,k=1,j\neq k}^{n+1}(-1)^{i+j+1}\left(\begin{vmatrix}\alpha_j & \alpha_0\\ d_i(\alpha_j) & d_i(\alpha_0)\end{vmatrix}+\begin{vmatrix}\alpha_0 & \alpha_i\\ d_j(\alpha_0) & d_j(\alpha_i)\end{vmatrix}\right)Jac(x)Wr_{i,j}(y)$$

$$+\sum_{k=1,k\neq i,j}^{n+1}\sum_{1\le i<j\le n+1}(-1)^{i+j+k}\left(\begin{vmatrix}\alpha_i & \alpha_j\\ d_k(\alpha_i) & d_k(\alpha_j)\end{vmatrix}+\begin{vmatrix}\alpha_j & \alpha_k\\ d_i(\alpha_j) & d_i(\alpha_k)\end{vmatrix}+\begin{vmatrix}\alpha_k & \alpha_k\\ d_j(\alpha_i) & d_j(\alpha_k)\end{vmatrix}\right)\times$$

$$Wr_k(x)Wr_{i,j}(y).$$

2-natija. $\langle\mathcal{A},Wr(-,\cdots,-)_\alpha\rangle$ *algebra n-Li algebra bo'lishi uchun quyidagi shartlar o'rinli bo'lishi zarur va yetarli:*

$$\begin{vmatrix}\alpha_j & \alpha_0\\ d_i(\alpha_j) & d_i(\alpha_0)\end{vmatrix}+\begin{vmatrix}\alpha_0 & \alpha_i\\ d_j(\alpha_0) & d_j(\alpha_i)\end{vmatrix}=0,$$

$$\begin{vmatrix}\alpha_i & \alpha_j\\ d_k(\alpha_i) & d_k(\alpha_j)\end{vmatrix}+\begin{vmatrix}\alpha_j & \alpha_k\\ d_i(\alpha_j) & d_i(\alpha_k)\end{vmatrix}+\begin{vmatrix}\alpha_k & \alpha_k\\ d_j(\alpha_i) & d_j(\alpha_k)\end{vmatrix}=0,$$

bu yerda, $i,j,k\in\{1,\dots,n+1\}$ *va* $i\neq j$, $k\neq i$, $k\neq j$.

To'rtinchi bobning ikkinchi bo'limida cheksiz o'lchamli sodda n-Li algebralariga misollar keltirilgan.

$\mathcal{WA}_D^\alpha$ algebrada $\alpha=(1,0,\dots,0,1)$ bo'lgan holni qaraylik. Bu algebrani $\mathcal{WA}_D^{1,0,1}$ kabi belgilaymiz.

Ushbu bobning asosiy natijalari quyidagi teoremalardan iborat:

9-teorema. $\mathcal{WA}_D^{1,0,1}$ *algebra sodda n-Li algebra bo'ladi.*

$\mathcal{A}_n=\mathbb{F}[x_1^{\pm},x_2^{\pm},\dots,x_{n+1}^{\pm}]$ assotsiativ kommutativ algebrani $d_i=\dfrac{\partial}{\partial x_i}$ differensiallashlar hamda $\alpha_i=\beta_i\boldsymbol{x}x_i^{-1}$ parametrlar bilan qaraylik, bu yerda $\beta_i\in\mathbb{F}$, $1\le i\le n+1$, $\boldsymbol{x}=x_1\dots x_{n+1}$.

10-teorema. *Agar* β_i *haqiqiy sonlar orasida* $\beta_i \notin \mathbb{Q}$, $\beta_j \in \mathbb{Q} \setminus \{0\}$ *shartlarni qanoatlantiruvchi sonlar mavjud bo'lsa, u holda* $\mathcal{JA}_n^{\beta}$ *algebra sodda n-Li algebra bo'ladi*.

11-teorema. *Agar biror* $i \in \mathbb{N}$ *uchun* $\beta_i \notin \mathbb{Z}$ *bo'lsa,* $\mathcal{WA}_n^{\beta}$ *algebra sodda n-Li algebra bo'ladi.*

XULOSA

Ushbu dissertatsiya kompleks sonlar maydoni ustida 4-o'lchamli nilpotent nokommutativ Yordan algebralar, 5-o'chamli kommutativ $\mathfrak{CD}$-algebralar hamda 5-o'chamli kommutativ algebralarning algebraik tasniflari olingan. Bundan tashqari, 4-o'lchamli nilpotent nokommutativ Yordan algebralar hamda 5-o'chamli kommutativ $\mathfrak{CD}$-algebralarning geometrik tasniflari aniqlangan. Ya'ni, kompleks sonlar maydoni ustida aniqlangan 4-o'lchamli nilpotent nokommutativ Yordan algebralar uchun zaruriy barcha degeneratsiyalar qurilib, ushbu ko'pxillik 5 ta keltirilmas komponentalarning birlashmasidan iborat ekanligi isbotlangan. Bu kabi natija 5-o'chamli nilpotent kommutativ $\mathfrak{CD}$-algebralar uchun ham olingan bo'lib, 5-o'chamli kommutativ $\mathfrak{CD}$-algebralar ko'pxilligi 10 ta keltirilmas komponentalarning birlashmasidan iboratligi isbotlangan.

Dissertatsiyaning to'rtinchi bobida Yakobian va Vronskian operatorlarini umumlashtirgan holda yangi n-ar algebralar qurilgan. Ushbu n-ar algebralarning n-Li algebrasi bo'lishi uchun shartlar topilib, yangi sodda n-Li algebralari qurilgan. Aniqroq aytganda Yakobian operatorini umumlashtirgan holda bitta sodda n-Li algebrasi, Vronskian operatorini umumlashtirib esa ikkita yangi sodda *n*-Li algebralari qurilgan.

Dissertatsiyaning asosiy natijalari quyidagilardan iborat:

1. 4-o'lchamli nilpotent nokommutativ Yordan algebralarning algebraik tasnifi keltirilgan;
2. 5-o'lchamli nilpotent kommutativ $\mathfrak{CD}$-algebralarning algebraik tasnifi olingan;
3. 5-o'lchamli nilpotent kommutativ algebralarning algebraik tasnifi keltirilgan;
4. 4-o'lchamli nilpotent nokommutativ Yordan algebralarning geometrik tasnifi olingan;
5. 5-o'lchamli nilpotent kommutativ $\mathfrak{CD}$-algebralarning geometrik tasnifi hosil qilingan;
6. Cheksiz o'lchamli n-Li algebralar qurilgan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 AT V.I.ROMANOVSKIY INSTITUTE OF MATHEMATICS

INSTITUTE OF MATHEMATICS

JUMANIYOZOV DOSTON ERKABOY UGLI

DESCRIPTION OF SMALL DIMENSIONAL NON-ASSOCIATIVE ALGEBRAS AND CONSTRUCTION OF SIMPLE *n*-LIE ALGEBRAS

01.01.06 – algebra

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT-2023

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Ministry of Higher education, Science and Innovations of the Republic of Uzbekistan under number B2023.3.PhD/FM915.

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Abstract of dissertation sent out on «5th» december
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INTRODUCTION (abstract of PhD thesis)

Actuality and demand of the theme of the dissertation. Today, as well as in theoretical physics, many properties of mathematical objects are studied through methods defined by elements of non-associative algebras. For instance, non-associative algebras are applied in general relativity, quantum field theory, quantum mechanics and string theory. The problems in the theory of non-associative algebras has been essential for many years and Lie algebras are the most important and useful mathematical objects among them. Algebraic classification of algebras of small dimension from a certain variety defined by a family of polynomial identities is a classic problem in the theory of non-associative algebras. The algebraic classification of nilpotent algebras can be obtained by the calculation of central extensions of algebras from the same variety which have a smaller dimension. An interesting approach of studying algebras of a fixed dimension is to study them from a geometric point of view, in which the study of degenerations leads us to define all irreducible components of this variety.

Another interesting direction in the theory of non-associative algebras is the theory of n-Lie algebras. An n-Lie algebra defined by the operator of Jacobian is one of the most important examples of infinite-dimensional n-Lie algebras and it enabled to construct other infinite-dimensional n-Lie algebras. So far, there exist two non-isomorphic infinite-dimensional simple n-Lie algebras which are defined by the operators of Jacobian and Wronskian and it is still actual to answer whether there exists another infinite-dimensional simple n-Lie algebra in the theory of n-ary algebras.

In recent years our country has paid increasing attention to mathematics, physics, geology and biological sciences, which have a scientific and practical applications of fundamental sciences. In particular, special attention was paid to the development of fundamental research on non-associative algebras. Within the frame of this fundamental research, significant results have been obtained in the study of finite dimensional non-associative algebras. The main task and direction of scientific work are conducting research[2] at the level of international standards on the priority areas “Algebra and Functional Analysis”. To ensure the implementation of the task, it is important to develop the theory of finite dimensional non-associative algebras for the application of scientific results in the relevant field of science.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees of the President of the Republic of Uzbekistan UP-4947 of February 7, 2017 “On the Strategy of actions for the Further Development of the Republic of Uzbekistan for 2017-2021”, UP-2789 dated February 17, 2017 “On measures to further improve the activities of the Academy of Sciences, organization, management and financing of research activities”, PP-3682 from April 27, 2018 “On measures to further improve the system of practical

[2] Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May « On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan» № 292 dated May 17, 2017.

implementation of innovative ideas, technologies and projects” and PP-4387 from July 9, 2019 “On the State support for the further development of Mathematical education and science, as well as radical improvement of the activities of V.I.Romanovskiy Institute of Mathematics of the Academy of Sciences of the Republic of Uzbekistan”, as well as in other regulations related to basic science.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV, Mathematics, Mechanics and Computer Science.

The degree of scrutiny of the problem. Lie algebras were introduced to study the concept of infinitesimal transformations by M. Sophus Lie in the 1870s. Jordan algebras were introduced by Pascual Jordan in 1933 in an effort to formalize the notion of an algebra of observables in quantum electrodynamics. After that generalizations of Jordan algebras such as Malcev algebras, Leibniz algebras quadratic Jordan algebras, noncommutative Jordan algebras, commutative $\mathfrak{CD}$-algebras were introduced by A.I.Malcev, J.L.Loday, K.McCrimmon, M.Osborn, I.Kaygorodov, P.Zusmanovich.

The theory of Lie algebras and their generalizations evolved as a special direction in the theory of non-associative algebras. Namely, the theory of Leibniz algebras has been important direction in recent years and many results related to the structure and classification problems have been obtained by Sh.Ayupov, S.Albeverio, K.Kudaybergenov, B.Omirov, I.Rahimov. Noncommutative Jordan algebras were introduced initially by A.Albert in 1948. He noted that the structure theories of alternative and Jordan algebras share so many nice properties that it is natural to conjecture that these algebras are members of a more general class with a similar theory. So he introduced the variety of noncommutative Jordan algebras. In 1955, R.Schafer proved that noncommutative Jordan algebras were defined by the Jordan identity and the flexibility identity. Later, a new class of non-associative algebras ($\mathfrak{CD}$-algebras) introduced by I.Kaygorodov and P.Zusmanovich. The idea of the definition of a $\mathfrak{CD}$-algebra comes from the following property of Jordan and Lie algebras: the commutator of any pair of multiplication operators is a derivation. In addition, many researchers have been paying attention to the algebraic classification nilpotent non-associative algebras in recent years. Main reason for this is to have clearer information about the variety of algebras. Moreover, the algebraic classification allows us to investigate this variety from geometric point of view. That is, by geometric classification of algebras the problem of describing all irreducible components of the variety can be obtain. There are many scientific works concerning algebraic and geometric classification of nilpotent non-associative algebras. For example, the algebraic classification of nilpotent associative commutative algebras over the field of complex numbers was obtained by I.Kaygorodov, I.Rakhimov and Sh.K.Said Hussain. A.Hegazi and H.Abdelwahab classified 5-dimensional nilpotent non-associative Jordan algebras. I.Kashuba and M.Martin investigated their geometric properties 5-dimensional nilpotent non-associative Jordan algebras. Moreover, the classification of orbit closures of 4-dimensional Lie algebras over the field of complex numbers was obtained by D.Burde and C.Steinhoff.

The theory of n-Lie algebras is another important direction in the theory of non-associative algebras. The general concept of Ω-algebra were introduced by A.G.Kurosh and after that V.T.Filippov determined n-Lie algebra that is defined by the operator of Jacobian. Moreover, A.Dzhumadildayev gave an example of n-Lie algebra which defined by the operator of Wronskian. N.Cantarini and V.Kac proved the simplicity of n-Lie algebras of Jacobian and Wronskian and gave example of another infinite-dimensional simple n-Lie algebra. They stated that whether there exists another infinite-dimensional simple n-Lie algebra.

Connection of the theme of the dissertation with the research works of higher education, where the dissertation is carried out. The dissertation work is carried out in accordance with the given topic of scientific research FZ-2020093065 «Automorphisms of operator algebras, classification of infinite-dimensional non-associative algebras and superalgebras» at V.I. Romanovskiy Institute of Mathematics (2021-2025).

The aim of research work is to classify the varieties of small-dimensional niplotent noncommutative Jordan and commutative $\mathfrak{CD}$-algebras, to obtain all irreducible components of these varieties and to construct infinite-dimensional simple n-Lie algebras.

Research problems:

the algebraic classification of four dimensional nilpotent noncommutative Jordan and five dimensional nilpotent commutative algebras is obtained;

all irreducible components of the variety of four dimensional are found and it is proved that there exist five rigid algebras in this variety;

it is proved that there exist ten irreducible components in the variety of algebras in which commutator of any right or left multiplication operators being derivation;

it is constructed new n-Lie algebras by generalizing known n-Lie algebras defined by the operators of Jacobian and Wronskian and it is proved the simplicity of some of these algebras.

The research object: noncommutative Jordan algebras, commutative $\mathfrak{CD}$-algebras, infinite-dimensional n-Lie algebras.

The research subject: four-dimensional niplotent noncommutative Jordan algebras, five-dimensional nilpotent commutative $\mathfrak{CD}$-algebras, infinite-dimensional simple n-Lie algebras.

Research methods: the method of central extensions, classification methods, method of degenerations, as well as the methods of invariant theory are applied in the dissertation.

Scientific novelty of the research work consists of the following:

algebraic classification of 4-dimensional niplotent noncommutative Jordan algebras is obtained;

algebraic classification of 5-dimensional nilpotent commutative $\mathfrak{CD}$-algebras and 5-dimensional niplotent commutative algebras is obtained;

geometric classification of 4-dimensional nilpotent noncommutative Jordan algebras is accomplished;

geometric classification of 5-dimensional nilpotent commutative $\mathfrak{CD}$-algebras is obtained;

new examples of infinite-dimensional simple n-Lie algebras are constructed.

Practical results of the research. Obtained results and used methods in the dissertation can be taught as a graduate course for masters and doctoral students of higher education institutions. In addition, the results of the dissertation concerning infinite-dimensional simple n-Lie algebras allow to verify number of hypothesis about simple n-Lie algebras algebras.

The reliability of the results of the study. Our results have been obtained by using invatiant theory, the known methods of investigation of other classes of algebras, fundamental results of the theory of algebras. The proofs of all obtained results are mathematically correct.

Scientific and practical significance of the research results. The scientific significance of the results of research can be used for further research of varieties of other algebras. In particular, the techniques and methods developed in this dissertation can be used to solve the hypothesis on infinite-dimensional n-Lie algebras.

The practical significance of the dissertation is that the obtained results can be used in the theory of algebraic structure. All results and used methods in the dissertation can be used in the graduate courses for masters and doctoral students of higher education institutions.

Implementation of the research results. Based on the results of the classification of small dimensional non-associative algebras and construction of simple n-Lie algebras:

results obtained in the classification of algebras in which commutator of any right or left multiplication operators being derivation were used in the research project «Nonregular sets in dynamic systems», № AP08051987 (Reference № 1.1-11/1141, Suleyman Demirel University, dated November 1, 2023, Kazakhstan) to obtain the classification of five dimensional one-generated nilpotent asymmetric algebras. The results made it possible to discover irreducible components of varieties of such algebras;

results obtained in the algebraic classification of nilpotent noncommutative Jordan algebras were applied in international scientific journals (Journal of Algebra and its Applications, 20, 2021, 2150198, Communications in Mathematics, 29, 2021, 215-226, Journal of Pure and Applied Algebra, 226, 2022, 106850 and Journal of Algebra and its Applications, 21, 2022, 2250031) to find irreducible components of nilpotent algebras and to obtain the algebraic and geometric classification of nilpotent right-commutative algebras. These applications made it possible to get full classification of five and six dimensional one-generated nilpotent complex bicommutative algebras.

Approbation of the research results. The main results of the research have been discussed at 4 international and 4 national scientific conferences.

Publications of the research results. On the topic of the dissertation, 11 scientific papers were published, 6 of which are included in the list of scientific publications proposed by the Higher Attestation Commission of the Republic of Uzbekistan for the defense of thesis of the Doctor of Philosophy, including 5 of them published in international journals and 1 in national scientific journals and 5 abstracts.

The structure and volume of the dissertation. The dissertation consists of the introduction, four chapters with conclusions, general conclusion and bibliography. The total volume of the work is 120 pages.

THE MAIN CONTENT OF THE DISSERTATION

The **Introduction** part of the dissertation includes the actuality and the demand of the research, the relevance of the research to the priority areas of science and technology. Moreover, in this part, the degree of scrutiny of the problem, the aim of the research work, the object and subject of the research and scientific novelty of the research work are presented. The theoritical and practical significance of the research is revealed and implementation and approbation of the research results, publications of the research results and the structure and volume of the dissertation are presented in this part.

The first chapter of the dissertation, entitled "**Preliminaries and basic concepts**" contains basic information about theory of non-associative algebra as well as definitions of some important types of non-associative algebras. Furthermore, this chapter includes basic concepts of n-Lie algebras and methods to obtain classification of non-associative algebras and irreducible components of algebras.

In the first section of the first chapter, definitions of Jordan, noncommutative Jordan and $\mathfrak{CD}$-algebras are presented.

An *algebra over a field* (often simply called an *algebra*) is a vector space equipped with a bilinear product. Let **A** and **B** be two algebras. A linear map $\varphi : \mathbf{A} \to \mathbf{B}$ is said to be a homomorphism if the following condition holds:

$$\varphi(xy) = \varphi(x)\varphi(y),$$

for all $x, y \in \mathbf{A}$. If φ is bijective, then it is called an isomorphism between **A** and **B**. An isomorphism from algebra to itself is said to be an automorphism.

Definition 1. *An algebra* **A** *is called Jordan algebra if it satisfies the following identities*:

$$xy = yx, \qquad x^2(yx) = (x^2y)x,$$

for all $x, y \in \mathbf{A}$.

Definition 2. *An algebra* **A** *is called noncommutative Jordan algebra if the following identities are satisfied*:

$$x(yx) = (xy)x, \qquad x^2(yx) = (x^2y)x,$$

for all $x, y \in \mathbf{A}$.

It should be noted that any Jordan algebra is a noncommutative Jordan algebra.

For the algebra **A** let us define the following sequences:

$$\mathbf{A}^1 = \mathbf{A},\ \mathbf{A}^{k+1} = \mathbf{A}^k\mathbf{A} + \mathbf{A}^{k-1}\mathbf{A}^2 + \ldots + \mathbf{A}\mathbf{A}^k,\ k \geq 1,$$

so-called *the lower central series* of the algebra $\mathbf{A}$.

Definition 3. *An algebra* **A** *is called nilpotent, if there exists* $s \in \mathbb{N}$ *such that* $\mathbf{A}^s = 0$. *The minimal number s with such property is called index of nilpotency of* **A**. *If the index of nilpotency of algebra is equal to* 3 *then it is called* 2*-step nilpotent*.

A linear map $d : \mathbf{A} \to \mathbf{A}$ is said to be a *derivation* if the following condition holds:

$$d(xy) = d(x)y + xd(y),$$

for all $x, y \in \mathbf{A}$. We denote by Der(**A**) the set of all derivations of algebra **A**.

Definition 4. *An algebra* **A** *is called* $\mathfrak{CD}$*-algebra if* $[R_a, R_b], [R_a, L_b], [L_a, L_b]$ *are derivations of* **A**.

Equivalently, an algebra **A** is $\mathfrak{CD}$-algebra if it satisfies the following identities:

$$\big((xy)a\big)b + \big((xb)a\big)y + x\big((yb)a\big) = \big((xy)b\big)a + \big((xa)b\big)y + x\big((ya)b\big),$$
$$b\big((xy)a\big) + \big((bx)a\big)y + x\big((by)a\big) = \big(b(xy)\big)a + \big(b(xa)\big)y + x\big(b(ya)\big),$$
$$b\big(a(xy)\big) + \big(a(bx)\big)y + x\big(a(by)\big) = a\big(b(xy)\big) + \big(b(ax)\big)y + x\big(b(ay)\big).$$

In the second section of the first chapter, the method of central extensions is given.

Let (**A**, ·) be an algebra with a linear identity $F(x_1, \ldots, x_n) = 0$ and $\mathbb{V}$ be a complex vector space of dimension *k*. Consider a bilinear map θ:**A**×**A**→$\mathbb{V}$. By this map, define a bilinear product "[–,–]" on a linear space $\mathbf{A}_\theta = \mathbf{A} \oplus \mathbb{V}$ by

$$[x + x_0, y + y_0] = x \cdot y + \theta(x, y) \text{ for all } x, y \in \mathbf{A}, x_0, y_0 \in \mathbb{V}.$$

The algebra $\mathbf{A}_\theta$ is called a *k-dimensional central extension* of **A** by $\mathbb{V}$.

Obviously, $\mathbf{A}_\theta$ algebra satisfies the identity $F(x_1, \ldots, x_n) = 0$ if and only if it is satisfied with the identity $\theta(F(x_1, \ldots, x_n)) = 0.$ For example, if $F(x, y, z) = x(yz) + z(yx) - (xy)z - (zy)x$, then the identity $\theta(F(x_1, \ldots, x_n)) = 0$ will be of the following form:

$$\theta(F(x, y, z)) = \theta(x, yz) + \theta(z, yx) - \theta(xy, z) - \theta(zy, x).$$

Then, one can define $Z^2(\mathbf{A}, \mathbb{V})$ as the set of all bilinear maps which satisfy with the identity $\theta(F(x_1, \ldots, x_n)) = 0$. These elements will be called *trivial* 2-*cocycles*. For a linear map *f* from **A** to $\mathbb{V}$, if we define δf:**A**×**A**→$\mathbb{V}$ by $\delta f(x,y) = f(xy)$, then one can check that $\delta f \in Z^2(\mathbf{A},\mathbb{V})$. Thus, the vector space $B^2(\mathbf{A},\mathbb{V})=\{\theta=\delta f : f$ is a linear map$\}$ is a linear subspace of $Z^2(\mathbf{A},\mathbb{V})$. Then, it allows us to define the *second cohomology space* $H^2(\mathbf{A},\mathbb{V})$ as the quotient space $Z^2(\mathbf{A},\mathbb{V})/B^2(\mathbf{A},\mathbb{V})$.

Let $Aut(\mathbf{A})$ be the group of automorphisms of $\mathbf{A}$ and let $\phi \in Aut(\mathbf{A})$. For $\theta \in Z^2(\mathbf{A},\mathbb{V})$ we define the action of the group $Aut(\mathbf{A})$ on $Z^2(\mathbf{A},\mathbb{V})$ by $\phi.\theta(x,y)=\theta(\phi(x),\phi(y))$. It is easy to verify that $B^2(\mathbf{A},\mathbb{V})$ is invariant under the action of $Aut(\mathbf{A})$. So, we have an induced action of $Aut(\mathbf{A})$ on $H^2(\mathbf{A},\mathbb{V})$.

Call the set $Ann(\theta)=\{x \in \mathbf{A} : \theta(x,\mathbf{A})=\theta(\mathbf{A},x)=0\}$ the *annihilator* of θ. We recall that the *annihilator* of an algebra $\mathbf{A}$ is defined as the ideal $Ann(\mathbf{A})=\{x \in \mathbf{A} : x\mathbf{A}=\mathbf{A}x=0\}$. Observe that $Ann(\mathbf{A}_\theta)=(Ann(\theta)\cap Ann(\mathbf{A}))\oplus\mathbb{V}$.

The following result was proved for Jordan algebras by A.Hegazi and H.Abdelwahab. However, it can be applied to other types of non-associative algebras. It shows that every algebra with a non-zero annihilator is a central extension of a smaller-dimensional algebra.

Lemma 1. *Let* $\mathbf{A}$ *be an n-dimensional algebra of certain type such that* $dim(Ann(\mathbf{A}))=m\neq 0$. *Then there exists, up to isomorphism, a unique* $(n\text{-}m)$*-dimensional algebra* $\mathbf{A}'$ *of the same type and a bilinear map* $\theta \in Z^2(\mathbf{A},\mathbb{V})$ *with* $Ann(\theta)\cap Ann(\mathbf{A})=0$, *where* $\mathbb{V}$ *is a vector space of dimension m, such that* $\mathbf{A}\cong \mathbf{A}'_\theta$ *and* $\mathbf{A}/Ann(\mathbf{A})\cong \mathbf{A}'$.

Definition 5. *Let* $\mathbf{A}$ *be an algebra and* $\boldsymbol{I}$ *be a subspace of* $Ann(\mathbf{A})$. *If* $\mathbf{A}=\mathbf{A}_0\oplus\boldsymbol{I}$ *then* $\boldsymbol{I}$ *is called an annihilator component of* $\mathbf{A}$. *A central extension of an algebra* $\mathbf{A}$ *without annihilator component is called a non-split central extension.*

Let us fix a bases $e_1,e_2,\dots,e_s$ of $\mathbb{V}$, and $\theta_1, \theta_2,\dots, \theta_s$ of $Z^2(\mathbf{A},\mathbb{V})$. Then θ can be uniquely written as $\theta(x,y)=\sum_{i=1}^{s}\theta_i(x,y)e_i$, where $\theta_i \in Z^2(\mathbf{A}, \mathbb{C})$. Moreover, $Ann(\theta)=Ann(\theta_1)\cap Ann(\theta_2)\cap\dots\cap Ann(\theta_s)$. Furthermore, $\theta \in B^2(\mathbf{A},\mathbb{V})$ if and only if all $\theta_i \in B^2(\mathbf{A}, \mathbb{C})$.

The following lemma proved by Hegazi and Abdelhawab for Jordan algebras and it can be proved for any non-associative algebra.

Lemma 2. *Let* $\theta(x,y)=\sum_{i=1}^{s}\theta_i(x,y)e_i \in Z^2(\mathbf{A},\mathbb{V})$ *and* $Ann(\theta)\cap Ann(\mathbf{A})=0$, *then* $\mathbf{A}_\theta$ *has an annihilator component if and only if* $[\theta_1],[\theta_2],\dots,[\theta_s]$ *are linearly* *dependent in* $\mathrm{H}^2(\mathbf{A},\mathbb{C})$.

This shows that there exists a one-to-one correspondence between the set of $Aut(\mathbf{A})$-orbits on $T_s(\mathbf{A})$ and the set of isomorphism classes of $E(\mathbf{A},\mathbb{V})$. Consequently we have a procedure that allows us to construct all non-split central extensions of an algebra $\mathbf{A}'$ of dimension n-s. This procedure is:

1. For a given algebra $\mathbf{A}'$ of dimension n-s, determine $\mathrm{H}^2(\mathbf{A},\mathbb{C})$, $Ann(\mathbf{A}')$ and $Aut(\mathbf{A}')$.
2. Determine the set of $Aut(\mathbf{A}')$-orbits on of subspace of $\mathrm{H}^2(\mathbf{A},\mathbb{C})$.
3. For each orbit, construct the algebra associated with a representative of it.

Let us introduce the following notations. Let $\mathbf{A}$ be a nilpotent algebra with a basis $e_1, e_2, \ldots, e_n$. Then by Δ_{ij} we will denote the bilinear form

$$\Delta_{ij}:\mathbf{A}\times\mathbf{A}\to\mathbb{C},\ \Delta_{ij}(e_l,e_m)=\delta_{il}\delta_{jm}.$$

The set $\left\{\Delta_{ij} : 1\leq i,j\leq n\right\}$ is a basis for the linear space of bilinear forms on $\mathbf{A}$, so every $\theta\in \mathrm{Z}^2(\mathbf{A}, \mathbb{V})$ can be uniquely written as $\theta=\sum\limits_{1\leq i\leq j\leq n} c_{ij}\Delta_{ij}$, where $c_{ij}\in\mathbb{C}$.

The second chapter of the dissertation, entitled "**The algebraic claasification of small dimensional nilpotent non-associative algebras**" is devoted to obtain a complete algebraic description of the variety of all 4-dimensional nilpotent noncommutative Jordan algebras, 5-dimensional nilpotent commutative $\mathfrak{CD}$-algebras and 5-dimensional nilpotent commutative algebras over the field of complex numbers.

In the first section of the second chapter, we give algebraic classification of four-dimensional nilpotent noncommutative Jordan algebras.

Let $(\mathbf{J},\cdot)$ be a noncommutative Jordan algebra over $\mathbb{C}$ and $\mathbb{V}$ be a vector space over $\mathbb{C}$. The $\mathbb{C}$-linear space $\mathrm{Z}^2(\mathbf{J}, \mathbb{V})$ is defined as the set of all bilinear maps $\theta:\mathbf{J}\times\mathbf{J}\to\mathbb{V}$ such that

$$\theta(x,yz)+\theta(z,yx)=\theta(xy,z)+\theta(zy,x), \tag{2}$$

$$\theta(xt,yz)+\theta(tx,yz)+\theta(tz,yx)+\theta(xz,yt)+\theta(zt,yx)+\theta(zx,yt)=$$
$$\theta((xt)y,z)+\theta((tx)y,z)+\theta((xz)y,t)+\theta((tz)y,x)+\theta((zx)y,t)+\theta((zt)y,x). \tag{3}$$

These elements will be called *trivial* 2-*cocycles* of noncommutative Jordan algebra $\mathbf{J}$.

It is well-known that there does not exist any nontrivial 1-dimensional nilpotent Jordan algebra, and there is only one nontrivial 2-dimensional nilpotent Jordan algebra:

$$\mathbf{J}_{01}^2 : e_1e_1=e_2.$$

Thanks to A.Calderon Martin, A.Fernandez Ouaridi and I.Kaygorodov, we have the classification of all non-split 3-dimensional nilpotent noncommutative

Jordan algebras:

$\mathbf{J}_{02}^{3}$: $e_1e_2 = e_3 \quad e_2e_1 = e_3$,

$\mathbf{J}_{03}^{3}$: $e_1e_2 = e_3 \quad e_2e_1 = -e_3$,

$\mathbf{J}_{04}^{3}(\lambda)$: $e_1e_1 = \lambda e_3 \quad e_1e_2 = e_3 \quad e_2e_1 = e_3$,

$\mathbf{J}_{05}^{3}$: $e_1e_1 = e_2 \quad e_1e_2 = e_3 \quad e_2e_1 = e_3$.

In the following proposition we give the description of the second cohomology space of 3-dimensional nilpotent nontrivial noncommutative Jordan algebras.

Proposition 1. *The following elements are bases of the second cohomology spaces of* 3*-dimensional nilpotent nontrivial noncommutative Jordan algebras*:

A	$Z^2(\mathbf{A},\mathbb{C})$	$B^2(\mathbf{A},\mathbb{C})$	$H^2(\mathbf{A},\mathbb{C})$
$\mathbf{J}_{01}^{3}$	$\langle \Delta_{11},\Delta_{12}+\Delta_{21},\Delta_{13}, \Delta_{23}+\Delta_{32},\Delta_{31},\Delta_{33}\rangle$	$\langle \Delta_{11}\rangle$	$\langle [\Delta_{12}+\Delta_{21}],[\Delta_{13}], [\Delta_{23}+\Delta_{32}],[\Delta_{31}],[\Delta_{33}]\rangle$
$\mathbf{J}_{02}^{3}$	$\langle \Delta_{11},\Delta_{12},\Delta_{13}+\Delta_{31}, \Delta_{21},\Delta_{22},\Delta_{23}+\Delta_{32}\rangle$	$\langle \Delta_{12}+\Delta_{21}\rangle$	$\langle [\Delta_{11}],[\Delta_{13}+\Delta_{31}], [\Delta_{21}],[\Delta_{22}],[\Delta_{23}+\Delta_{32}]\rangle$
$\mathbf{J}_{03}^{3}$	$\langle \Delta_{11},\Delta_{12},\Delta_{13}-\Delta_{31}, \Delta_{21},\Delta_{22},\Delta_{23}-\Delta_{32}\rangle$	$\langle \Delta_{12}-\Delta_{21}\rangle$	$\langle [\Delta_{11}],[\Delta_{13}-\Delta_{31}], [\Delta_{21}],[\Delta_{22}],[\Delta_{23}-\Delta_{32}]\rangle$
$\mathbf{J}_{04}^{3}(\lambda)$	$\langle \Delta_{11},\Delta_{12},\Delta_{21},\Delta_{22}\rangle$	$\langle \lambda\Delta_{11}+\Delta_{21}+\Delta_{22}\rangle$	$\langle [\Delta_{11}],[\Delta_{12}],[\Delta_{22}]\rangle$
$\mathbf{J}_{05}^{3}$	$\langle \Delta_{11},\Delta_{12}+\Delta_{21}, \Delta_{13}+\Delta_{22}+\Delta_{31}\rangle$	$\langle \Delta_{11},\Delta_{21}+\Delta_{21}\rangle$	$\langle [\Delta_{13}+\Delta_{22}+\Delta_{31}]\rangle$

where $\mathbf{J}_{01}^{3} = \mathbf{J}_{01}^{2} \oplus \mathbb{C}e_3$.

Now, we present the main result of this section.

Theorem 1. *Any complex 4-dimensional nilpotent noncommutative Jordan algebra is* 2*-step nilpotent or is isomorphic to one of the following pairwise non-isomorphic algebras:*

$\mathbf{J}_{01}^{4}$: $e_1e_1 = e_2 \quad e_1e_2 = e_3 \quad e_2e_1 = e_3$

$\mathbf{J}_{02}^{4}$: $e_1e_1 = e_2 \quad e_2e_3 = e_4 \quad e_3e_2 = e_4 \quad e_3e_1 = e_4$

$\mathbf{J}_{03}^{4}$: $e_1e_1 = e_2 \quad e_1e_2 = e_4 \quad e_2e_1 = e_4 \quad e_3e_1 = e_4 \quad e_3e_3 = e_4$

$\mathbf{J}_{04}^{4}$: $e_1e_1 = e_2 \quad e_1e_2 = e_4 \quad e_2e_1 = e_4 \quad e_3e_1 = e_4$

$\mathbf{J}_{05}^{4}$: $e_1e_1 = e_2 \quad e_2e_3 = e_4 \quad e_3e_2 = e_4$

$\mathbf{J}_{06}^{4}$: $e_1e_1 = e_2 \quad e_1e_2 = e_4 \quad e_2e_1 = e_4 \quad e_3e_3 = e_4$

$\mathbf{J}_{07}^{4}$: $e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = e_3 + e_4 \quad e_2e_3 = e_4 \quad e_3e_1 = e_4 \quad e_3e_2 = e_4$

$\mathbf{J}_{08}^{4}$: $e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = e_3 + e_4 \quad e_2e_2 = e_4 \quad e_3e_1 = e_4$

$\mathbf{J}_{09}^{4}$: $e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = e_3 + e_4 \quad e_3e_1 = e_4$

$\mathbf{J}_{10}^{4}$: $e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = e_3 \quad e_2e_3 = e_4 \quad e_3e_1 = e_4 \quad e_3e_2 = e_4$

$\mathbf{J}_{11}^{4}$: $e_1e_2 = e_3 \quad e_1e_3 = e_4 \quad e_2e_1 = e_3 \quad e_3e_1 = e_4$

$\mathbf{J}_{12}^4:$ $e_1e_2=e_3$ $e_1e_3=e_4$ $e_2e_1=e_3$ $e_2e_2=e_4$ $e_3e_1=e_4$

$\mathbf{J}_{13}^4:$ $e_1e_2=e_3$ $e_1e_3=e_4$ $e_2e_1=-e_3$ $e_3e_1=-e_4$

$\mathbf{J}_{14}^4:$ $e_1e_1=e_4$ $e_1e_2=e_3$ $e_1e_3=e_4$ $e_2e_1=-e_3$ $e_3e_1=-e_4$

$\mathbf{J}_{15}^4:$ $e_1e_2=e_3$ $e_1e_3=e_4$ $e_2e_1=-e_3+e_4$ $e_3e_1=-e_4$

$\mathbf{J}_{16}^4:$ $e_1e_2=e_3$ $e_1e_3=e_4$ $e_2e_1=-e_3$ $e_2e_2=e_4$ $e_3e_1=-e_4$

$\mathbf{J}_{17}^4:$ $e_1e_1=e_4$ $e_1e_2=e_3$ $e_1e_3=e_4$ $e_2e_1=-e_3$ $e_2e_2=e_4$ $e_3e_1=-e_4$

$\mathbf{J}_{18}^4:$ $e_1e_1=e_2$ $e_1e_2=e_3$ $e_1e_3=e_4$ $e_2e_1=e_3$ $e_2e_2=e_4$ $e_3e_1=e_4$

In the second section of the second chapter, we give an algebraic classification of nilpotent commutative $\mathfrak{CD}$-algebras.

Let $(\mathbf{A},\cdot)$ be complex commutative $\mathfrak{CD}$-algebra and $\mathbb{V}$ be a complex vector space. The $\mathbb{C}$-linear space $\mathrm{Z}^2(\mathbf{A},\mathbb{V})$ is defined as the set of all bilinear maps θ : $\mathbf{A}\times\mathbf{A}\to\mathbb{V}$ such that

$$\theta(x,y)=\theta(y,x), \tag{4}$$

$$\theta((xy)a,b)+\theta((xb)a,y)+\theta(x,(yb)a)=\theta((xy)b,a)+\theta((xa)b,y)+\theta(x,(ya)b). \tag{5}$$

These elements will be called *trivial second cocycles* of commutative $\mathfrak{CD}$-algebras. Note that the variety of Jordan algebras is subvariety of commutative $\mathfrak{CD}$-algebras. Therefore, any trivial second cocycle which satisfies the identities (3) and (4) is a trivial second cocycle of the variety of commutative $\mathfrak{CD}$-algebras. Thus, we denote by $\mathrm{H}^2_{\mathfrak{J}}(\mathbf{A})$ and $\mathrm{H}^2_{\mathfrak{CD}}(\mathbf{A})$ the second cocycles of algebra $\mathbf{A}$ respect to the identities of Jordan and $\mathfrak{CD}$-algebras, respectively.

Let us fix the following notations:

C_j^i – *j th i-dimensional* commutative $\mathfrak{CD}$ (*non-Jordan*) *algebra.*

C_j^{i*} – *j th i-dimensional* commutative $\mathfrak{CD}$ (*Jordan*) *algebra.*

Thanks to A.Calderon Martin, A.Fernandez Ouaridi and I.Kaygorodov, we have the algebraic classification of all complex 4-dimensional nilpotent commutative $\mathfrak{CD}$-algebras with 2- and 3-dimensional annihilator:

C_{01}^{2*} : $e_1e_1=e_2$

C_{02}^{3*} : $e_1e_1=e_2$ $e_1e_2=e_3$

C_{03}^{3*} : $e_1e_2=e_3$

C_{01}^{3} : $e_1e_1=e_2$ $e_2e_2=e_3$

C_{04}^{4*} : $e_1e_1=e_3$ $e_2e_2=e_4$

C_{05}^{4*} : $e_1e_1=e_3$ $e_1e_2=e_4$

C_{06}^{4*} : $e_1e_1=e_4$ $e_2e_3=e_4$

$\mathrm{C}_{02}^{4}(0)$: $e_1e_1=e_2$ $e_1e_2=e_4$ $e_2e_2=e_3$

Moreover, thanks to A.Fernandez Ouaridi, I.Kaygorodov, M.Khrypchenko and Y.Volkov, we have the following list of four-dimensional commutative $\mathfrak{CD}$-algebras which are obtained by one dimensional central extensions of 3-dimensional $\mathfrak{CD}$-algebra:

C_{07}^{4*}	:	$e_1e_1=e_2$	$e_2e_3=e_4$		
C_{08}^{4*}	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_3e_3=e_4$	
C_{03}^{4}	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_4$	
C_{04}^{4}	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_3e_3=e_4$	
C_{09}^{4*}	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=e_4$
$C_{02}^{4}(\alpha)$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=\alpha e_4$	$e_2e_2=(\alpha+1)e_4$
C_{10}^{4*}	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=e_4$	
C_{11}^{4*}	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_3=e_4$	
C_{12}^{4*}	:	$e_1e_2=e_3$	$e_1e_3=e_4$		
C_{05}^{4}	:	$e_1e_2=e_3$	$e_3e_3=e_4$		
C_{06}^{4}	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_2e_2=e_4$	$e_3e_3=e_4$
C_{07}^{4}	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_3e_3=e_4$	

It is obvious that any three-dimensional extension of two-dimensional algebra C_{01}^{2*} is split. Thus, to obtain full description of 5-dimensional non-split commutative $\mathfrak{CD}$-algebras it is enough to consider 2-dimensional extensions of the three dimensional algebras $C_{01}^{3*}=C_{01}^{2*}\oplus\mathbb{C}e_3$, C_{02}^{3*}, C_{03}^{3*}, C_{01}^{3} and 1-dimensional extensions of the nilpotent four dimensional $\mathfrak{CD}$-algebras.

Below, we present the second cohomologies of three- and four-dimensional commutative $\mathfrak{CD}$-algebras. Since we have intended to obtain only non-Jordan commutative $\mathfrak{CD}$-algebras we give the description of Jordan and $\mathfrak{CD}$ cohomologies of these algebras.

Proposition 2. *The following elements are basis of the second cohomology spaces of* 3*-dimensional nilpotent commutative* $\mathfrak{CD}$*-algebras*:

Algebras	Jordan cohomology	$\mathfrak{CD}$-cohomology
C_{01}^{3*}	$H_{\mathfrak{J}}^2(C_{01}^{3*})=\langle[\Delta_{12}],[\Delta_{13}],[\Delta_{23}],[\Delta_{33}]\rangle$	$H_{\mathfrak{CD}}^2(C_{01}^{3*})=H_{\mathfrak{J}}^2(C_{01}^{3*})\oplus\langle[\Delta_{22}]$
C_{02}^{3*}	$H_{\mathfrak{J}}^2(C_{02}^{3*})=\langle[\Delta_{13}+\Delta_{22}]\rangle$	$H_{\mathfrak{CD}}^2(C_{02}^{3*})=H_{\mathfrak{J}}^2(C_{02}^{3*})\oplus\langle[\Delta_{22}]\rangle$
C_{03}^{3*}	$H_{\mathfrak{J}}^2(C_{03}^{3*})=\langle[\Delta_{11}],[\Delta_{22}],[\Delta_{13}],[\Delta_{23}]\rangle$	$H_{\mathfrak{CD}}^2(C_{03}^{3*})=H_{\mathfrak{J}}^2(C_{03}^{3*})\oplus\langle[\Delta_{33}]\rangle$
C_{01}^{3}	–	$H_{\mathfrak{CD}}^2(C_{01}^{3})=\langle[\Delta_{12}]\rangle$

Proposition 3. *The algebras* $C_{07}^{4*}, C_{11}^{4*}, C_{12}^{4*}, C_{01}^{4}, C_{03}^{4}, C_{04}^{4}, C_{05}^{4}, C_{06}^{4}, C_{07}^{4}$ *have only trivial extension and second cohomologies of commutative* $\mathfrak{CD}$*-algebras* C_{01}^{4*}, C_{02}^{4*}, ..., C_{09}^{4*} *and* $C_{02}^{4}(\alpha)$ *has the following bases.*

Algebras	Jordan cohomology	$\mathfrak{CD}$-cohomology
C_{01}^{4*}	$H_{\mathfrak{J}}^2(C_{01}^{4*}) = \langle [\Delta_{ij}] \rangle_{1\le i\le j\le 4}^{(i,j)\notin\{(1,1),(2,2)\}}$	$H_{\mathfrak{CD}}^2(C_{01}^{4*}) = H_{\mathfrak{J}}^2(C_{01}^{4*}) \oplus \langle [\Delta_{22}]$
C_{02}^{4*}	$H_{\mathfrak{J}}^2(C_{02}^{4*}) = \left\langle \begin{array}{c} [\Delta_{13}+\Delta_{22}],[\Delta_{14}], \\ [\Delta_{24}],[\Delta_{44}] \end{array} \right\rangle$	$H_{\mathfrak{CD}}^2(C_{02}^{4*}) = H_{\mathfrak{J}}^2(C_{02}^{4*}) \oplus \langle [\Delta_{13}] \rangle$
C_{03}^{4*}	$H_{\mathfrak{J}}^2(C_{03}^{4*}) = \langle [\Delta_{ij}] \rangle_{1\le i\le j\le 4}^{(i,j)\notin\{(1,2),(3,3)\}}$	$H_{\mathfrak{CD}}^2(C_{03}^{4*}) = H_{\mathfrak{J}}^2(C_{03}^{4*}) \oplus \langle [\Delta_{33}] \rangle$
C_{04}^{4*}	$H_{\mathfrak{J}}^2(C_{04}^{4*}) = \left\langle \begin{array}{c} [\Delta_{12}],[\Delta_{13}],[\Delta_{14}], \\ [\Delta_{23}],[\Delta_{24}] \end{array} \right\rangle$	$H_{\mathfrak{CD}}^2(C_{04}^{4*}) = H_{\mathfrak{J}}^2(C_{04}^{4*}) \oplus \langle [\Delta_{33}],[\Delta_{34}],[\Delta_{44}] \rangle$
C_{05}^{4*}	$H_{\mathfrak{J}}^2(C_{05}^{4*}) = \left\langle \begin{array}{c} [\Delta_{13}],[\Delta_{14}],[\Delta_{22}], \\ [\Delta_{23}],[\Delta_{24}] \end{array} \right\rangle$	$H_{\mathfrak{CD}}^2(C_{05}^{4*}) = H_{\mathfrak{J}}^2(C_{05}^{4*}) \oplus \langle [\Delta_{33}],[\Delta_{34}],[\Delta_{44}] \rangle$
C_{06}^{4*}	$H_{\mathfrak{J}}^2(C_{06}^{4*}) = \langle [\Delta_{ij}] \rangle_{1\le i\le j\le 4}^{(i,j)\notin\{(1,1),(4,4)\}}$	$H_{\mathfrak{CD}}^2(C_{06}^{4*}) = H_{\mathfrak{J}}^2(C_{06}^{4*}) \oplus \langle [\Delta_{44}] \rangle$
C_{08}^{4*}	$H_{\mathfrak{J}}^2(C_{08}^{4*}) = \left\langle \begin{array}{c} [\Delta_{13}],[\Delta_{14}+\Delta_{22}], \\ [\Delta_{23}],[\Delta_{33}] \end{array} \right\rangle$	$H_{\mathfrak{CD}}^2(C_{08}^{4*}) = H_{\mathfrak{J}}^2(C_{08}^{4*}) \oplus \langle [\Delta_{14}] \rangle$
C_{09}^{4*}	$H_{\mathfrak{J}}^2(C_{09}^{4*}) = \langle [\Delta_{14}]+[\Delta_{23}] \rangle$	$H_{\mathfrak{CD}}^2(C_{09}^{4*}) = H_{\mathfrak{J}}^2(C_{09}^{4*}) \oplus \langle [\Delta_{22}] \rangle$
$C_{02}^{4}(\alpha)$	–	$H_{\mathfrak{CD}}^2(\mathfrak{C}_{02}^4(\alpha)_{\alpha\ne 0,1}) = \langle [\Delta_{22}],[\Delta_{14}+(3+\alpha)\Delta_{23}] \rangle$ $H_{\mathfrak{CD}}^2(\mathfrak{C}_{02}^4(1)) = \langle [\Delta_{22}],[\Delta_{14}+4\Delta_{23}],[\Delta_{24}] \rangle$ $H_{\mathfrak{CD}}^2(\mathfrak{C}_{02}^4(0)) = \langle [\Delta_{13}],[\Delta_{14}+3\Delta_{23}] \rangle$

where $C_{01}^{4*} = C_{01}^{2*} \oplus \mathbb{C}e_3 \oplus \mathbb{C}e_4$.

Here, we formulate the main result of this section.

Theorem 2. *Let* C *be a complex* 5*-dimensional nilpotent commutative* $\mathfrak{CD}$*-algebra. Then* C *is a Jordan algebra or it is isomorphic to one algebra from the* **Appendix A**(*there are* 81 *algebras*, 12 *of them are one parametric and* 2 *of them are two parametric families of algebras*).

In the third section of the second chapter, we intend to obtain full classification of 5-dimensional nilpotent commutative algebras. Since we classified all 5-dimensional nilpotent commutative $\mathfrak{CD}$-algebras in previous section, we will focus on to the classification of non $\mathfrak{CD}$ commutative algebras.

Let $(\mathbf{A},\cdot)$ be complex commutative algebra and $\mathbb{V}$ be a complex vector space. The $\mathbb{C}$-linear space $Z^2(\mathbf{A},\mathbb{V})$ is defined as the set of all bilinear maps $\theta{:}\mathbf{A}\times\mathbf{A}\to\mathbb{V}$ such that $\theta(x,y)=\theta(y,x)$. These elements will be called *trivial second cocycles* of commutative algebras.

Here is the main result of this section.

Theorem 3. *Every complex 5-dimensional nilpotent commutative algebra is $\mathfrak{CD}$-algebra or isomorphic to one algebra from the* **Appendix B**(*there are* 409 *algebras*, 99 *of them are one parametric*, 63 *of them are two parametric*, 28 *of them are three parametric*, 11 *of them are four parametric*, 6 *of them are five parametric and one of them is six parametric families of algebras*).

The third chapter of the dissertation, entitled "**The geometric classification of small dimensional non-associative algebras**" deals with the geometric properties of 4-dimensional noncommutative Jordan algebras and 5-dimensional commutative Jordan algebras.

Let $\mathbb{V}$ be an n-dimensional vector space, then the set $\mathrm{Hom}(\mathbb{V}\otimes\mathbb{V},\mathbb{V})$ is a vector space of dimension n^3. This space has the structure of the affine variety $\mathbb{C}^{n^3}$. Indeed, let us fix a basis $e_1, e_2, \ldots, e_n$ of $\mathbb{V}$. Then any $\mu\in\mathrm{Hom}(\mathbb{V}\otimes\mathbb{V},\mathbb{V})$ is determined by n^3 structure constants $c_{ij}^k\in\mathbb{C}$ such that $\mu(e_i\otimes e_j)=\sum_{k=1}^{n}c_{ij}^k e_k$. A subset of $\mathrm{Hom}(\mathbb{V}\otimes\mathbb{V},\mathbb{V})$ is *Zariski-closed* if it can be defined by a set of polynomial equations in the variables $c_{ij}^k\ (1\le i,j,k\le n)$.

Let T be a set of polynomial identities. The set of algebra structures on $\mathbb{V}$ satisfying polynomial identities from T forms a Zariski-closed subset of the variety $\mathrm{Hom}(\mathbb{V}\otimes\mathbb{V},\mathbb{V})$. We denote this subset by $\mathbb{L}(T)$. The general linear group $\mathrm{GL}(\mathbb{V})$ acts on $\mathbb{L}(T)$ by conjugations:

$$(g*\mu)(x\otimes y)=g\mu(g^{-1}x\otimes g^{-1}y)$$

for $x,y\in\mathbb{V}$, $\mu\in\mathbb{L}(T)\subset\mathrm{Hom}(\mathbb{V}\otimes\mathbb{V},\mathbb{V})$ and $g\in\mathrm{GL}(\mathbb{V})$. Thus, $\mathbb{L}(T)$ is decomposed into $\mathrm{GL}(\mathbb{V})$-orbits that correspond to the isomorphism classes of algebras. Let $O(\mu)$ denote the orbit of $\mu\in\mathbb{L}(T)$ under the action of $\mathrm{GL}(\mathbb{V})$ and $\overline{O(\mu)}$ denote the Zariski closure of $O(\mu)$.

Let **A** and **B** be two n-dimensional algebras satisfying the identities from T, and let $\mu,\lambda\in\mathbb{L}(T)$ represent **A** and **B**, respectively.

Definition 6. *We say that* **A** *degenerates to* **B** *and write* $\lambda\to\mu$ *if* $\lambda\in\overline{O(\mu)}$.

Note that in this case we have $\overline{O(\lambda)}\subset\overline{O(\mu)}$. Hence, the definition of a degeneration does not depend on the choice of μ and λ. If $\mathbf{A}\not\cong\mathbf{B}$, then the assertion $\lambda\to\mu$ is called a *proper degeneration*. We write $\lambda\not\to\mu$ if $\lambda\notin\overline{O(\mu)}$.

Let **A** be represented by $\mu\in\mathbb{L}(T)$. Then **A** is said to be *rigid* in $\mathbb{L}(T)$ if $O(\mu)$

is an open subset of $\mathbb{L}(T)$. Recall that a subset of a variety is called irreducible if it cannot be represented as a union of two non-trivial closed subsets. A maximal irreducible closed subset of a variety is called an *irreducible component*. It is well known that any finite-dimensional affine variety can be represented uniquely as a finite union of its irreducible components. The algebra **A** is rigid in $\mathbb{L}(T)$ if and only if $\overline{O(\mu)}$ is an irreducible component of $\mathbb{L}(T)$.

The main result of the present section is the following theorem.

Theorem 4. *The variety of complex* 4*-dimensional nilpotent noncommutative Jordan algebras has dimension* 14. *It is defined by* 3 *rigid algebras and two one-parametric families of algebras, and can be described as the closure of the union of* $\mathrm{GL}_4(\mathbb{C})$*-orbits of the following algebras* ($\alpha\in\mathbb{C}$):

$$
\begin{array}{llllllll}
\mathfrak{N}_2(\alpha): & e_1e_1=e_3 & e_1e_2=e_4 & e_2e_1=-\alpha e_3 & e_2e_2=-e_4, & & & \\
\mathfrak{N}_3(\alpha): & e_1e_1=e_4 & e_1e_2=\alpha e_4 & e_2e_1=-\alpha e_4 & e_2e_2=e_4 & e_3e_3=e_4, & & \\
\mathbf{J}_{07}^4: & e_1e_2=e_3 & e_1e_3=e_4 & e_2e_1=e_3+e_4 & e_2e_3=e_4 & e_3e_1=e_4 & e_3e_2=e_4, & \\
\mathbf{J}_{17}^4: & e_1e_1=e_4 & e_1e_2=e_3 & e_2e_1=-e_3 & e_1e_3=e_4 & e_2e_2=e_4 & e_3e_1=-e_4 & \\
\mathbf{J}_{18}^4: & e_1e_1=e_2 & e_1e_2=e_3 & e_2e_1=e_3 & e_1e_3=e_4 & e_2e_2=e_4 & e_3e_1=e_4. &
\end{array}
$$

In the second section of the third chapter, we obtain all irreducible components of the variety of 5-dimensional nilpotent commutative $\mathfrak{CD}$-algebras. Note that irreducible components of 5-dimensional nilpotent associative commutative algebras were found by G.Mazzola and all irreducible components of 5-dimensional nilpotent Jordan algebras were described by I.Kashuba and M.Martin.

Theorem 5. *The variety of complex* 5*-dimensional nilpotent commutative* $\mathfrak{CD}$*-algebras is* 24*-dimensional and it has* 10 *irreducible components. In particular, there are* 6 *rigid algebras: non-Jordan algebras* $\mathfrak{C}_{69}^5, \mathfrak{C}_{72}^5, \mathfrak{C}_{76}^5, \mathfrak{C}_{77}^5, \mathfrak{C}_{81}^5$ *and Jordan algebra* $\mathfrak{J}_{21}$.

The fourth chapter of the dissertation, entitled "**Infinite-dimensional simple *n*-Lie algebras**" is devoted to investigate infinite-dimensional *n*-Lie algebras. The main goal in this chapter is to construct infinite-dimensional *n*-Lie algebras. We provide several examples and for three them we prove their simplicity.

Definition 7. *A vector space* **A** *equipped with skew-symmetric n-ary bracket* $[-,\ldots,-]$ *is called n-Lie algebra if for any* $x_1,\ldots,x_n,y_1,\ldots,y_{n-1}\in\mathbf{A}$ *the following identity holds true*

$$[[x_1,\ldots,x_n],y_1,\ldots,y_{n-1}]=\sum_{s=1}^{n}[x_1,\ldots,x_{s-1},[x_s,y_1,\ldots,y_{n-1}],x_{s+1},\ldots,x_n].$$

It should be noted that 2-Lie algebras are nothing but Lie algebras.

Let **A** be an associative commutative $\mathbb{F}$-algebra. For any fixed mutually commuting derivations $d_1,\dots,d_n$ of the algebra $\mathbf{A}$ and any $x_1,\dots,x_n \in \mathbf{A}$ the Jacobian $Jac(x_1,\dots,x_n)$ is defined as follows

$$Jac(x_1,\dots,x_n) = \begin{vmatrix} d_1(x_1) & \cdots & d_1(x_n) \\ \vdots & \ddots & \vdots \\ d_n(x_1) & \cdots & d_n(x_n) \end{vmatrix}.$$

We denote by $\mathbf{A}^*(d_1,\dots,d_n)$ the algebra of Jacobians, which is equipped with the n-ary bracket $[x_1,\dots,x_n]_J = Jac(x_1,\dots,x_n)$ on a vector space $\mathbf{A}$.

Now, we present a theorem proved by V.T.Filippov.

Theorem 6. *The algebra of Jacobians* $\mathbf{A}^*(d_1,\dots,d_n)$ *for any associative and commutative* $\mathbb{F}$*-algebra* A *with its commuting derivations* $d_1,\dots,d_n$ *is an n-Lie algebra.*

Similarly, if one considers an associative commutative $\mathbb{F}$-algebra $\mathbf{A}$ and its commuting derivations $d_1,\dots,d_{n-1}$, then due to the result of A.Dzhumadildayev the vector space $\mathbf{A}$ with the following n-ary bracket $[x_1,\dots,x_n]_W = Wr(x_1,\dots,x_n)$ where

$$Wr(x_1,\dots,x_n) = \begin{vmatrix} x_1 & x_2 & \cdots & x_n \\ d_1(x_1) & d_1(x_2) & \cdots & d_1(x_n) \\ \vdots & \vdots & \ddots & \cdots \\ d_{n-1}(x_1) & d_{n-1}(x_2) & \cdots & d_{n-1}(x_n) \end{vmatrix}.$$

forms an n-Lie algebra, which is called the n-Lie algebra of Wronskians.

Let $\mathcal{A}$ be an associative commutative $\mathbb{F}$-algebra and $d_1,\dots,d_{n+1}$ be mutually commuting derivations of $\mathcal{A}$. Fix $\alpha = (\alpha_1,\dots,\alpha_{n+1})$ where $\alpha_i \in \mathcal{A}$ and for any $x_1,\dots,x_n \in \mathcal{A}$ define the following n-ary bracket on $\mathcal{A}$:

$$[x_1,\dots,x_n]_\alpha = Jac(x)_\alpha = \begin{vmatrix} d_1(x_1) & \cdots & d_1(x_n) & \alpha_1 \\ \vdots & \ddots & \vdots & \vdots \\ d_{n+1}(x_1) & \cdots & d_{n+1}(x_n) & \alpha_{n+1} \end{vmatrix}.$$

Theorem 7. *For any* $x_1,\dots,x_n,y_1,\dots,y_{n-1} \in \mathcal{A}$ *the following equality holds true:*

$$\sum_{s=1}^{n}[x_1,\dots,x_{s-1},[x_s,y_1,\dots,y_{n-1}]_\alpha,x_{s+1},\dots,x_n]_\alpha - [[x_1,\dots,x_n]_\alpha,y_1,\dots,y_{n-1}]_\alpha$$

$$= \sum_{k=1,k\neq i,j}^{n+1} \sum_{1\le i<j\le n+1} (-1)^{i+j+k} \left(\begin{vmatrix} \alpha_i & \alpha_j \\ d_k(\alpha_i) & d_k(\alpha_j) \end{vmatrix} + \begin{vmatrix} \alpha_j & \alpha_k \\ d_i(\alpha_j) & d_i(\alpha_k) \end{vmatrix} + \begin{vmatrix} \alpha_k & \alpha_k \\ d_j(\alpha_i) & d_j(\alpha_k) \end{vmatrix} \right) \times$$

$$Jac_k(x) Jac_{i,j}(y).$$

Corollary 1. *The algebra* $\langle \mathcal{A},[-,\dots,-]_\alpha \rangle$ *is an n-Lie algebra if and only if the following conditions hold true:*

$$\begin{vmatrix} \alpha_i & \alpha_j \\ d_k(\alpha_i) & d_k(\alpha_j) \end{vmatrix} + \begin{vmatrix} \alpha_j & \alpha_k \\ d_i(\alpha_j) & d_i(\alpha_k) \end{vmatrix} + \begin{vmatrix} \alpha_k & \alpha_k \\ d_j(\alpha_i) & d_j(\alpha_k) \end{vmatrix} = 0.$$

where, $i,j,k\in\{1,\ldots,n+1\}$ and $i\neq j$, $k\neq i$, $k\neq j$.

Let $\mathcal{A}$ be an associative commutative $\mathbb{F}$-algebra and $d_1,\ldots,d_n$ be pairwise commuting derivations of $\mathcal{A}$. Fix $\alpha_0,\alpha_1,\ldots,\alpha_n\in\mathcal{A}$ and for any $x_1,\ldots,x_n\in\mathcal{A}$ define the following n-ary bracket on $\mathcal{A}$ as follows

$$\{x_1,\ldots,x_n\}_\alpha = Wr(x)_\alpha = \begin{vmatrix} x_1 & \cdots & x_n & \alpha_0 \\ d_1(x_1) & \cdots & d_1(x_n) & \alpha_1 \\ \vdots & \ddots & \vdots & \vdots \\ d_n(x_1) & \cdots & d_n(x_n) & \alpha_n \end{vmatrix}.$$

Theorem 8. *For any* $x_1,\ldots,x_n,y_1,\ldots,y_{n-1}\in\mathcal{A}$ *the following equality holds true:*

$$\sum_{s=1}^{n}\{x_1,\ldots,x_{s-1},\{x_s,y_1,\ldots,y_{n-1}\}_\alpha,x_{s+1},\ldots,x_n\}_\alpha - \{\{x_1,\ldots,x_n\}_\alpha,y_1,\ldots,y_{n-1}\}_\alpha$$

$$= \sum_{j,k=1,j\neq k}^{n+1}(-1)^{j+k+1}\left(\begin{vmatrix} \alpha_0 & \alpha_j \\ d_k(\alpha_0) & d_k(\alpha_j) \end{vmatrix} + \begin{vmatrix} \alpha_j & \alpha_0 \\ d_i(\alpha_j) & d_i(\alpha_0) \end{vmatrix}\right) Wr_k(x)Jac_j(y)$$

$$+ \sum_{j,k=1,j\neq k}^{n+1}(-1)^{i+j+1}\left(\begin{vmatrix} \alpha_j & \alpha_0 \\ d_i(\alpha_j) & d_i(\alpha_0) \end{vmatrix} + \begin{vmatrix} \alpha_0 & \alpha_i \\ d_j(\alpha_0) & d_j(\alpha_i) \end{vmatrix}\right) Jac(x)Wr_{i,j}(y)$$

$$+ \sum_{k=1,k\neq i,j}^{n+1}\sum_{1\le i<j\le n+1}(-1)^{i+j+k}\left(\begin{vmatrix} \alpha_i & \alpha_j \\ d_k(\alpha_i) & d_k(\alpha_j) \end{vmatrix} + \begin{vmatrix} \alpha_j & \alpha_k \\ d_i(\alpha_j) & d_i(\alpha_k) \end{vmatrix} + \begin{vmatrix} \alpha_k & \alpha_k \\ d_j(\alpha_i) & d_j(\alpha_k) \end{vmatrix}\right)\times$$

$$Wr_k(x)Wr_{i,j}(y).$$

Corollary 2. *The algebra* $\langle\mathcal{A},Wr(-,\cdots,-)_\alpha\rangle$ *is an n-Lie algebra if and only if the following conditions hold true:*

$$\begin{vmatrix} \alpha_j & \alpha_0 \\ d_i(\alpha_j) & d_i(\alpha_0) \end{vmatrix} + \begin{vmatrix} \alpha_0 & \alpha_i \\ d_j(\alpha_0) & d_j(\alpha_i) \end{vmatrix} = 0,$$

$$\begin{vmatrix} \alpha_i & \alpha_j \\ d_k(\alpha_i) & d_k(\alpha_j) \end{vmatrix} + \begin{vmatrix} \alpha_j & \alpha_k \\ d_i(\alpha_j) & d_i(\alpha_k) \end{vmatrix} + \begin{vmatrix} \alpha_k & \alpha_k \\ d_j(\alpha_i) & d_j(\alpha_k) \end{vmatrix} = 0,$$

where, $i,j,k\in\{1,\ldots,n+1\}$ and $i\neq j$, $k\neq i$, $k\neq j$.

In the second section of the fourth chapter, we give examples of infinite-dimensional simple n-Lie algebras.

Now, consider the algebra $\mathcal{WA}_D^\alpha$ with $\alpha=(1,0,\ldots,0,1)$. We denote this algebra by $\mathcal{WA}_D^{1,0,1}$.

Theorem 9. *The n-Lie algebra* $\mathcal{WA}_D^{1,0,1}$ *is simple.*

Consider $\mathcal{JA}_n^\alpha$ and $\mathcal{WA}_D^\alpha$ with $\mathcal{A}_n = \mathbb{F}[x_1^\pm, x_2^\pm, \ldots, x_{n+1}^\pm]$, derivations $d_i = \dfrac{\partial}{\partial x_i}$ and parameters $\alpha_i = \beta_i \boldsymbol{x} x_i^{-1}$, $\beta_i \in \mathbb{F}$, $1 \le i \le n+1$, $\boldsymbol{x} = x_1 \ldots x_{n+1}$.

Theorem 10. *Let there exist i,j such that* $\beta_i \notin \mathbb{Q}$ *and* $\beta_j \in \mathbb{Q} \setminus \{0\}$. *Then* $\mathcal{JA}_n^\alpha$ *is simple.*

Theorem 11. *Let* $\beta_i \notin \mathbb{Z}$ *for some i. Then the n-Lie algebra* $\mathcal{WA}_D^\alpha$ *is simple.*

CONCLUSION

In this dissertation, the algebraic classification of of 4-dimensional nilpotent noncommutative Jordan algebras, 5-dimensional nilpotent commutative $\mathfrak{CD}$-algebras and 5-dimensional nilpotent commutative algebras over the complex field is obtained. Moreover, the geometric classification of 4-dimensional nilpotent noncommutative Jordan algebras, 5-dimensional nilpotent commutative $\mathfrak{CD}$-algebras is determined. That is, all necessary degeneration for the variety of 4-dimensional nilpotent noncommutative Jordan algebras are constructed and it is shown that this variety can be described as a union of 5 irreducible components. Similar results are obtained for the variety of 5-dimensional nilpotent commutative $\mathfrak{CD}$-algebras and it is shown that this variety can be described as a union of 10 irreducible components. In the fourth chapter new n-ary algebras are constructed by generalizing the operators of Jacobian and Wronskian. The necessary and sufficient conditions are found for this n-ary algebras to be n-Lie algebras. It is constructed one example of infinite-dimensional simple n-Lie algebra by generalization of Jacobian and two examples of infinite-dimensional simple n-Lie algebras by generalization of Wronskian. It is constructed one example of infinite-dimensional simple n-Lie algebra by generalization of Jacobian and two examples of infinite-dimensional simple n-Lie algebras by generalization of Wronskian.

Main results of the dissertation are the following:

1. algebraic classification of 4-dimensional nilpotent noncommutative Jordan algebras;
2. algebraic classification of 5-dimensional nilpotent commutative $\mathfrak{CD}$-algebras;
3. algebraic classification of 5-dimensional nilpotent commutative algebras;
4. geometric classification of 4-dimensional nilpotent noncommutative Jordan algebras;
5. geometric classification of 5-dimensional nilpotent commutative $\mathfrak{CD}$-algebras;
6. construction of infinite-dimensional simple n-Lie algebras.

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ИНСТИТУТ МАТЕМАТИКИ

ДЖУМАНИЯЗОВ ДОСТОН ЭРКАБОЙ УГЛИ

ОПИСАНИЕ НЕАССОЦИАТИВНЫХ АЛГЕБР МАЛЫХ РАЗМЕРНОСТЕЙ И ПОСТРОЕНИЕ ПРОСТЫХ *n*-ЛИЕВЫХ АЛГЕБР

01.01.06 – Алгебра

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПОФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

ТАШКЕНТ-2023

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования является классификация ниплотентных некоммутативных йордановых и коммутативных $\mathfrak{CD}$-алгебр малых размерностей, а также геометрическую классификацию ниплотентных некоммутативных йордановых и коммутативных $\mathfrak{CD}$-алгебр, построение бесконечномерные простые n-Лиевых алгебр.

Научная новизна исследования состоит в следующем:

получена алгебраическая классификация 4-мерных нильпотентных некоммутативных йордановых алгебр.

получена алгебраическая классификация 5-мерных нильпотентных коммутативных алгебр и 5-мерных нильпотентных коммутативных алгебр.

осуществлена геометрическая классификация 4-мерных нильпотентных некоммутативных йордановых алгебр.

получена геометрическая классификация 5-мерных нильпотентных коммутативных $\mathfrak{CD}$-алгебр.

построены новые примеры бесконечномерных простых n-Лиевых алгебр.

Внедрение результатов исследования. По результатам классификации маломерных неассоциативных алгебр и построения простых n-Лиевых алгебр:

Результаты, полученные в диссертации были применены в исследовательском проекте «Нерегулярное множества в динамических системах», № AP08051987, (Справка университета им. Сулеймана Демиреля от 1-ноября 2023 года, № 1.1-11/1141, Казахстан). Научные результаты диссертации были применены для получения классификации пятимерных однопорожденных нильпотентных асимметричных алгебр. Кроме того, эти результаты позволили обнаружить неприводимые компоненты многообразий таких алгебр.

Результаты, полученные при алгебраической классификации нильпотентных некоммутативных йордановых алгебр, применялись в научных работах (Journal of Algebra and its Applications, 20, 2021, 2150198, Communications in Mathematics, 29, 2021, 215- 226, Journal of Pure and Applied Algebra, 226, 2022, 106850 и Journal of Algebra and its Applications, 21, 2022, 2250031) для исследования алгебраической и геометрической классификации нильпотентных алгебр; Эти результаты позволили получить полную классификацию пятимерных и шестимерных однопорожденных нильпотентных бикоммутативных алгебр.

Структура и объем диссертации. Диссертация состоит из введения, четырех глав, заключения и списка использованной литературы. Объем диссертации составляет 121 страниц.

E’LON QILINGAN ISHLAR RO‘YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS
I бўлим (I часть; part I)

1. Jumaniyozov D., Kaygorodov I., Khudoyberdiyev A., The algebraic and geometric classification of nilpotent noncommutative Jordan algebras, Journal of Algebra and Its Applications, 2020, Vol. 20(11), 2150202. (3. Scopus. IF=0.54).
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II бўлим (II часть; part II)

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