**V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI
HUZURIDAGI ILMIY DARAJALAR BERUVCHI
DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH**

NAMANGAN DAVLAT UNIVERSITETI

TO‘XTABAYEV AKBARXO‘JA MAMAJONOVICH

STATISTIK MEXANIKANING BA’ZI KLASSIK MODELLARI UCHUN p-ADIK DAVRIY GIBBS O‘LCHOVLARI

01.01.01 – Matematik analiz

**FIZIKA-MATEMATIKA FANLARI bo‘yicha falsafa doktori (PhD) dissertatsiyasi
AVTOREFERATI**

Toshkent - 2023 yil

UDK: 517.98 – 519.21

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi avtoreferati mundarijasi

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

Contents of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

V.I.ROMANOVSKIY NOMIDAGI MATEMATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

NAMANGAN DAVLAT UNIVERSITETI

TO‘XTABAYEV AKBARXO‘JA MAMAJONOVICH

STATISTIK MEXANIKANING BA’ZI KLASSIK MODELLARI UCHUN p-ADIK DAVRIY GIBBS O‘LCHOVLARI

01.01.01 – Matematik analiz

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

Namangan – 2023 yil

Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, Fan va Innovatsiyalar Vazirligi huzuridagi Oliy attestasiya komissiyasida № B2021.4.PhD/FM556 raqam bilan ro'yxatga olingan.

Dissertatsiya Namangan davlat universitetida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, ingliz, rus (rezyume)) Ilmiy kengash veb-sahifasi (https://kengash.mathinst.uz) va "ZiyoNet" ta'lim axborot tarmog'ida (http://www.ziyonet.uz) joylashtirilgan.

Ilmiy rahbar: **Raxmatullayev Muzaffar Muxammadjanovich**
fizika-matematika fanlari doktori, katta ilmiy xodim

Rasmiy opponentlar: **Kudaybergenov Karimbergen Kadirbergenovich**
fizika-matematika fanlari doktori, professor

Botirov G`olibjon Isroilovich
fizika-matematika fanlari doktori, katta ilmiy xodim

Yetakchi tashkilot: **O'zbekiston Milliy universiteti**

Dissertatsiya himoyasi V.I. Romanovskiy nomidagi Matematika instituti huzuridagi DSc.02/30.12.2019.FM.86.01 raqamli Ilmiy kengashning 2023 yil " 21 " noyabr kuni soat 16:00 dagi majlisida bo'lib o'tadi. (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko'chasi, 9-uy. Tel.: (+998 71) 207 91 40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

Dissertatsiya bilan V.I. Romanovskiy nomidagi Matematika institutining Axborot-resurs markazida tanishish mumkin (170 -raqami bilan ro'yxatga olingan). (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko'chasi, 9-uy. Tel.: (+998 71) 207 91 40).

Dissertatsiya avtoreferati 2023-yil " 27 " oktabr kuni tarqatildi.
(2023 yil " 27 " oktabrdagi 2 -raqamli reestr bayonnomasi).

U.A. Rozikov
Ilmiy darajalar beruvchi
Ilmiy kengash raisi,
f.-m.f.d., professor

J.K. Adashev
Ilmiy darajalar beruvchi
Ilmiy kengash ilmiy kotibi,
f.-m.f.d., katta ilmiy xodim

U.U. Jamilov
Ilmiy darajalar beruvchi
Ilmiy kengash huzuridagi
Ilmiy seminar raisi,
f.-m.f.d., katta ilmiy xodim

KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Jahon miqyosida olib borilayotgan fizik, biologik sistemalarning termodinamik xossalarini tadqiq qilishga bag'ishlangan ilmiy-amaliy masalalar ko'pincha statistik mexanikaning Arximed va noarximed modellarini tadqiq qilishga keltiriladi. p-adik Gibbs o'lchovlari statistik mexanikaning noarximed modellari uchun faza almashishlari nazariyasining asosiy ob'yekti hisoblanadi. Kimyo, fizika, xizmat ko'rsatish nazariyasi, materialshunoslik, biologiya kabi fan va texnikaning turli sohalaridagi muammolarni hal qilishda Gibbs o'lchovlari to'plamini tavsiflash muhim ahamiyat kasb etadi. Statistik mexanikaning noarximed modellariga mos barcha p-adik Gibbs o'lchovlari to'plamini to'liq tavsiflash murakkabligidan berilgan modelga mos yetarlicha ko'p Gibbs o'lchovlarini qurish masalasi dolzarbligicha qolmoqda.

Hozirgi kunda panjarali sistema, xususan daraxt strukturasiga ega sanoqli graflarda berilgan Gamiltonian uchun p-adik Gibbs o'lchovlarining mavjudligini aniqlash, barcha p-adik Gibbs o'lchovlari to'plamini tavsiflash dolzarb masalalardan biridir. Statistik fizikada faza almashishlar nazariyasining muhim masalalaridan biri barcha p-adik Gibbs o'lchovlari to'plamida kamida ikkita p-adik davriy Gibbs o'lchovlari mavjudligini aniqlash masalasidir. Shu bois berilgan Gamiltonian uchun barcha davriy va translyatsion-invariant p-adik Gibbs o'lchovlari to'plamini tavsiflash, bunday o'lchovlarni chegaralanganlikka tekshirish hamda umumlashgan p-adik Gibbs o'lchovlari uchun faza almashish mavjudligini aniqlash maqsadli ilmiy tadqiqotlardan hisoblanadi.

Ma'lumki, mamlakatimizda taraqqiyotning asosiy tayanchlaridan biri bo'lgan fundamental fanlarning ilmiy va amaliy tatbiqiga ega bo'lgan sohalariga e'tibor kuchaytirilmoqda. Jumladan, oxirgi yillarda Keli daraxtida aniqlangan statistik mexanikaning klassik modellari uchun haqiqiy va p-adik sonlar maydonida translyatsion-invariant, davriy va konstruktiv usulda qurilgan ba'zi davriy bo'lmagan Gibbs o'lchovlarining xossalarini tadqiq qilish borasida salmoqli natijalarga erishildi. "Matematik fizika, noarximed analiz, noarximed o'lchovlar va dinamik sistemalar nazariyalari" fanlarining ustuvor yo'nalishlari bo'yicha xalqaro standartlar darajasida ilmiy tadqiqotlar olib borish matematika fanining asosiy vazifalari va faoliyat yo'nalishlari etib belgilandi[1]. Qaror ijrosini ta'minlashda ilmiy natijalardan ilm-fanning turdosh sohalarida foydalanish maqsadida statistik mexanikaning klassik modellari uchun faza almashishlar nazariyasini rivojlantirish muhim ahamiyatga ega.

O'zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-son "O'zbekiston Respublikasini yanada rivojlantirish bo'yicha harakatlar strategiyasi to'g'risida"gi va 2022-yil 28-yanvardagi PF-60-son "2022-2026-yillarga mo'ljallangan Yangi O'zbekistonning Taraqqiyot strategiyasi to'g'risida"gi Farmonlari, 2019-yil 9-iyuldagi PQ-4387-son "Matematika ta'limi va fanlarini

[1] O'zbekiston Respublikasi Vazirlar mahkamasining 2017-yil 18-maydagi "O'zbekiston Respublikasi Fanlar akademiyasining yangidan tashkil etilgan ilmiy tadqiqotlar muassasalari faoliyatini tashkil etish to'g'risida"gi 292-sonli qarori.

yanada rivojlantirishni davlat tomonidan qo‘llab-quvvatlash, shuningdek O‘zbekiston Respublikasi Fanlar akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to‘g‘risida”gi va 2020-yil 7-maydagi PQ-4708-son “Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarorlari hamda mazkur faoliyatga tegishli boshqa normativ-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishi ustuvor yo‘nalishlariga bog‘liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. “Matematika, mexanika va informatika” ustuvor yo‘nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Doimiy harorat saqlanadigan va atrof-muhit bilan issiqlik muvozanatida bo‘lgan sistemalarning holatini tavsiflovchi Gibbs taqsimoti tushunchasi amerikalik olim J.U. Gibbs tomonidan kiritilgan. Haqiqiy qiymatli limit Gibbs o‘lchovlarining umumiy xarakteristikasi R.L. Dobrushin, O. Lenford va D. Ryuellarning ishlarida berilgan. Haqiqiy qiymatli Gibbs o‘lchovlarining zamonaviy nazariyasi R. Bekster, X.O. Georgi, V.A. Malishov, R.A. Minlos, K. Preston, D. Ryuel, Ya.G. Sinay, G. Gallavotti, F. Bonetto, J. Jentile, Jin Zin-Jastin, N.N. G‘anixodjayev O‘.A. Roziqov va F.M. Muxamedovlarning ishlarida yoritib berilgan. R.L. Dobrushin tomonidan haqiqiy qiymatli limit Gibbs o‘lchovining mavjudligi haqidagi teorema isbotlangan. Statistik mexanikaning klassik modellari uchun Gibbs o‘lchovlari R.L. Dobrushin, G.R. Braytvell, P. Uinkler, Yu.M. Suxov, J. Martin, N.N. G‘anixodjayev, O‘.A. Roziqov, F.M. Muxammedov, D. Gandolfo, M.M. Raxmatullayev, R.M. Xakimov, G‘.I. Botirov, O.N. Hakimov, E. Normatov, Sh. Shoyusupov, F. Haydarov, M. Rasulova va boshqalarning ishlarida o‘rganilgan.

N.N. G‘anixodjayev, F.M. Muxamedov va O‘.A. Roziqovlarning ishida $\mathbb{Z}$ panjarada Izing modeli uchun p-adik Gibbs taqsimotining yagonaligi isbotlangan. Izing-Vannimenus modeli uchun p-adik Gibbs o‘lchovining yagona emasligi N.N. G‘anixodjayev, H. Akinlarning ishida ko‘rsatilgan. Vannimenus model uchun O.N. Hakimovning ishlarida ikkinchi tartibli Keli daraxtida translyatsion-invariant va davriy p-adik kvazi Gibbs o‘lchovlari o‘rganilgan. Ushbu ishlar F.M. Muxamedov, M.K. Saburov va O.N. Hakimovning ishlarida Izing-Vannimenus modeli uchun davom ettirilgan.

Izing modeli uchun yagona p-adik Gibbs o‘lchovi mavjud ekanligi M. Xamrayev, F.M. Muxamedovlarning ishlarida isbotlangan. Translyatsion-invariant p-adik umumlashgan Gibbs o‘lchovlari F.M. Muxamedov va O.N. Hakimovning ishlarida o‘rganilgan. Translyatsion-invariant p-adik umumlashgan Gibbs o‘lchovlarining chegaralanganlik mezoni topilgan va bu o‘lchovlar uchun kuchli faza alamashish mavjud emasligi isbotlangan. p-adik umumlashgan Gibbs o‘lchovlari uchun xaotiklik masalalari F.M. Muxamedov, O.N. Hakimov, H. Akin, M. Dogan ishlarida o‘rganilgan.

Haqiqiy sonlar maydonida Izing modeli uchun konstruksiya usulidan foydalanib, davriy bo‘lmagan sanoqsiz sondagi chetki Gibbs o‘lchovlarining mavjudligi P. Blexer va N.N. G‘anixodjayev ishlarida isbotlangan. H. Akin, O‘.A. Roziqov va S. Temir tomonlaridan ART Gibbs o‘lchovlari deb ataluvchi o‘lchovlar o‘rganilgan. O‘.A. Roziqov va M.M. Raxmatullayevlar tomonidan (k_0)-translyatsion-invariant va (k_0)-davriy Gibbs o‘lchovlari o‘rganilgan.

Yuqoridagi kabi ko‘plab ilmiy ishlar bajarilganiga qaramasdan, Keli daraxtida hozirgacha birorta ham model uchun limit Gibbs o‘lchovlarining to‘liq tasnifi olinmaganligini ta’kidlash joiz.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya tadqiqoti Namangan davlat universitetining ilmiy-tadqiqot ishlari rejasining “Fundamental tadqiqotlar” tarmog‘i doirasida bajarilgan.

Tadqiqot maqsadi Keli daraxtida spin qiymatlari to‘plami chekli bo‘lgan Izing va Potts modellari uchun limit p-adik translyatsion-invariant, davriy va davriy bo‘lmagan Gibbs o‘lchovlari to‘plamini tavsiflash hamda bunday o‘lchovlarning chegaralanganligini tahlil qilishdan iborat.

Tadqiqotning vazifalari:

uchinchi tartibli Keli daraxtida Izing modeliga mos translyatsion-invariant *p*-adik umumlashgan Gibbs o‘lchovlari mavjud bo‘lishi uchun yetarli shartlarni aniqlash;

ikkinchi tartibli Keli daraxtida uch holatli Potts modeli uchun G_2-davriy *p*-adik kvazi Gibbs o‘lchovlarining mavjudlik shartlarini topish hamda faza almashishlari mavjudligini tekshirish;

ikkinchi tartibli Keli daraxtida uch holatli Potts modeliga mos funksional tenglamaning translyatsion-invariant yechimlaridan foydalanib yuqori tartibli Keli daraxtida ART *p*-adik kvazi Gibbs o‘lchovlarini qurish hamda bu o‘lchovlarni chegaralanganlikka tekshirish;

ikkinchi va uchinchi tartibli Keli daraxtida Izing modeliga mos translyatsion-invariant va $G_k^{(2)}$-davriy *p*-adik Gibbs o‘lchovlari yordamida yuqori tartibli Keli daraxtida davriy bo‘lmagan ART, (k_0)-translyatsion-invariant va (k_0)-davriy *p*-adik umumlashgan Gibbs o‘lchovlarini qurish.

Tadqiqot ob’ekti: Keli daraxtida p-adik Izing va Potts modellari.

Tadqiqot predmeti. Gruppalar va graflar nazariyasi, Gibbs o‘lchovlari nazariyasi, algebra va sonlar nazariyasi, nochiziqli dinamik sistemalar nazariyasi, p-adik analiz, Markov jarayonlari.

Tadqiqot usullari. Tadqiqot ishida p-adik analiz, sonlar nazariyasi, funksional analiz, kombinatorika, gruppalar nazariyasi, o‘lchovlar nazariyasi, chiziqli algebra va dinamik sistemalar nazariyasi usullaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

uchinchi tartibli Keli daraxtida Izing modeliga mos translyatsion-invariant *p*-adik umumlashgan Gibbs o‘lchovlari mavjud bo‘lishi uchun yetarli shartlar topilgan;

ikkinchi tartibli Keli daraxtida uch holatli Potts modeli uchun davriy *p*-adik kvazi Gibbs o‘lchovlarining mavjudlik shartlari topilgan hamda faza almashishlari mavjudligi isbotlangan;

kichik tartibli Keli daraxtida berilgan uch holatli Potts modeliga mos funksional tenglamaning topilgan translyatsion-invariant yechimlaridan foydalanib yuqori tartibli Keli daraxtida *p*-adik kvazi Gibbs o‘lchovlarini qurish konstruksiyasi berilgan hamda bunday o‘lchovlarning chegaralanganlik shartlari topilgan;

kichik tartibli Keli daraxtida Izing modeliga mos translyatsion-invariant va ikki davriy *p*-adik Gibbs o‘lchovlari yordamida yuqori tartibli Keli daraxtida davriy bo‘lmagan turli *p*-adik umumlashgan Gibbs o‘lchovlari qurilgan.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

Olingan natijalar va dissertatsiyada qo‘llanilgan usullar oliy o‘quv yurti magistratura talabalari va oliy ta’limdan keyingi ta’limda o‘quv kursi sifatida o‘qitilishi mumkin. Shuningdek, davriy *p*-adik umumlashgan Gibbs o‘lchovlari to‘plamida turli faza almashishlari mavjudligini ta’minlaydigan parametr qiymatlarining aniq ifodasidan xizmat ko‘rsatish nazariyasi masalalarini yechishda foydalanish mumkin.

Tadqiqot natijalarining ishonchliligi. p-Adik analiz, noarximed funksional analiz, noarximed Markov tasodifiy maydonlar nazariyasi, p-adik ehtimollar nazariyasi, sonlar nazariyasi, diskret vaqtli dinamik sistemalar, Gibbs va noarximed o‘lchovlar nazariyasi usullaridan foydalanilgan. Olingan natijalar qat’iy matematik mulohazalarga asoslanib isbotlangan.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati statistik mexanikaning turli p-adik modellari uchun Gibbs o‘lchovlari nazariyasining rivojlantirishda qo‘llanilishi bilan izohlanadi.

Tadqiqot natijalarining amaliy ahamiyati noarximed fizik sistemalar holatining o‘zgarishi tadqiq qilinganligi hamda kombinatorika va telekommunikatsiyaning ba’zi masalalarini yechish imkonini berganligi bilan izohlanadi.

Tadqiqot natijalarining joriy qilinishi. Statistik mexanikaning ba’zi klassik modellari uchun p-adik davriy Gibbs o‘lchovlari bo‘yicha olingan natijalar asosida:

Keli daraxtida p-adik Izing modeli uchun olingan natijalar G0003247 raqamli “Panjara modellarining qayta normalizatsiya qilingan gruppalari bilan bog‘liq xaotik va aralash p-adik dinamik sistemalari” mavzusidagi xorijiy grant loyihasida Keli daraxtlarida Izing modeliga mos p-adik umumlashgan Gibbs o‘lchovlarini tadqiq qilishda foydalanilgan (Birlashgan Arab Amirliklari universitetining 2023 yil 4 sentabrdagi ma’lumotnomasi, BAA). Ilmiy natijalarning qo‘llanilishi ba’zi *p*-adik ratsional funksiyalarning dinamikasini va fizik sistemalarning termodinamik xossalarini tavsiflash imkonini bergan.

Keli daraxtida statistik mexanikaning ba’zi klassik modellari uchun translyatsion-invariant va davriy p-adik umumlashgan Gibbs o‘lchovlari mavjudligini isbotlash usullaridan YOT-FTEX-2018-154 raqamli “Z^d panjaralarda va Γ^k Keli daraxtlarida gamiltonianlar spektrlari va Gibbs o‘lchovlari” mavzusidagi grant loyihasida statistik mexanikaning spin qiymatlari to‘plami kontinuum bo‘lgan ayrim klassik modellari uchun translyatsion-invariant va davriy Gibbs o‘lchovlarini tadqiq qilishda foydalanilgan (Mirzo Ulug‘bek nomidagi O‘zbekiston Milliy universitetining 2023-yil 9-sentabrdagi 04/11-5431 sonli ma’lumotnomasi). Ushbu ilmiy natijaning qo‘llanilishi statistik mexanikaning spin qiymatlari to‘plami kontinuum bo‘lgan ba’zi modellari uchun translyatsion-invariant va davriy Gibbs o’lchovlari to‘plamini kengaytirish imkonini bergan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 7 ta ilmiy-amaliy anjumanlarda, jumladan, 3 ta xalqaro va 4 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Dissertatsiya tadqiqoti mavzusi bo‘yicha jami 13 ta ilmiy ish chop etilgan, shulardan, O‘zbekiston Respublikasi Oliy Attestatsiya Komissiyasining falsafa doktorlik dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 6 ta, jumladan, 5 tasi xorijiy va 1 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, o‘nta paragraf, xulosa va foydalanilgan adabiyotlar ro‘yxatidan tashkil topgan. Dissertatsiyaning umumiy hajmi 111 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustivor yo‘nalishlariga mosligi ko‘rsatilgan, muammoning o‘rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, ob’yekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning “**p-Adik analiz va p-adik o‘lchovlar**” deb nomlanuvchi birinchi bobida asosiy tushunchalar, ta’riflar, shuningdek dissertatsiya ishida olingan natijalarni yoritishda foydalaniladigan muhim teoremalar keltirilgan va ikkinchi tartibli Keli daraxtida uch holatli Potts modeli uchun G_2-davriy p-adik kvazi Gibbs o‘lchovlari tadqiq qilingan, bu o‘lchovlarni mavjudlik shartlarini aniqlangan, ularni chegaralanganlikka tekshirilgan va faza almashishining mavjudlik shartlari topilgan.

p-biror fiksirlangan tub son bo‘lsin. Ixtiyoriy $x \neq 0$ ratsional sonni, $x = p^r \frac{m}{n}$ ko‘rinishda tasvirlash mumkin. Bunda, $m \in \mathbb{Z}$, $n \in \mathbb{N}$, m va n lar p tub songa bo‘linmaydi, ya’ni $(m, p) = (n, p) = 1$. Ushbu x ratsional sonni p-adik normasi quyidagicha aniqlanadi

$$|x|_p = \begin{cases} p^{-r}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Ushbu norma $|x+y|_p \leq \max\{|x|_p, |y|_p\}$ kuchli uchburchak tengsizligini qanoatlantirgani uchun noarximed normasi bo‘ladi. $\mathbb{Q}$ ratsional sonlar maydonining p-adik norma bo‘yicha to‘ldirmasiga p-adik sonlar maydoni deyiladi va bu maydon $\mathbb{Q}_p$ kabi belgilanadi.

Dissertatsiya ishida ko‘p foydalanilgan quyidagi muhim to‘plamlari keltiramiz.

$a \in \mathbb{Q}_p$ va $r > 0$ sonlar berilgan bo‘lsin.

a markazli r radiusli ochiq shar

$$B(a,r) = \{x \in \mathbb{Q}_p : |x-a|_p < r\},$$

a markazli r radiusli yopiq shar

$$\overline{B(a,r)} = \{x \in \mathbb{Q}_p : |x-a|_p \leq r\},$$

a markazli r radiusli sfera

$$S(a,r) = \{x \in \mathbb{Q}_p : |x-a|_p = r\},$$

p-adik butun sonlar

$$\mathbb{Z}_p = \overline{B(0,1)},$$

p-adik birlar

$$\mathbb{Z}_p^* = S(0,1)$$

kabi aniqlanadi.

p-adik Gibbs o‘lchovlar nazariyasida muhim ahamiyatga ega bo‘lgan quyidagi funksiyalarni keltiramiz. *p-adik logarifm* yaqinlashish sohasi $B(1,1)$ bo‘lgan quyidagi qator orqali aniqlanadi

$$\log_p(x) = \log_p(1+(x-1)) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}.$$

p-adik eksponenta yaqinlashish sohasi $B\left(1, p^{-1/(p-1)}\right)$ bo‘lgan quyidagi qator bilan beriladi

$$\exp_p(x) = \sum_{n=1}^{\infty} \frac{x^n}{n!}.$$

Quyidagi to‘plamni aniqlaymiz

$$\mathcal{E}_p = \left\{ x \in \mathbb{Q}_p : |x-1|_p < p^{-\frac{1}{p-1}} \right\}.$$

Shuni ta’kidlash joizki, $\mathcal{E}_p$ to‘plam p-adik eksponentaning qiymatlari to‘plami bo‘lib, ko‘paytirish amaliga nisbatan kommutativ gruppadir.

Faraz qilaylik $(X, \mathfrak{B})$-o‘lchovli fazo, $\mathfrak{B}$ to‘plamlar oilasi X ning qism to‘plamlaridan tuzilgan algebra va $\mathbb{K}$-noarximed maydon bo‘lsin. Agar

$\mu:\mathfrak{B}\to\mathbb{K}$ to‘plam funksiyasi chekli additiv bo‘lsa, ya’ni har qanday chekli yig‘indi uchun $A=\bigcup_{k=1}^{n}A_k,\ A_k\in\mathfrak{B},\ k=\overline{1,n},\ A_i\cap A_j=\varnothing,\ i\neq j$ uchun $\mu(A)=\sum_{k=1}^{n}\mu(A_k)$ tenglik o‘rinli bo‘lsa, u holda bu funksiya *noarximed taqsimoti* deyiladi.

Agar μ noarximed taqsimoti uchun $\mu(X)=1$ shart o‘rinli bo‘lsa, bunday taqsimot *noarximed ehtimollik taqsimoti* deyiladi. Chegaralangan ya’ni, $\sup\left\{|\,\mu(A)|_p : A\in\mathfrak{B}\right\}<\infty$ shartni qanoatlantiruvchi noarximed taqsimoti μ ga *noarximed o‘lchovi* deyiladi. Agar μ noarximed o‘lchovi uchun $\mu(X)=1$ shart o‘rinli bo‘lsa, bunday o‘lchov *noarximed ehtimollik o‘lchovi* deyiladi.

Ma’lumki, $\Gamma^k=(V,L)$ k-tartibli Keli daraxti har bir uchidan $k+1$ ta qirra chiqadigan siklga ega bo‘lmagan cheksiz grafdir. Bu yerda V daraxtning uchlari to‘plami, L uning qirralari to‘plami. Agar $x,y\in V$ uchlar uchun ularni tutashtiruvchi $l\in L$ qirra mavjud bo‘lsa, u holda x va y uchlar yaqin qo‘shnilar deyiladi va $l=\langle x,y\rangle$ kabi belgilanadi. Keli daraxtida $x,\,y\in V$ uchlar orasidagi $d(x,y)$ masofa ushbu

$$d(x,y)=\min\{d\mid \exists x=x_0,x_1,\ldots,x_{d-1},x_d=y\in V,\quad \langle x_0,x_1\rangle,\ldots,\langle x_{d-1},x_d\rangle\}$$

formula orqali aniqlanadi.

Fiksirlangan $x^0\in V$ uchun quyidagicha belgilashlarni kiritamiz:

$$W_n=\left\{x\in V\mid d(x,x^0)=n\right\},\ \ V_n=\bigcup_{j=0}^{n}W_j,\ \ L_n=\left\{l=\langle x,y\rangle\in L\mid x,y\in V_n\right\}.$$

$x\in W_n$ uchun $S(x)=\{y\in W_{n+1}:\ d(x,y)=1\}$ to‘plam x uchning to‘g‘ri avlodlari to‘plami deyiladi.

G_k-barpo etuvchilari mos ravishda $a_1,a_2,...,a_{k+1}$ bo‘lgan $k+1$ ta ikkinchi tartibli siklik gruppalarning ozod ko‘paytmasidan iborat bo‘lgan gruppa bo‘lsin.

Quyidagi tasdiq N.N. G‘anixodjayevning ishlaridan ma’lum.

1-tasdiq. k-tartibli Keli daraxtining V uchlar to‘plami bilan G_k gruppa o‘rtasida o‘zaro bir qiymatli moslik mavjud.

$\Gamma^k=(V,L)$ Keli daraxti va $\Phi\subset\mathbb{Z}$ chekli to‘plam berilgan bo‘lsin. $\Lambda\subset V$ to‘plamda aniqlangan $x\in\Lambda\to\sigma_\Lambda(x)\in\Phi$ funksiya konfiguratsiya deyiladi. Λ to‘plamda aniqlangan barcha konfiguratsiyalar to‘plami $\Omega_\Lambda=\Phi^\Lambda$ kabi belgilanadi. Xususan, $\Omega=\Omega_V$.

Berilgan $\sigma\in\Omega_\Lambda$ va $\varphi\in\Omega_{V\setminus\Lambda}$ konfiguratsiyalar uchun quyidagini aniqlaymiz

$$(\sigma\vee\varphi)(x)=\begin{cases}\sigma(x), & \text{agar } x\in\Lambda,\\ \varphi(x), & \text{agar } x\in V\setminus\Lambda.\end{cases}$$

$\sigma\in\Omega$ konfiguratsiyaning energiyasi ushbu

$$H(\sigma)=\sum_{\substack{\Lambda\subset V:\\ \mathrm{diam}(\Lambda)\le r}} I(\sigma_\Lambda),$$

gamiltonian orqali beriladi, bu yerda $r\in\mathbb{N},\ \mathrm{diam}(\Lambda)=\max\limits_{x,y\in\Lambda}d(x,y)$ va $I:\Omega_\Lambda\to\mathbb{Q}_p$ berilgan potensial.

Ω ning silindrik qism to‘plamlaridan hosil qilingan algebra $\mathfrak{B}$ bo‘lsin.

1-ta’rif. $(\Omega,\mathfrak{B})$ da aniqlangan p-adik ehtimollik taqsimoti μ berilgan bo‘lsin. Faraz qilaylik ixtiyoriy $\Lambda\subset V$ uchun

$$\mu(\sigma\,|\,\varphi)=\frac{\exp_p\left\{H\left(\sigma\vee\varphi\right)\right\}}{Z_{\Lambda,\varphi}},\ \forall\sigma\in\Omega_\Lambda,\ \forall\varphi\in\Omega_{V\setminus\Lambda},$$

o‘rinli bo‘lsin. Bu yerda $Z_{\Lambda,\varphi}$ normallovchi o‘zgarmas bo‘lib, u quyidagicha aniqlanadi

$$Z_{\Lambda,\varphi}=\sum_{\omega\in\Omega_\Lambda}\exp_p\left\{H\left(\omega\vee\varphi\right)\right\}.$$

Bunday taqsimotga p-*adik Gibbs taqsimot* deyiladi.

1.4 paragrafda ikkinchi tartibli yarim Keli daraxtida p-adik Potts modeli uchun G_2-davriy kvazi Gibbs o‘lchovlari o‘rganilgan.

$\mathbb{Q}_p$ p-adik sonlar maydoni va $\Phi=\{1,2,...,q\}$ bo‘lsin. p-adik Potts modelining formal gamiltoniani

$$H(\sigma)=J\sum_{\langle x,y\rangle\in L}\delta_{\sigma(x)\sigma(y)},$$

orqali aniqlanadi. Bunda $J\in B(0,p^{-1/(p-1)})$ o‘zgarmas, δ_{ij} Kroneker simvoli.

Faraz qilaylik $\mathbf{h}:V\setminus\{x^0\}\to\mathbb{Q}_p^q$ akslantirish berilgan ya’ni, $\mathbf{h}_x=(h_{1,x},h_{2,x},...,h_{q,x})$, bo‘lsin, bunda $h_{i,x}\in\mathbb{Q}_p$ $(i\in\Phi)$ va $x\in V\setminus\{x^0\}$. Har bir $n\in\mathbb{N}$ uchun Ω_{V_n} da p-adik ehtimollik taqsimoti $\mu_{\mathbf{h}}^{(n)}$ quyidagicha aniqlanadi

$$\mu_{\mathbf{h}}^{(n)}(\sigma)=\frac{1}{Z_n^{(\mathbf{h})}}\exp_p\{H_n(\sigma)\}\prod_{x\in W_n}h_{\sigma(x),x},\qquad(1)$$

bu yerda $\sigma\in\Omega_{V_n}$, $Z_n^{(\mathbf{h})}$ normallovchi o‘zgarmas bo‘lib, u

$$Z_n^{(\mathbf{h})}=\sum_{\sigma\in\Omega_{V_n}}\exp_p\{H_n(\sigma)\}\prod_{x\in W_n}h_{\sigma(x),x},$$

formula bilan aniqlanadi.

p-adik ehtimollik taqsimoti ixtiyoriy $n\in\mathbb{N}$ va $\sigma_{n-1}\in\Omega_{V_{n-1}}$ uchun

$$\sum_{\omega_n\in\Omega_{W_n}}\mu_{\mathbf{h}}^{(n)}(\sigma_{n-1}\vee\omega_n)=\mu_{\mathbf{h}}^{(n-1)}(\sigma_{n-1}),\qquad(2)$$

shartni bajarsa, ushbu p-adik ehtimollik taqsimoti muvofiqlik shartini qanoatlantiradi deyiladi. Agar (1) p-adik ehtimollik taqsimoti (2) muvofiqlik shartini qanoatlantirsa, N.N. G‘anixodjayev, F.M. Muxamedov, O‘.A. Roziqovlarning ishida isbotlangan Kolmogorov teoremasining noarximed muqobiliga ko‘ra, Ω da yagona limit p-adik o‘lchov mavjud bo‘lib, bu o‘lchov ixtiyoriy $n\in\mathbb{N}$ va $\sigma_n\in\Omega_{V_n}$ uchun

$$\mu(\sigma\in\Omega:\sigma|_{V_n}\equiv\sigma_n)=\mu_{\mathbf{h}}^{(n)}(\sigma_n),$$

shartni qanoatlantiradi. Bu o‘lchovni p-adik kvazi Gibbs o‘lchovi deyiladi.

F.M. Muxamedov ishlarida p-adik Gibbs o‘lchovlari uchun faza almashish tushunchasi keltirilgan. Agar berilgan model uchun kamida ikkita p-adik Gibbs o‘lchovlari mavjud bo‘lib, ulardan biri chegaralangan, boshqa biri esa chegaralanmagan bo‘lsa, u holda berilgan model uchun faza almashish mavjud deyiladi.

Ma’lumki, Keli daraxtida Potts modeli uchun (1) ko‘rinishda aniqlangan $\mu_{\mathbf{h}}^{(n)}$ $(n=1,2,\ldots)$ ehtimollik taqsimotlari ketma-ketligi (2) muvofiqlik shartini qanotlantirish uchun ixtiyoriy $x\in V\setminus\{x^0\}$ da $\mathbf{h}=\{\mathbf{h}_{\mathbf{x}}, x\in V\}$ miqdorlar to‘plami ushbu

$$\mathbf{h}_x=\prod_{y\in S(x)}F(\mathbf{h}_y,\theta), \tag{3}$$

tenglikni qanoatlantirishi zarur va yetarli, bu yerda $\theta=\exp_p\{J\}$,
$\hat{h}=\left(\hat{h}_1,\hat{h}_2,...,\hat{h}_{q-1}\right)\in\mathbb{Q}_p^{q-1}$ vektor $\mathbf{h}=(h_1,h_1,...,h_q)\in\mathbb{Q}_p^q$ vektor orqali quyidagicha hosil qilinadi

$$\hat{h}_i=\frac{h_i}{h_q}, i=1,2,...,q-1,$$

$F:\mathbb{Q}_p^{q-1}\times\mathbb{Q}_p\to\mathbb{Q}_p^{q-1}$ akslantirish esa $F(\mathbf{x};\theta)=\left(F_1(\mathbf{x};\theta),...,F_{q-1}(\mathbf{x};\theta)\right)$ ko‘rinishda aniqlanadi, bunda

$$F_i(\mathbf{x};\theta)=\frac{(\theta-1)x_i+\sum_{j=1}^{q-1}x_j+1}{\sum_{j=1}^{q-1}x_j+\theta}, \mathbf{x}=(x_1,x_2,...,x_{q-1})\in\mathbb{Q}_p^{q-1},\ i=1,2,...,q-1.$$

Γ_+^k yarim Keli daraxtining ildizi x^0 bo‘lsin. Ma’lumki, yarim Keli daraxtining koordinatali tuzilishi surish binar amali bilan $(G^k,\circ)$ yarim gruppa hosil qiladi. $G\subset G^k$ to‘plam G^k ning qism yarim gruppa bo‘lsin. $h:G^k\to Y$ akslantirish G^k da aniqlangan Y to‘plamdan qiymat oluvchi funksiya bo‘lsin. Agar h akslantirish $\forall g\in G$ va $\forall x\in G^k$ uchun $h(\tau_g(x))=h(x)$ shartni qanoatlantirsa, bu akslantirish G-*davriy* deyiladi, bunda $\tau_g(x)=g\circ x$. Ixtiyoriy G^k-davriy akslantirish *translyatsion-invariant* deyiladi.

$m\geq 2$ bo‘lsin. Ma’lumki, quyidagi to‘plam

$$G_m=\{x\in G^k: d(x,x^0)\equiv 0(\mathrm{mod}\, m)\}$$

to‘plam G^k ning qism yarim gruppasi bo‘ladi.

Quyidagi

$$I_m=\left\{\mathbf{h}:\mathbf{h}=(\underbrace{1,...,1,h}_{m},1,...,1)\right\}, m=1,...,q-1$$

to‘plam (3) uchun invariant to‘plam bo‘lishini oson tekshirish mumkin. Quyidagi teoremalarda o‘lchovlar deganda ushbu I_m invariant to‘plamdagi $\mathbf{h}$ vektorlarga mos kelgan o‘lchovlarni nazarda tutamiz.

I_m invariant to‘plamda $h:=h_0=1$ ga mos kelgan translyatsion-invariant p-adik kvazi Gibbs o‘lchovini μ_{h_0} orqali belgilaymiz.

1-teorema. Ikkinchi tartibli Keli daraxtida uch holatli Potts modeli uchun quyidagi tasdiqlar o‘rinli

1) Agar $p=2$ yoki $p\equiv 5(\mathrm{mod}\,8)$ yoki $p\equiv 7(\mathrm{mod}\,8)$ bo‘lsa, u holda yagona chegaralangan p-adik G_2-davriy kvazi Gibbs o‘lchovi μ_{h_0} mavjud;
2) Agar $p\equiv 1(\mathrm{mod}\,8)$ yoki $p\equiv 3(\mathrm{mod}\,8)$ bo‘lsa, u holda beshta p-adik G_2-davriy kvazi Gibbs o‘lchovlari mavjud bo‘lib, ulardan ikkitasi translyatsion-invariant bo‘lmagan p-adik G_2-davriy kvazi Gibbs o‘lchovlaridir. Qolaversa,

2.1) $p=3$ bo‘lsa, ularning barchasi chegaralanmagan,

2.2) $p\neq 3$ bo‘lsa, faqat μ_{h_0} o‘lchovgina chegaralangan.

2-teorema. Agar $p\equiv 1(\mathrm{mod}\,8)$ yoki $p\equiv 3(\mathrm{mod}\,8)$, $p\neq 3$ bo‘lsa, u holda ikkinchi tartibli Keli daraxtida uch holatli p-adik Potts modeli uchun faza almashish mavjud.

1-izoh. F.M. Muxamedov, O.N. Hakimov ishlarida Γ_+^k yarim Keli daraxtida p-adik Potts modeli uchun ixtiyoriy $m\in\mathbb{N}$ uchun G_m-davriy kvazi Gibbs o‘lchovlari mavjudligi isbotlangan. Ammo o‘lchovlarini oshkor ko‘rinishini topilmagan. Biz 2-tartibli yarim Keli daraxtida G_2-davriy p-adik kvazi Gibbs o‘lchovlarini oshkor ko‘rinishini topdik. Ushbu ko‘rinish o‘lchovlari chegaralanganlikka tekshirishni osonlashtiradi.

2-bob p-adik Izing modeliga bag‘ishlangan bo‘lib, 2.1 paragrafda asosiy tushunchalar, ma’lum natijalar keltirilgan. 2.2 va 2.3 paragraflarda 3-tartibli Keli daraxtida Izing modeli uchun mos ravishda translyatsion-invariant va $G_k^{(2)}$-davriy p-adik umumlashgan Gibbs o‘lchovlari o‘rganilgan.

G_k^* gruppa G_k gruppaning normal qism gruppasi bo‘lsin.

2-ta’rif. Agar $\forall x\in G_k,\ y\in G_k^*$ uchun $h_{yx}=h_x$ bo‘lsa, u holda $h=\{h_x, x\in G_k\}$ miqdorlar to‘plami G_k^*-davriy deyiladi.

G_k-davriy miqdorlar to‘plami translyatsion-invariant deyiladi.

G_k^*-davriy (translyatsion-invariant) $h=\{h_x, x\in G_k\}$ miqdorlar to‘plamiga mos kelgan Gibbs o‘lchovlari G_k^*-davriy (translyatsion-invariant) Gibbs o‘lchovlari deyiladi.

Ma’lumki, barcha juft uzunlikdagi so‘zlardan iborat bo‘lgan $G_k^{(2)}$ to‘plam G_k-gruppaning ikki indeksli normal qism gruppasi bo‘ladi ya’ni,

$$G_k^{(2)}=\left\{x\in G_k: |x|-\text{juft}\right\}$$

to‘plam uchun $G_k/G_k^{(2)}=\left\{G_k^{(2)}, G_k\setminus G_k^{(2)}\right\}$ bo‘ladi.

$\mathbb{Q}_p$ p-adik sonlar maydoni va $\Phi=\{-1,1\}$ bo‘lsin. p-adik Izing modelining formal gamiltoniani

$$H(\sigma)=J\sum_{\langle x,y\rangle\in L}\sigma(x)\sigma(y),$$

orqali aniqlanadi. Bunda $J\in B(0,p^{-1/(p-1)})$ o‘zgarmas parametr.

Faraz qilaylik $h: x\in V\setminus\{x^0\}\to h_x\in\mathbb{Q}_p$ akslantirish berilgan bo‘lsin. Har bir $n\in\mathbb{N}$ uchun Ω_{V_n} da p-adik ehtimollik taqsimoti $\mu_{\mathbf{h}}^{(n)}$ quyidagicha aniqlanadi

$$\mu_h^{(n)}(\sigma)=\frac{1}{Z_n^{(h)}}\exp_p\{H_n(\sigma)\}\prod_{x\in W_n}h_x^{\sigma(x)}, \tag{4}$$

bu yerda $\sigma\in\Omega_{V_n}$, $Z_n^{(h)}$ normallovchi o‘zgarmas.

Agar (4) p-adik ehtimollik taqsimoti (2) muvofiqlik shartini qanoatlantirsa, Kolmogorov teoremasining noarximed muqobiliga ko‘ra Ω da yagona limit p-adik o‘lchov mavjud. Bu o‘lchovni p-adik umumlashgan Gibbs o‘lchovi deyiladi.

Agar $\forall x\in V$ uchun $h_x\in\mathcal{E}_p$ bo‘lsa, bunday $h=\{h_x,\, x\in V\}$ miqdorlarga mos kelgan p-adik umumlashgan Gibbs o‘lchovi p-adik Gibbs o‘lchovi bo‘ladi.

Ma’lumki, Keli daraxtida Izing modeli uchun (4) ko‘rinishda aniqlangan $\mu_h^{(n)}$ $(n=1,2,\dots)$ ehtimollik taqsimotlari ketma-ketligi (2) muvofiqlik shartini qanotlantirish uchun ixtiyoriy $x\in V\setminus\{x^0\}$ da $h=\{h_x, x\in V\}$ miqdorlar to‘plami ushbu

$$h_x^2=\prod_{y\in S(x)}\frac{\theta h_y^2+1}{h_y^2+\theta}, \tag{5}$$

tenglikni qanoatlantirishi zarur va yetarli, bu yerda $\theta=\exp_p\{2J\}$.

3-teorema. N uchinchi tartibli Keli daraxtida Izing modeli uchun translyatsion-invariant p-adik umumlashgan Gibbs o‘lchovlari soni bo‘lsin. U holda quyidagi tasdiqlar o‘rinli:

$$N=\begin{cases}1, & \text{agar } p\not\equiv 1 (\text{mod}\, 4) \text{ bo'lsa},\\ 2, & \text{agar } p\equiv 5 (\text{mod}\, 12) \text{ bo'lsa},\\ 4, & \text{agar } p\equiv 1 (\text{mod}\, 12) \text{ bo'lsa}.\end{cases}.$$

Ko‘rish mumkinki, $h:=h_0=1$ (5) funksional tenglamani yechimi bo‘ladi. Biz bu yechimga mos kelgan p-adik umumlashgan Gibbs o‘lchovini μ_{h_0} orqali belgilaymiz.

4-teorema. Uchinchi tartibli Keli daraxtida Izing modeli uchun translyatsion-invariant p-adik umumlashgan Gibbs o‘lchovlari haqidagi quyidagi tasdiqlar o‘rinli:

1) Agar $p=2$ bo‘lsa, yagona translyatsion-invariant chegaralanmagan μ_{h_0} p-adik umumlashgan Gibbs o‘lchovi mavjud.

2) Agar $p\neq 2$ bo‘lsa, translyatsion-invariant o‘lchovlar ichida faqat μ_{h_0} chegaralangan.

2-izoh. O‘.A. Rozikov va O.N. Hakimovning ishlaridan ma’lumki, p-adik Izing modeli uchun har qanday G_k^*-davriy Gibbs o‘lchovi yoki translyatsion-invariant yoki $G_k^{(2)}$-davriy Gibbs o‘lchovi bo‘ladi.

Uchinchi tartibli Keli daraxtida Izing modeli uchun davriy Gibbs o‘lchovlarini o‘rganishimiz uchun 2-izohga ko‘ra $G_k^{(2)}$-davriy Gibbs o‘lchovlarini o‘rganishimiz yetarli.

5-teorema. M uchinchi tartibli Keli daraxtida Izing modeli uchun translyatsion-invariant bo‘lmagan $G_k^{(2)}$-davriy p-adik umumlashgan Gibbs o‘lchovlari soni bo‘lsin. U holda quyidagi tasdiqlar o‘rinli:

$$M=\begin{cases}6, \text{ agar } p\equiv 1(\mathrm{mod}\,6) \text{ bo'lsa},\\ 2, \text{ agar } p=2, |\theta-1|_2=\frac{1}{4} \text{ bo'lsa},\\ 2, \text{ agar } p=3, \mathrm{ord}_3|\theta-1|_3 \text{ toq son}, |\theta-1|_3\le\frac{1}{27} \text{ va } |\theta-1|_3(\theta-1)\in\mathcal{E}_3 \text{ bo'lsa},\\ 0, \text{ boshqa hollarda}.\end{cases}.$$

6-teorema. Agar $p\equiv 1(\mathrm{mod}\,12)$ yoki $p=3$, $\mathrm{ord}_3|\theta-1|_3$ toq son $|\theta-1|_3\le\frac{1}{27}$, $|\theta-1|_3(\theta-1)\in\mathcal{E}_3$ bo‘lsa, u holda uchinchi tartibli Keli daraxtida p-adik Izing modeli uchun faza almashishi mavjud.

3-bob Keli daraxtida Izing va Potts modellari uchun davriy bo‘lmagan p-adik Gibbs o‘lchovlariga bag‘ishlangan. Biz ushbu bobda davriy bo‘lmagan p-adik Gibbs o‘lchovlarini konstruktiv usulda hosil qilamiz.

O.N. Hakimovning ishlaridan ma’lumki, $p\equiv 1(\mathrm{mod}\,4)$ bo‘lganda ikkinchi tartibli yarim Keli daraxtida Izing modeli uchun (5) funksional tenglamani translyatsion-invariant va G_2-davriy yechimlari quyidagicha bo‘ladi:

$$h_0^{(t)}=1,\ h_{1,2}^{(t)}=\frac{\theta-1\pm\sqrt{(\theta-3)(\theta+1)}}{2},\ h_{1,2}^{(p)}=\frac{1-\theta\pm\sqrt{(\theta-1)^2-4\theta^2}}{2\theta}. \tag{6}$$

$k\ge 3, k_0=2$ bo‘lsin. $x\in V$ uchun $S_{k_0}(x)$ orqali $S(x)$ dagi ixtiyoriy k_0 ta uchni belgilaymiz, qolgan $k-k_0$ ta uchni $S_{k-k_0}(x)$ orqali belgilaymiz. $k-k_0=a+b+c$, bo‘lsin, bunda a va b juft sonlar, c juft yoki toq son.

$h=\{h_x, x\in V\}$ (bunda $h_x\in\left\{1,h_1^{(t)},h_2^{(t)},h_1^{(p)},h_2^{(p)}\right\}$) miqdorlar to‘plamini quyidagi qoidalar orqali aniqlaymiz:

(A_1) x uchda $h_x=h_i^{(t)}$ $(i=1,2)$ bo‘lsa, u holda

$$h_y = \begin{cases} h_i^{(t)}, & S(x) \text{ ning } \dfrac{a}{2}+2 \text{ ta uchlari uchun,} \\ h_{3-i}^{(t)}, & S(x) \text{ ning } \dfrac{a}{2} \text{ ta uchlari uchun,} \\ h_i^{(p)}, & S(x) \text{ ning } \dfrac{b}{2} \text{ ta uchlari uchun,} \\ h_{3-i}^{(p)}, & S(x) \text{ ning } \dfrac{b}{2} \text{ ta uchlari uchun,} \\ 1, & S(x) \text{ ning } c \text{ ta uchlari uchun.} \end{cases}$$

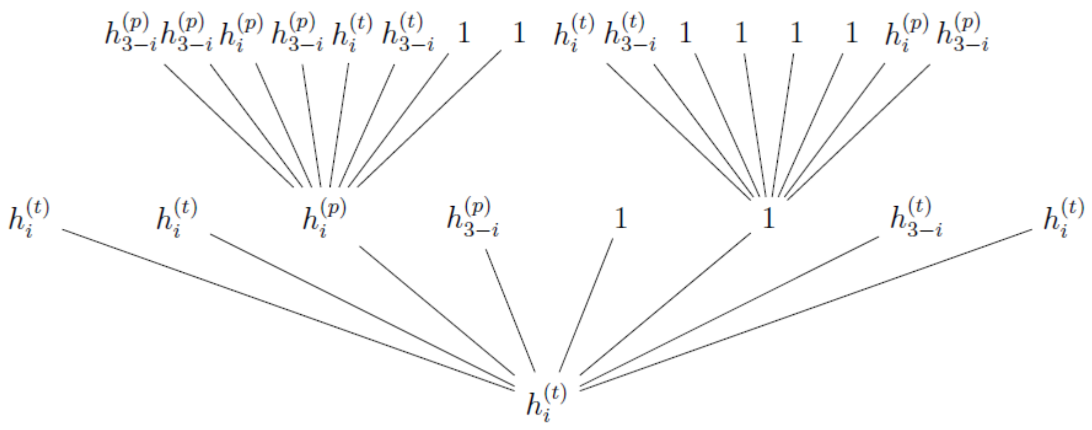
(A_2) x uchda $h_x = h_i^{(p)}$ $(i=1,2)$ bo‘lsa, u holda

$$h_y = \begin{cases} h_i^{(t)}, & S(x) \text{ ning } \dfrac{a}{2} \text{ ta uchlari uchun,} \\ h_{3-i}^{(t)}, & S(x) \text{ ning } \dfrac{a}{2} \text{ ta uchlari uchun,} \\ h_i^{(p)}, & S(x) \text{ ning } \dfrac{b}{2} \text{ ta uchlari uchun,} \\ h_{3-i}^{(p)}, & S(x) \text{ ning } \dfrac{b}{2}+2 \text{ ta uchlari uchun,} \\ 1, & S(x) \text{ ning } c \text{ ta uchlari uchun.} \end{cases}$$

(A_3) x uchda $h_x = 1$ bo‘lsa, u holda

$$h_y = \begin{cases} h_i^{(t)}, & S(x) \text{ ning } \dfrac{a}{2} \text{ ta uchlari uchun,} \\ h_{3-i}^{(t)}, & S(x) \text{ ning } \dfrac{a}{2} \text{ ta uchlari uchun,} \\ h_i^{(p)}, & S(x) \text{ ning } \dfrac{b}{2} \text{ ta uchlari uchun,} \\ h_{3-i}^{(p)}, & S(x) \text{ ning } \dfrac{b}{2} \text{ ta uchlari uchun,} \\ 1, & S(x) \text{ ning } c+2 \text{ ta uchlari uchun.} \end{cases}$$

$(i=1,2)$ (1-rasm).



1-rasm: 8-tartibli yarim Keli daraxtida h_x miqdorlarni $a=2, b=2, c=2, i=1,2$ holdagi joylashuvi.

7-teorema. $p \equiv 1 \,(\mathrm{mod}\, 4)$ bo‘lsin. U holda Γ_+^k yarim Keli daraxtida (A_1), (A_2) va (A_3) qoidalar yordamida aniqlangan h miqdorlar to‘plami (5) funksional tenglamani qanoatlantiradi. Qolaversa, agar $a^2+b^2 \neq 0$ bo‘lsa, u holda bu qoidalar yordamida qurilgan h miqdorlarga mos p-adik umumlashgan Gibbs o‘lchovlari chegaralanmagan bo‘ladi.

3-izoh. 1) Agar $(A_1), (A_2)$ qoidalarda $a=b=0, c\neq 0$ bo‘lsa, u holda h miqdorlarga mos Gibbs o‘lchovlari k-tartibli Keli daraxtida Izing modeli ART p-adik umumlashgan Gibbs o‘lchovlari bo‘ladi.

2) Agar (A_1) qoidada $b=c=0, a\neq 0$ bo‘lsa, u holda h miqdorlarga mos Gibbs o‘lchovlari k-tartibli Keli daraxtida Izing modeli uchun (k_0)-translyatsion-invariant p-adik umumlashgan Gibbs o‘lchovlari bo‘ladi.

3) Agar (A_2) qoidada $a=b=0, c\neq 0$ bo‘lsa, u holda h miqdorlarga mos Gibbs o‘lchovlari k-tartibli Keli daraxtida Izing modeli uchun (k_0)-davriy p-adik umumlashgan Gibbs o‘lchovlari bo‘ladi;

4) Qolgan barcha hollarda avvalgi ma’lum o‘lchovlardan farqli o‘lchovlar hosil bo‘ladi.

3.3 paragrafda Blexer-G‘anixodjayev konstruksiyasining p-adik muqobili ham qurilgan. Γ_+^k yarim Keli daraxtida biror $\pi = x^0 = x_0 < x_1 < \ldots$ yo‘lni tayinlaylik (Bunda $x<y$ belgi x^0 dan y ga boruvchi yo‘l x orqali o‘tishini anglatadi). Γ_+^k yarim Keli daraxtida π yo‘lga mos (5) tenglamani qanoatlantiruvchi $h^\pi = \{h_x^\pi : x\in V\}$ p-adik sonlar to‘plamini quyidagicha aniqlaymiz.

$x \in W_n$ bo‘lsin.

$$h_x^{\pi} = \begin{cases} \dfrac{1}{h_*}, \text{ agar } x \prec x_n, \ x \in W_n \text{ bo'lsa;} \\ h_*, \text{ agar } x_n \prec x, \ x \in W_n \text{ bo'lsa;} \\ h_{x_n}^{(n)}, \text{agar } x = x_n \text{ bo'lsa.} \end{cases} \quad (7)$$

Bunda $n = 1, 2, ...,$ $x \prec x_n$ $(x_n \prec x)$ belgi x π yo‘ldan chapda (o‘ngda) joylashganini anglatadi, h_* esa (5) funksional tenglamaning ixtiyoriy translyatsion-invariant yechimi va $h_{x_n}^{(n)}$ ketma-ketlik $\left(h_{x_n}^{(n)}\right)^2 \in \mathbb{Z}_p^* \setminus B(-1, 1)$ shartni qanoatlantiruvchi ixtiyoriy ketma-ketlikdir.

8-teorema. $p \geq 3$ bo‘lsin. Γ_+^k yarim Keli daraxtida ixtiyoriy π yo‘lga mos (5) va (7) tenglamalarni qanoatlantiruvchi yagona $h^{\pi} = \{h_x^{\pi}, x \in V\}$ p-adik sonlar to‘plami mavjud. Qolaversa, ushbu p-adik sonlar to‘plamiga mos kelgan p-adik umumlashgan Gibbs o‘lchovlari $h_* = 1$ bo‘lgandagina chegaralangan.

9-teorema. $p \geq 3$ bo‘lsin. Agar (5) funksional tenglamaning $h_* \notin \{-1, 1\}$ shartni qanoatlantiruvchi translyatsion-invariant yechimi mavjud bo‘lsa, u holda p-adik Izing modeli uchun faza almashish mavjud bo‘ladi.

10-teorema. 8-teoremadagi o‘lchovlar π yo‘lga bog‘liq bo‘lib, Keli daraxtida p-adik Izing modeli uchun sanoqsiz sondagi p-adik umumlashgan Gibbs o‘lchovlari mavjud.

XULOSA

Dissertatsiya chekli holatli p-adik Potts va Izing modellari uchun translyatsion-invariant, davriy va davriy bo‘lmagan p-adik Gibbs o‘lchovlariga, shuningdek, ularning chegaralanganlik muammolari hamda bu modellar uchun faza almashishlariga bag‘ishlangan.

Tadqiqotning asosiy natijalari quyidagilardan iborat:

1. Agar $p \equiv 1(\mathrm{mod}\, 8)$ yoki $p \equiv 3(\mathrm{mod}\, 8)$ bo‘lsa, ikkita translyatsion-invariant bo‘lmagan p-adik G_2-davriy kvazi Gibbs o‘lchovlari mavjudligi isbotlangan.

2. Agar $p \equiv 1(\mathrm{mod}\, 8)$ yoki $p \equiv 3(\mathrm{mod}\, 8)$, $p \neq 3$ bo‘lsa, u holda ikkinchi tartibli Keli daraxtida uch holatli p-adik Potts modeli uchun faza almashish mavjudligi isbotlangan.

3. Uchinchi tartibli Keli daraxtida Izing modeli barcha translyatsion invariant p-adik umumlashgan Gibbs o‘lchovlari topilgan. Jumladan, Agar $p \equiv 1(\mathrm{mod}\, 12)$ bo‘lsa, bunday o‘lchovlar soni to‘rtta bo‘lishi va faza almashishi ro‘y berishi isbotlangan.

4. Uchinchi tartibli Keli daraxtida Izing modeli uchun barcha translyatsion-invariant bo‘lmagan $G_k^{(2)}$-davriy p-adik umumlashgan Gibbs o‘lchovlari topilgan. Bu o‘lchovlarni chegaralanganlikka tekshirilgan.

5. Ikkinchi tartibli Keli daraxtida uch holatli Potts modeli uchun funksional tenglamaning topilgan translyatsion-invariant yechimlaridan foydalanib, yuqori tartibli Keli daraxtida ART p-adik kvazi Gibbs o‘lchovlari topilgan, bu o‘lchovlarni mavjudlik shartlari va ularning chegaralanganlikka tekshirilgan.

6. Ikkinchi va uchinchi tartibli Keli daraxtida Izing modeli uchun funksional tenglamaning topilgan translyatsion-invariant va $G_k^{(2)}$-davriy yechimlaridan foydalanib, yuqori tartibli Keli daraxtida davriy bo‘lmagan ART, (k_0)-translyatsion-invariant va (k_0)-davriy p-adik umumlashgan Gibbs o‘lchovlarini topilgan. Avvaldan ma’lum bo‘lgan ba’zi konstruksiyalarni umumlashtiradigan konstruksiya qurilgan va konstruksiya yordamida, qurilgan o‘lchovlarni mavjudlik shartlarini aniqlangan va ularning chegaralanganlikka tekshirilgan.

7. Izing modeli uchun sanoqsiz sondagi Gibbs o‘lchovlarni mavjudligini taminlovchi Blexer-G‘anixodjayev konstruksiyasining p-adik muqobili qurilgan va bu konstruksiyadan foydalanib topilgan o‘lchovlarni mavjudlik shartlarini aniqlangan va faza almashishlari mavjud bo‘lishi uchun yangi shart topilgan.

SCIENTIFIC COUNCIL AWARDING SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED AFTER V.I.ROMANOVSKY

NAMANGAN STATE UNIVERSITY

TUKHTABAEV AKBARKHUJA MAMAJONOVICH

p-ADIC PERIODIC GIBBS MEASURES FOR SOME CLASSICAL MODELS OF STATISTICAL MECHANICS

01.01.01-Mathematical analysis

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) ON PHYSICAL AND MATHEMATICAL SCIENCES

Tashkent – 2023

The theme of dissertation of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Ministry of Higher education, Science and Innovations of the Republic of Uzbekistan under number № B2021.4.PhD/FM556.

Dissertation has been prepared at Namangan state university.

The abstract of the dissertation is posted in three languages (Uzbek, English, Russian (resume)) on the website of Scientific Council (https://kengash.mathinst.uz) and the "ZiyoNet" information and educational portal (http://www.ziyonet.uz).

Scientific supervisor:	**Rahmatullaev Muzaffar Muxammadjanovich** Doctor of Physical and mathematical Sciences, senior researcher
Official opponents:	**Kudaybergenov Karimbergen Kadirbergenovich** Doctor of Physical and mathematical Sciences, professor
	Botirov Golibjon Isroilovich Doctor of Physical and mathematical Sciences, senior researcher
Leading organization:	**The National University of Uzbekistan**

Defense will take place on " 21 " November 2023 at 16:00 at the meeting of Scientific Council DSc.02/30.12.2019.FM.86.01 at Institute of Mathematics named after V.I.Romanovsky. (Address: University str. 9, Almazar district, Tashkent, 100174, Uzbekistan, Ph.: (+998 78) 207 91 40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

Dissertation is possible to review in Information-resource center at Institute of Mathematics named after V.I.Romanovsky (registered for No.170). (Address: University str. 9, Almazar district, Tashkent, 100174, Uzbekistan, Ph.: (+998 78) 207 91 40).

Abstract of dissertation sent out on " 27 " October 2023 year.
(Mailing report No.2 on " 27" October 2023 year).

U.A.Rozikov
Chairman of Scientific Council
on award of scientific degrees,
DSc., Professor

J.K.Adashev
Scientific secretary of Scientific Council
on award of scientific degrees,
DSc., senior researcher

U.U.Jamilov
Chairman of scientific Seminar under
Scientific Council on award of scientific degrees,
DSc., senior researcher

INTRODUCTION (abstract of PhD thesis)

Actuality and demand of the theme of dissertation. Many scientific and practical researches devoted to the study of thermodynamic properties of physical and biological systems conducted around the world are often leaded to the investigation of Archimedean and non-Archimedean models of statistical mechanics. p-adic Gibbs measures are the main object of the theory of phase transitions for non-Archimedean models of statistical mechanics. The study of Gibbs measures is important in solving problems in various fields of science and technology, such as chemistry, physics, service theory, material science, biology.
Due to the complexity of fully describing the set of all p-adic Gibbs measures corresponding to the non-Archimedean models of statistical mechanics, the problem of constructing a sufficient number of Gibbs measures corresponding to the given model remains relevant.

Nowadays in the world, one of the actual problems is to determine the existence of p-adic Gibbs measures for the Hamiltonian given on lattice systems, in particular, tree-structured countable graphs, and to describe the set of all p-adic Gibbs measures. One of the important problems of the theory of phase transitions in statistical physics is the problem of determining the existence of at least two -adic periodic Gibbs measures in the set of all p-adic Gibbs measures. Therefore, describing the set of all periodic and translational-invariant p-adic Gibbs measures for a given Hamiltonian, checking such measures for boundedness, and determining the existence of a phase transition for p-adic generalized Gibbs measures are purposeful scientific researches.

It is known that in our country, in our country, much attention has been paid to develop important directions of scientific and practical application of fundamental sciences, which is one of the main bases of development. In particular, in recent years, significant results have been achieved in the study of translation-invariant, periodic and some non-periodic Gibbs measures generated by constructive methods for classical models of statistical mechanics in the field of real and p-adic numbers on the Cayley tree. The investigations on the international level in such important areas as the mathematical physics, non-Archimedean analysis, theory of non-Archimedean measures and theory of dynamical systems are considered as the main task of fundamental research[1]. In this way, the development of phase transitions theory for the classical models of statistical mechanics plays a crucial role in the implementation this decree.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees and Resolutions of the President of the Republic of Uzbekistan of February 7, 2017, PF-4947 , “On the strategy of action for the

[1] Decree of Cabinet of Ministers of the Republic of Uzbekistan at the 2017 year 18 May “On measures on the organization of activities of the first created scientific research institutions of the Academy of Sciences of the Republic of Uzbekistan” № 292

further development of the Republic of Uzbekistan", PQ-4387 dated July 9, 2019 "On state support for the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of the Republic of Uzbekistan", PQ-4708 of May 7, 2020 "On measures to improve the quality of education and research in the field of mathematics" as well as in other regulations related to basic sciences.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of Republic of Uzbekistan IV, "Mathematics, Mechanics and Computer Science".

The degree of scrutiny of the problem. The concept of Gibbs distribution, which is important for systems with a constant temperature and in thermal equilibrium with the environment, was introduced by the American scientist J.U. Gibbs. The general characteristics of real-valued limit Gibbs measures are given in the works of R.L. Dobrushin, O. Lanford and D. Ruelle. Modern theory of real-valued Gibbs measures is highlighted in works of R. Baxter, H.O. Georgii, V.A. Malishov, R.A. Minlos, K. Preston, D. Ruelle, Ya.G. Sinai, G. Gallavotti, F. Bonetto, J. Gentile, Jin Zin-Justin, N.N. Ganikhodjaev, U.A. Rozikov and F.M. Mukhamedov. R.L. Dobrushin proved the theorem about the existence of a real-valued limit Gibbs measure. Gibbs measures for classical models of statistical mechanics are studied by R.L. Dobrushin, G.R. Brightwell, P. Winkler, Yu.M. Suhov, J. Martin, N.N. Ganikhodjaev, U.A. Rozikov, F.M. Mukhamedov, D. Gandolfo, M.M. Rahmatullaev , R.M. Khakimov, G.I. Botirov, O.N. Khakimov, E. Normatov, Sh. Shoyusupov, F. Haydarov, M. Rasulova and others.

In the work of N.N. Ganikhodjaev, F.M. Mukhamedov, and U.A. Rozikov, the uniqueness of the p-adic Gibbs distribution for the Ising model on the lattice $\mathbb{Z}$ was proved. The non-uniqueness of the p-adic Gibbs measure for the Ising-Vannimenus model was shown in the work of N.N. Ganikhodjaev and H.Akin. For the Vannimenus model, translation-invariant and periodic p-adic quasi Gibbs measures on the Cayley tree of order two were studied in the work of O.N. Khakimov. These works were continued for the Ising-Vannimenus model in the work of F.M.Mukhamedov, M.K. Saburov and O.N. Khakimov.

The existence of a unique p-adic Gibbs measure for the Ising model was proved by M. Khamraev, F.M. Mukhamedov. Translation-invariant p-adic generalized Gibbs measures were studied in the works of F.M. Mukhamedov and O.N. Khakimov. The boundedness criterion of the translation-invariant p-adic generalized Gibbs measures was found and it was proved that there does not occur the strong phase transition for these measures. Chaotic problems for p-adic generalized Gibbs measures was studied by F.M. Mukhamedov, O.N. Khakimov, H. Akin, M. Dogan.

In the field of real numbers, using the constructive method for the Ising model the existence of uncountable non-periodic extreme Gibbs measures was proved by P. Bleher and N.N. Ganikhodjaev. H. Akin, U.A. Rozikov and S. Temir studied Gibbs measures which are called ART measures. U.A. Rozikov and M.M. Rahmatullaev studied (k_0)-translation-invariant and (k_0)-periodic Gibbs measures.

Note that, despite there are many scientific works as mentioned above, the full classification of all limiting Gibbs measures for any model on the Cayley tree has not been obtained yet.

Connection of the theme of the dissertation with the research works of scientific research Institute, where the dissertation is carried out. Dissertation research was carried out within the "Fundamental research" branch of the research plan of Namangan state university.

The aim of research work contains describing the limit p-adic translation-invariant, periodic and non-periodic Gibbs measures for Ising and Potts models with a finite set of spin values on the Cayley tree, analyzing the boundedness of these measures.

Research problems are**:**

to determine sufficient conditions for the existence of translation-invariant *p*-adic generalized Gibbs measures for the Ising model on the Cayley tree of order three;

to find the conditions of the existence of G_2-periodic *p*-adic quasi Gibbs measures for the three-state Potts model on the Cayley tree of order two and to check the existence of the phase transitions;

to construct ART *p*-adic quasi Gibbs measures on the high-order Cayley tree using translation-invariant solutions of the functional equation corresponding to the three-state Potts model on the binary tree and to check these measures for boundedness;

to construct non-periodic ART, (k_0)-translation-invariant and (k_0)-periodic *p*-adic generalized Gibbs measures on a higher-order Cayley tree tree using translation-invariant and $G_k^{(2)}$-periodic *p*-adic Gibbs measures for the Ising model on the Cayley tree of order two and three.

The research object: p-adic Ising and Potts models on the Cayley tree.

The research subject: Theory of groups and graphs, Gibbs measures theory, Algebra and number theory, theory of non-linear discrete dynamical systems, p-adic analysis, non-linear Markov processes.

Research methods: In the research the methods of p-adic analysis, number theory, functional analysis, combinatorics, group theory, measures theory, theory of linear algebra and dynamical systems are used.

The scientific novelty of the research consists of the followings:

The sufficient conditions were found for the existence of the translation-invariant *p*-adic generalized Gibbs measures for the Ising model on the Cayley tree of order three;

the existence conditions of the periodic *p*-adic quasi Gibbs measures for the three-state Potts model on the Cayley tree of order two were found and the existence of phase transitions was proved;

using translation-invariant solutions of the functional equations for the 3-state Potts model on the lower-order Cayley tree, the construction of p-adic quasi Gibbs measures on a higher-order Cayley tree was given and the boundedness conditions of these measures was found;

using translation-invariant and two periodic solutions of the functional equations for the Ising model on the lower-order Cayley tree, non-periodic various p-adic generalized Gibbs measures on the higher-order Cayley tree were constructed.

Practical results of the research consists of the followings:

The obtained results and the methods used in the dissertation can be taught as a course for graduate students of higher education and after the higher education. Also, the exact expression of parameter values that ensure the existence of the different phase transitions in a set of periodic *p*-adic generalized Gibbs measures can be used to solve service theory problems.

The reliability of the results of the study. The results have been obtained by using the methods of p-adic analysis, non-Archimedean functional analysis, non-Archimedean Markov theory of random fields, p-adic probability theory, number theory, discrete dynamical systems, Gibbs and non-Archimedean measure theory. The obtained results are mathematically strongly proved.

Scientific and practical significance of research results. The scientific importance of the results of the research work is explained by the fact that various *p*-adic models of statistical mechanics can be used in the developing of Gibbs measures theory.

The practical significance of the research determines by the changing of state of physical systems is investigated and it gives opportunity to solve some problems of combinatorics and telecommunications.

Implementation of the research results. The scientific results obtained during the research of the dissertation are implemented in the following research projects:

The obtained results for the p-adic Ising model on the Cayley tree in the foreign grant project number G0003247 “Chaotic and mixed p-adic dynamical systems related to renormalized groups of lattice models” is used to investigation of p-adic generalized Gibbs measures for the Ising model on Cayley trees (United Arab Emirates University Bulletin dated 4 Septembert 2023, UAE). The application of scientific results made it possible to open new insight into the theory of p-adic dynamical systems and practical studies to study the thermodynamic properties of physical systems.

The proof methods of the existence of translation-invariant and periodic p-adic generalized Gibbs measures for the Potts and Ising models are used in the fundamental project YOT-FTEX-2018-154 “Spectra of Hamiltonians and Gibbs measures on lattices Z^d and Cayley trees Γ^k” to investigate existence of translation-invariant and periodic p-adic generalized Gibbs measures for some classical models of statistical mechanics with continuum spin value (Reference No. 04/11-5431 of the National university of Uzbekistan named after Mirzo Ulugbek dated September 9, 2023). The application of the scientific work made it possible to expand the set of translation-invariant and periodic Gibbs measures for some models of statistical mechanics with continuum spin values.

Approbation of the research results. The main results of the research have been discussed in 3 international and 4 national scientific conferences.

Publications of the research results. On the topic of the dissertation 6 research papers have been published in the scientific journals, all of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis, including 5 in foreign and 1 in Republic journals.

The structure and volume of the dissertation. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The volume of the thesis is 111 pages.

THE MAIN CONTENT OF THE DISSERTATION

In the introduction besides the motivation of research theme and correspondence to the priority research areas of science and technology of the Republic, we present the degree of scrutiny of the problem, formulate our goals and objectives, identify the object and subject of study, and state scientific novelty and practical results of the research. Moreover, we reduce the theoretical and practical importance of the obtained results, and give information on the implementation of the research results, the published works and the structure of dissertation.

The first chapter of the thesis, titled **“On p-adic analysis and p-adic measures”,** is devoted to present the main concepts, definitions, important theorems that used to explain the obtained results in the dissertation and to investigate the G_2-periodic p-adic quasi Gibbs measures for the three-state Potts model on the Cayley tree of order two, to check the boundedness of obtained measures, and to determine the existence of the phase transitions.

Let $\mathbb{Q}$ be the field of rational numbers. For a fixed prime number p, every rational number $x \neq 0$ can be represented in the form $x = p^r \frac{m}{n}$ where, $m \in \mathbb{Z}$,

$n \in \mathbb{N}$, and m, n are relatively prime with p, i.e. $(m,p)=(n,p)=1$. The p-adic norm of $x \in \mathbb{Q}$ is given by

$$|x|_p = \begin{cases} p^{-r}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

This norm is non-Archimedean, i.e., it satisfies the strong triangle inequality $|x+y|_p \leq \max\{|x|_p, |y|_p\}$ for all $x, y \in \mathbb{Q}$.

The completion of $\mathbb{Q}$ with respect to the p-adic norm defines the p-adic field $\mathbb{Q}_p$.

Let $a \in \mathbb{Q}_p$ and $r > 0$. We present the following important sets that are often used in the dissertation work.

The open ball of radius r centred at a is the set

$$B(a,r) = \{x \in \mathbb{Q}_p : |x-a|_p < r\},$$

the closed ball of radius r centred at a is the set

$$\overline{B(a,r)} = \{x \in \mathbb{Q}_p : |x-a|_p \leq r\},$$

the sphere of radius r centred at a is the set

$$S(a,r) = \{x \in \mathbb{Q}_p : |x-a|_p = r\},$$

the p-adic integers as the set

$$\mathbb{Z}_p = \overline{B(0,1)},$$

the p-adic units is the set

$$\mathbb{Z}_p^* = S(0,1).$$

We give two functions that are important in the p-adic Gibbs measures theory.

p*-adic logarithm* is defined by the series

$$\log_p(x) = \log_p\left(1+(x-1)\right) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n},$$

which converges for $x \in B(1,1)$.

p-adic *exponential* is defined by

$$\exp_p(x) = \sum_{n=0}^{\infty}\frac{x^n}{n!},$$

which converges for $x \in B\left(0,\ p^{-\frac{1}{p-1}}\right)$.

We set

$$\mathcal{E}_p = \left\{x \in \mathbb{Q}_p : |x-1|_p < p^{-\frac{1}{p-1}}\right\}.$$

It should be noted that the set is the range of the p-adic exponential function, moreover, the set is a commutative group under multiplication.

Let $(X,\mathfrak{B})$ be a measurable space, $\mathbb{K}$ be a non-archimedean field, where $\mathfrak{B}$ is an algebra of subsets X. A function $\mu:\mathfrak{B}\to\mathbb{K}$ is said to be a non-archimedean distribution if for any $A=\bigcup_{k=1}^{n}A_k,\ A_k\in\mathfrak{B},\ k=\overline{1,n}$ such that $A_i\cap A_j=\varnothing,\ i\neq j$, the following holds:

$$\mu\left(\bigcup_{i=1}^{n}A_i\right)=\sum_{i=1}^{n}\mu(A_i).$$

A non-archimedean distribution is called *probabilistic* if $\mu(X)=1$. A non-archimedean distribution μ is called bounded if $\sup\{|\mu(A)|_p: A\in B\}<\infty$. A bounded non-archimedean distribution is called non-archimedean measure. A non-archimedean measure is called *probabilistic* if $\mu(X)=1$.

Cayley tree $\Gamma^k=(V,L)$ of order $k\geq 1$ is an infinite tree, i.e. a graph without cycles, such that exactly $k+1$ edges originate from each vertex, where V is the set of vertices, L is the set of edges. Two vertices x and y are called nearest neighbors, if there exist an edge $l\in L$ connecting them. We denote by $l=\langle x,y\rangle$. The distance $d(x,y)$ on the Cayley tree defined by

$$d(x,y)=\min\{d\,|\,\exists x=x_0,x_1,\ldots,x_{d-1},x_d=y\in V,\ \ \langle x_0,x_1\rangle,\ldots,\langle x_{d-1},x_d\rangle\}.$$

For fixed $x^0\in V$, called the root, we set

$$W_n=\left\{x\in V\,|\,d(x,x^0)=n\right\},\ \ V_n=\bigcup_{j=0}^{n}W_j,\ \ L_n=\left\{l=\langle x,y\rangle\in L|\ x,y\in V_n\right\}.$$

and denote

$$S(x)=\{y\in W_{n+1}:\ d(x,y)=1\},\ x\in W_n,$$

direct successors of x.

Let G_k be a free product of $k+1$ cyclic groups of the second order with generators $a_1,a_2,...,a_{k+1}$, respectively.

The following proposition proved by N.N. Ganikhodjaev.

Proposition 1. There exists a one-to-one correspondence between the set of vertices V of the Cayley tree $\Gamma^k=(V,L)$ and the group G_k.

Let $\Gamma^k=(V,L)$ be Cayley tree and $\Phi\subset\mathbb{Z}$ be finite set. For $\Lambda\subset V$ a configuration σ_Λ on Λ is defined as a function

$$x\in\Lambda\to\sigma_\Lambda(x)\in\Phi.$$

The set all configurations on Λ is denoted by $\Omega_\Lambda=\Phi^\Lambda$, in particular, $\Omega=\Omega_V$.

For given configurations $\sigma\in\Omega_\Lambda$ and $\varphi\in\Omega_{V\setminus\Lambda}$ we define a configuration in Ω as follows

$$(\sigma\vee\varphi)(x)=\begin{cases}\sigma(x), & \text{if } x\in\Lambda,\\ \varphi(x), & \text{if } x\in V\setminus\Lambda.\end{cases}$$

The energy of the configuration $\sigma\in\Omega$ is given by the formal Hamiltonian

$$H(\sigma)=\sum_{\substack{\Lambda\subset V:\\ \operatorname{diam}(\Lambda)\le r}} I(\sigma_\Lambda),$$

where $r\in\mathbb{N}$, $\operatorname{diam}(\Lambda)=\max_{x,y\in\Lambda} d(x,y)$, $I:\Omega_\Lambda\to\mathbb{Q}_p$ is a given potensial.

Let $\mathfrak{B}$ be algebra which contains cylindrical subsets of Ω.

Definition 1. Let μ be p-adic probability distributions on $(\Omega,\mathfrak{B})$. Assume that for all $\Lambda\subset V$ and $\sigma\in\Omega_\Lambda$, $\varphi\in\Omega_{V\setminus\Lambda}$ the following equality holds

$$\mu(\sigma\,|\,\varphi)=\frac{\exp_p\left\{H\left(\sigma\vee\varphi\right)\right\}}{Z_{\Lambda,\varphi}},$$

where $Z_{\Lambda,\varphi}$ is the normalizing constant, i.e.

$$Z_{\Lambda,\varphi}=\sum_{\omega\in\Omega_\Lambda}\exp_p\left\{H\left(\omega\vee\varphi\right)\right\}.$$

Such a distribution is called *p-adic Gibbs distribution.*

In paragraph 1.4, we study G_2-periodic quasi Gibbs measures for the three-state Potts model on the semi-infinite Cayley tree of order two.

Let $\mathbb{Q}_p$ be the field of p-adic numbers and $\Phi=\{1,2,...,q\}$ be a finite set. The formal Hamiltonian of p-adic Potts model is

$$H(\sigma)=J\sum_{\langle x,y\rangle\in L}\delta_{\sigma(x)\sigma(y)}$$

where $J\in B(0,p^{-1/(p-1)})$ is a coupling constant, and δ_{ij} is the Kronecker's symbol.

Assume that $\mathbf{h}:V\setminus\{x^0\}\to\mathbb{Q}_p^\Phi$ is a mapping, i.e. $\mathbf{h}_x=(h_{1,x},h_{2,x},...,h_{q,x})$, where $h_{i,x}\in\mathbb{Q}_p$ $(i\in\Phi)$ and $x\in V\setminus\{x^0\}$. Given $n\in\mathbb{N}$, we consider p-adic probability distributions $\mu_{\mathbf{h}}^{(n)}$ on Ω_{V_n} defined by

$$\mu_{\mathbf{h}}^{(n)}(\sigma)=\frac{1}{Z_n^{(\mathbf{h})}}\exp_p\{H_n(\sigma)\}\prod_{x\in W_n}h_{\sigma(x),x}\,, \qquad (1)$$

here, $\sigma\in\Omega_{V_n}$, and $Z_n^{(\mathbf{h})}$ is the corresponding normalizing factor

$$Z_n^{(\mathbf{h})}=\sum_{\sigma\in\Omega_{V_n}}\exp_p\{H_n(\sigma)\}\prod_{x\in W_n}h_{\sigma(x),x}\,.$$

We say that p-adic probability distributions (1) are compatible if all $n\in\mathbb{N}$ and $\sigma_{n-1}\in\Omega_{V_{n-1}}$:

$$\sum_{\omega_n\in\Omega_{W_n}}\mu_{\mathbf{h}}^{(n)}(\sigma_{n-1}\vee\omega_n)=\mu_{\mathbf{h}}^{(n-1)}(\sigma_{n-1}). \qquad (2)$$

We note that if p-adic probability distributions (1) satisfies the compatible condition (2) then according to non-Archimedean analogue of the Kolmogorov extension theorem, which was proved by N.N. Ganikhodjaev, F.M. Mukhamedov, U.A. Rozikov, there exists a unique p-adic measure μ_h on Ω such that

$$\mu(\sigma\in\Omega:\sigma|_{V_n}\equiv\sigma_n)=\mu_{\mathbf{h}}^{(n)}(\sigma_n),$$

for all $\sigma_n\in\Omega_{V_n}, n\in\mathbb{N}$. Such measure is called a *p-adic quasi Gibbs measure* corresponding to the Hamiltonian H and vector-valued function $\mathbf{h}_x$, $x\in V\setminus\{x^0\}$.

In the works of F.M. Mukhamedov, the concept of phase transitions for p-adic Gibbs measures is presented. If there are at least two distinct p-adic Gibbs measures μ and ν such that μ is bounded and ν is unbounded, then one says that a *phase transition* occurs.

It is known that, the probability distributions $\mu_{\mathbf{h}}^{(n)}$, $n=1,2,\dots$, defined by (1) satisfy the compatibility condition (2) if and only if for any $n\in\mathbb{N}$, $x\in V\setminus\{x^0\}$ the following equation holds:

$$\mathbf{h}_x=\prod_{y\in S(x)}F(\mathbf{h}_y,\theta), \tag{3}$$

here and below a vector
$\hat{h}=\left(\hat{h}_1,\hat{h}_2,\dots,\hat{h}_{q-1}\right)\in\mathbb{Q}_p^{q-1}$ is defined by a vector $\mathbf{h}=(h_1,h_1,\dots,h_q)\in\mathbb{Q}_p^q$ as follows

$$\hat{h}_i=\frac{h_i}{h_q},i=1,2,\dots,q-1,\ \theta=\exp_p\{J\},$$

and mapping
$F:\mathbb{Q}_p^{q-1}\times\mathbb{Q}_p\to\mathbb{Q}_p^{q-1}$ is defined by $F(\mathbf{x};\theta)=\left(F_1(\mathbf{x};\theta),\dots,F_{q-1}(\mathbf{x};\theta)\right)$ with

$$F_i(\mathbf{x};\theta)=\frac{(\theta-1)x_i+\sum_{j=1}^{q-1}x_j+1}{\sum_{j=1}^{q-1}x_j+\theta},\mathbf{x}=(x_1,x_2,\dots,x_{q-1})\in\mathbb{Q}_p^{q-1},\ i=1,2,\dots,q-1.$$

Let $\Gamma_+^k=(V,L)$ be a semi-infinite Cayley tree of order $k\geq 1$ with the root x^0 It is known that the coordinate structure of Γ_+^k with translations, as a binary operation, becomes a non-commutative semigroup with a unit. We denote this group by $(G^k,\circ)$. Let $G\subset G^k$ be a sub-semigroup of G^k and $h:G^k\to Y$ be a Y-valued function defined on G^k. We say that h is G- *periodic* if $h(\tau_g(x))=h(x)$ for all $g\in G$ and $x\in G^k$, where $\tau_g(x)=g\circ x$. Any G^k-periodic function is called *translation-invariant*.

Now for each $m\geq 2$ we put

$$G_m=\{x\in G^k:d(x,x^0)\equiv 0(\mathrm{mod}\,m)\}.$$

One can check that G_m is a sub-semigroup of G^k.

It is easy to check that the set

$$I_m=\left\{\mathbf{h}:\mathbf{h}=(\underbrace{1,\dots,1,h}_{m},1,\dots,1)\right\},\ m=1,\dots,q-1$$

is invariant set for (3). Note that, measures which are given the following three theorems, correspond to set of quantities $\mathbf{h}$, defined on the invariant set I_m.

We denote by μ_{h_0} translation invariant quasi Gibbs measure for the Potts model corresponding to $h:=h_0=1$ on invariant set I_m.

Theorem 1. For 3-state p-adic Potts model on the Cayley tree of order two the following assertions hold:

1) if $p=2$ or $p\equiv 5(\mathrm{mod}\,8)$ or $p\equiv 7(\mathrm{mod}\,8)$, then there exists a unique G_2-periodic p-adic quasi Gibbs measure μ_{h_0}. Moreover, the measure is bounded.
2) if $p\equiv 1(\mathrm{mod}\,8)$ or $p\equiv 3(\mathrm{mod}\,8)$, then then there exist five G_2-periodic p-adic quasi Gibbs measures, two of them are non-translation-invariant G_2-periodic p-adic quasi Gibbs measures. Furthermore,

2.1) if $p=3$, all measures are unbounded,

2.2) if $p\neq 3$, only the measure μ_{h_0} is bounded.

Theorem 2. Let $p\equiv 1(\mathrm{mod}\,8)$ or $p\equiv 3(\mathrm{mod}\,8)$, $p\neq 3$. Then for p-adic 3-state Potts model on a Cayley tree of order two there exists a phase transition.

Remark 1. In work of F. Mukhamedov and O. Khakimov the existence of G_m-periodic quasi Gibbs measures was found. However, the explicit form of that measures is not given. We find the explicit form of G_2-periodic quasi Gibbs measures. The explicit form makes easier to check existence and boundedness of the measures.

The second chapter is devoted to p-adic Ising model on the Cayley tree. The main concepts and known importat results are presented in the paragraph 2.1. The translyatsion-invariant and $G_k^{(2)}$-periodic p-adic generalized Gibbs measures for the Ising model on the Cayley tree of order three are studied in the paragraphs 2.2 and 2.3, respectively.

Let G_k^* be a normal subgroup of the group G_k.

Definition 2. A function h_x (a configuration $\sigma(x), x\in V$) is called G_k^*-periodic if $h_{yx}=h_x$ (resp., $\sigma(yx)=\sigma(x)$) for any $x\in G_k$ and $y\in G_k^*$.
A G_k-periodic function is called *translation-invariant*.

The Gibbs measure is called G_k^*-periodic (translation-invariant) if it corresponds to G_k^*-periodic (translation-invariant) function h.

Let $G_k^{(2)}$ be subset of G_k, consisting of all words of even length. Clearly, $G_k^{(2)}$ is a normal subgroup of index two, i.e. if

$$G_k^{(2)}=\{x\in G_k : |x| - \text{even}\}$$

then $G_k/G_k^{(2)}=\{G_k^{(2)}, G_k\setminus G_k^{(2)}\}$.

Let $\mathbb{Q}_p$ be a field of p-adic numbers and $\Phi=\{-1,1\}$. A formal Hamiltonian $H:\Omega\to\mathbb{Q}_p$ of the p-adic Ising model is defined by

$$H(\sigma)=J\sum_{\langle x,y\rangle\in L}\sigma(x)\sigma(y),$$

where $J\in B\left(0,\ p^{\frac{1}{1-p}}\right)$ is a constant parameter.

Let

$$h: x\in V\setminus\{x^0\}\to h_x\in\mathbb{Q}_p$$

be a function. We define p-adic probability generalized Gibbs distributions $\mu_h^{(n)}$, $n\in\mathbb{N}$ on Ω_{V_n}, by

$$\mu_h^{(n)}(\sigma_n)=\frac{1}{Z_n^{(h)}}\exp_p\left\{H_n(\sigma_n)\right\}\prod_{x\in W_n}h_x^{\sigma(x)}, \qquad (4)$$

where $\sigma_n\in\Omega_{V_n}$, $Z_n^{(h)}$ is corresponding normalizing constant.

If p-adic probability generalized Gibbs distributions (4) satisfy the compatible condition (2) then according to non-Archimedean analogue of the Kolmogorov extension theorem there exists a unique limit p-adic measure μ_h on $\Omega=\Phi^V$. Such measure is called a *p-adic generalized Gibbs measure* corresponding to the Hamiltonian H and vector-valued function $\mathbf{h}_x$, $x\in V\setminus\{x^0\}$.

If for $\forall x\in V$ $h_x\in\mathcal{E}_p$, then p-adic generalized Gibbs measure corresponding to the quantities $h=\{h_x, x\in V\}$ is p-adic Gibbs measure.

It is known that, p-adic probability generalized Gibbs distributions $\mu_h^{(n)}(\sigma_n)$ ($n=1,2,\ldots$), defined by (4) are compatible iff for any $x\in V\setminus\{x^0\}$ the following equation holds:

$$h_x^{\,2}=\prod_{y\in S(x)}\frac{\theta h_y^{\,2}+1}{h_y^{\,2}+\theta}, \qquad (5)$$

where $\theta=\exp_p\{2J\}$.

Theorem 3. Let N be a number of translation-invariant p-adic generalized Gibbs measures for the Ising model on the Cayley tree of order three. Then the following assertions hold:

$$N=\begin{cases}1, & \text{if } p\not\equiv 1(\mathrm{mod}\,4);\\ 2, & \text{if } p\equiv 5(\mathrm{mod}\,12);\\ 4, & \text{if } p\equiv 1(\mathrm{mod}\,12).\end{cases}.$$

Clearly, $h:=h_0=1$ is a solution of (5). We denoted by μ_{h_0} the p-adic generalized Gibbs measure corresponding to $h_0=1$.

Theorem 4. For the p-adic Ising model on the Cayley tree of order three the following assertions hold:

1) if $p=2$, then there exists a unique translyatsion-invariant p-adic generalized Gibbs measure μ_{h_0}. Moreover the measure is unbounded.
2) if $p\neq 2$, among the translation-invariant p-adic generalized Gibbs measures only μ_{h_0} is bounded.

Remark 2. By the work of U.A. Rozikov and O.N. Khakimov, it is known that each G_k^*-periodic p-adic generalized Gibbs measure for Ising model is either translation-invariant or $G_k^{(2)}$-periodic.

To study G_k^*-periodic p-adic generalized Gibbs measures for Ising model on the Cayley tree of order three, according to Remark 2, now it is sufficient to investigate $G_k^{(2)}$-periodic p-adic generalized Gibbs measures of this model.

Theorem 5. Let M be a number of non-translation-invariant $G_k^{(2)}$-periodic p-adic generalized Gibbs measures for the Ising model on the Cayley tree of order three. Then the following assertions hold:

$$M=\begin{cases}6, \text{ if } \ p\equiv 1 (\operatorname{mod} 6);\\ 2, \text{ if } \ p=2, |\theta-1|_2=\dfrac{1}{4};\\ 2, \text{ if } \ p=3, \operatorname{ord}_3|\theta-1|_3 \text{ is odd}, |\theta-1|_3\le\dfrac{1}{27} \text{ and } |\theta-1|_3(\theta-1)\in\mathcal{E}_3;\\ 0, \text{ otherwise.}\end{cases}$$

Theorem 6. Let $k=3$. If either $p=3$, $\operatorname{ord}_3|\theta-1|_3$ is odd, $|\theta-1|_3\le\frac{1}{27}$, $|\theta-1|_3(\theta-1)\in\mathcal{E}_3$ or $p\equiv 1(\operatorname{mod} 12)$ then there exists a phase transition.

The third chapter is devoted to non-periodic p-adic Gibbs measures for the Ising and Potts models on the Cayley tree. In this chapter, by constructive methods we construct non-periodic p-adic Gibbs measures.

On the semi-infinite Cayley tree of order two, we denote by $h_i^{(t)}$ $(i=0,1,2)$ and $h_j^{(p)}$ $(j=1,2)$ the translation-invariant and G_2-periodic solutions of the equation (5), respectively. By O.N. Khakimov's work, it is known that if $p\equiv 1(\operatorname{mod} 4)$ then

$$h_0^{(t)}=1,\ h_{1,2}^{(t)}=\frac{\theta-1\pm\sqrt{(\theta-3)(\theta+1)}}{2},\ h_{1,2}^{(p)}=\frac{1-\theta\pm\sqrt{(\theta-1)^2-4\theta^2}}{2\theta}. \tag{6}$$

Let $k\ge 3, k_0=2$. For $x\in V$, by $S_{k_0}(x)$ we denote an arbitrary set of k_0 vertices of the set $S(x)$, and the remaining $k-k_0$ vertices are denoted by $S_{k-k_0}(x)$. Let $k-k_0=a+b+c$, where a and b are even, c is even or odd. We define the set of quantities $h=\{h_x, x\in V\}$ (where $h_x\in\{1,h_1^{(t)},h_2^{(t)},h_1^{(p)},h_2^{(p)}\}$) as follows:

(A_1) if at vertex x we have $h_x=h_i^{(t)}$ $(i=1,2)$, then the function h_y, which gives p-adic values to each vertex $y\in S(x)$ by the following rule

$$h_y = \begin{cases} h_i^{(t)}, & \text{on } \frac{a}{2}+2 \text{ vertices of } S(x); \\ h_{3-i}^{(t)}, & \text{on } \frac{a}{2} \text{ vertices of } S(x); \\ h_i^{(p)}, & \text{on } \frac{b}{2} \text{ vertices of } S(x); \\ h_{3-i}^{(p)}, & \text{on } \frac{b}{2} \text{ vertices of } S(x); \\ 1, & \text{on } c \text{ vertices of } S(x). \end{cases}$$

(A_2) if at vertex x we have $h_x = h_i^{(p)}$ $(i=1,2)$, then

$$h_y = \begin{cases} h_i^{(t)}, & \text{on } \frac{a}{2} \text{ vertices of } S(x); \\ h_{3-i}^{(t)}, & \text{on } \frac{a}{2} \text{ vertices of } S(x); \\ h_i^{(p)}, & \text{on } \frac{b}{2} \text{ vertices of } S(x); \\ h_{3-i}^{(p)}, & \text{on } \frac{b}{2}+2 \text{ vertices of } S(x); \\ 1, & \text{on } c \text{ vertices of } S(x). \end{cases}$$

(A_3) if at vertex x we have $h_x = 1$, then

$$h_y = \begin{cases} h_i^{(t)}, & \text{on } \frac{a}{2} \text{ vertices of } S(x); \\ h_{3-i}^{(t)}, & \text{on } \frac{a}{2} \text{ vertices of } S(x); \\ h_i^{(p)}, & \text{on } \frac{b}{2} \text{ vertices of } S(x); \\ h_{3-i}^{(p)}, & \text{on } \frac{b}{2} \text{ vertices of } S(x); \\ 1, & \text{on } c+2 \text{ vertices of } S(x). \end{cases}$$

(see Figure 1).

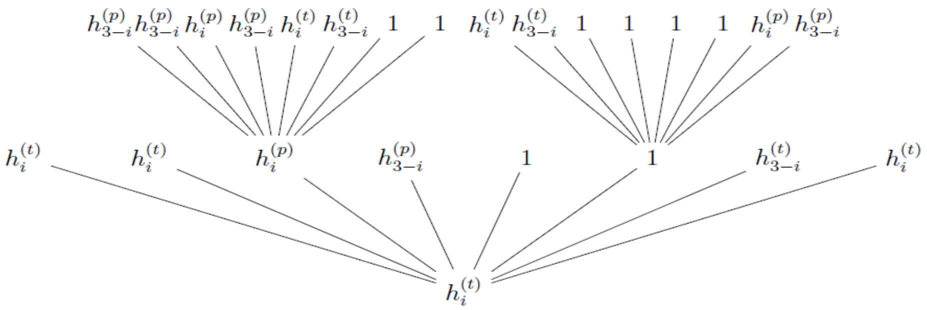


Figure 1: This is an example of the function h_x *on the vertices of the semi-infinite Cayley tree of order 8 in the case* $a=2, b=2, c=2, i=1,2$.

Theorem 7. Let $p \equiv 1(\mathrm{mod}\, 4)$. Then any set of quantities according to the rules (A_1), (A_2) and (A_3) on the semi-infinite Cayley tree Γ_+^k satisfies the functional equation (5). Futhermore, if $p \equiv 1(\mathrm{mod}\, 4)$ and $a^2+b^2 \neq 0$ then these measures corresponding to the set of quantities h are unbounded.

Remark 3. 1) In cases (A_1), (A_2), if $a=b=0$, $c \neq 0$ then p-adic generalized Gibbs measures corresponding to set of quantities h_x are ART Gibbs measures;

2) In case (A_1), if $b=c=0$, $a \neq 0$ then p-adic generalized Gibbs measures corresponding to set of quantities h_x are (k_0)-translation-invariant;

3) In case (A_2), if $a=c=0$, $b \neq 0$ then p-adic generalized Gibbs measures corresponding to set of quantities h_x are (k_0)-periodic;

4) In other cases we get new measures except for previous known ones.

In the paragraph 3.3 we construct a p-adic analogue of the Bleher-Ganikhodjaev construction. We consider an infinite path $\pi = x^0 = x_0 < x_1 < \dots$ on the semi-infinite Cayley tree Γ_+^k (the notation $x<y$ meaning that paths from the root x^0 to y go through x). We assign the set of p-adic numbers $h^\pi = \{h_x^\pi : x \in V\}$ satisfying the equation (5) to the path π. For $x \in W_n$, the set h^π is defined by

$$h_x^\pi = \begin{cases} \frac{1}{h_*}, \text{ if } & x \prec x_n, \; x \in W_n; \\ h_*, \text{ if } & x_n \prec x, \; x \in W_n; \\ h_{x_n}^{(n)}, \text{if } & x = x_n. \end{cases} \tag{7}$$

where $n=1,2,\dots$, $x \prec x_n$ (resp. $x_n \prec x$) means that x is on the left (resp. right) from the path π, h_* is translation-invariant solution of the equation (5) and $h_{x_n}^{(n)}$ is an arbitrary p-adic numbers such that $\left(h_{x_n}^{(n)}\right)^2 \in \mathbb{Z}_p^* \setminus B(-1,1)$.

Theorem 8. Let $p \geq 3$. For any infinite path π, there exists a unique set of numbers $h^{\pi} = \{h_x^{\pi}, x \in V\}$ satisfying (5) and (7). Moreover, the measures corresponding to the set of quantities h_x^{π} are bounded if and only if $h_* = 1$.

Theorem 9. Let $p \geq 3$. If there exists a translation-invariant solution h_* of the functional equation (5) such that $h_* \notin \{-1, 1\}$, then there exists a phase transition.

Theorem 10. The measure which is defined in Theorem 8 depends on the path π. Moreover, there exists a set of uncountable non-periodic p-adic generalized Gibbs measures for the Ising model on the Cayley tree.

CONCLUSION

The dissertation is devoted to investigate the translation-invariant, periodic and non-periodic p-adic Gibbs measures for finite state p-adic Potts and Ising models, furthermore, to check the boundedness of found measures and to study phase transitions for these models.

The main results of the research are as follows:

1. It was proved that if $p \equiv 1 (\bmod 8)$ or $p \equiv 3 (\bmod 8)$, then there exist two non-translation-invariant G_2-periodic p-adic quasi Gibbs measures for the 3-state Potts model on the Cayley tree of order two;
2. If $p \equiv 1 (\bmod 8)$ or $p \equiv 3 (\bmod 8)$, $p \neq 3$, then the existence of a phase transition for 3-state p-adic Potts model on the Cayley tree of order two was proved;
3. All translation-invariant p-adic generalized Gibbs measures for the Ising model on the Cayley tree of order three were found. In particular, if $p \equiv 1 (\bmod 12)$, it was proved that there are four translation-invariant p-adic generalized Gibbs measures for the Ising model on the Cayley tree of order three and a phase transition occurs.
4. All non-translation-invariant $G_k^{(2)}$-periodic p-adic generalized Gibbs measures for the Ising model on the Cayley tree of order three were found. The boundedness of these measures was checked;
5. Using translation-invariant and G_2-periodic solutions of the functional equations for the 3-state Potts model on the Cayley tree of order two, ART p-adic quasi Gibbs measures on a higher-order Cayley tree were constructed, the conditions of existence of these measures were determined and their boundedness was checked;
6. Using translation-invariant and $G_k^{(2)}$-periodic solutions of the functional equations for the Ising model on the Cayley tree of order two and three, ART, (k_0)-translation-invariant, (k_0)-periodic p-adic generalized Gibbs measures on a higher-order Cayley tree were constructed, It was proved that this construction generalizes some known constructions, i.e. at some specific values of the parameter, the construction coincide with one of the constructions, ART, (k_0)-translation-invariant, and (k_0)-periodic. Using this construction, some non-periodic p-adic generalized Gibbs measures were constructed. The conditions of existence of these measures were determined and boundedness of these measures was checked;
7. A p-adic analogue of the Bleher-Ganikhodjaev construction, providing the existence of uncountable non-periodic p-adic generalized Gibbs measures for the Ising model on the Cayley tree, was constructed. The conditions of existence of found measures were determined and boundedness of these measures was checked. The new condition of the existence a phase transition was found.

НАУЧНЫЙ СОВЕТ DSc.02/30.12.2019.FM.86.01 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО

НАМАНГАНСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

ТУХТАБАЕВ АКБАРХУЖА МАМАЖОНОВИЧ

p-АДИЧЕСКИЕ ПЕРИОДИЧЕСКИЕ МЕРЫ ГИББСА ДЛЯ НЕКОТОРЫХ КЛАССИЧЕСКИХ МОДЕЛЕЙ СТАТИСТИЧЕСКОЙ МЕХАНИКИ

01.01.01 – Математический анализ

АВТОРЕФЕРАТ ДИССЕРТАЦИИ
ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

Ташкент – 2023 год

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве Высшего образования, Науки и Инноваций Республики Узбекистан за № B2021.4.PhD/FM556.

Диссертация выполнена в Наманганском государственном университете.

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (https://kengash.mathinst.uz) и на Информационно-образовательном портале «Ziyonet» (http://www.ziyonet.uz).

Научный руководитель:	**Рахматуллаев Музаффар Мухаммаджанович** доктор физико-математических наук, старший научный сотрудник
Официальные оппоненты:	**Кудайбергенов Каримберген Кадирбергенович** доктор физико-математических наук, профессор
	Ботиров Голибжон Исроилович доктор физико-математических наук, старший научный сотрудник
Ведущая организация:	**Национальный университет Узбекистана им. Мирзо Улугбека**

Защита диссертации состоится « 21 » ноября 2023 года в 16:00 часов на заседании Научного совета DSc.02/30.12.2019.FM.86.01 при Институте Математики имени В.И.Романовского. (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9. Тел.: (+998 78) 207 91 40, e-mail: mathinst@umail.uz, Website: www.mathinst.uz).

С диссертацией можно ознакомиться в Информационно-ресурсном центре Института Математики имени В.И.Романовского (зарегистрирована за № 170). (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9. Тел.: (+998 78) 207 91 40).

Автореферат диссертации разослан « 27 » октября 2023 года.
(протокол рассылки № 2 от « 27 » октября 2023 года).

У.А. Розиков
Председатель Научного совета по присуждению ученых степеней, д.ф.-м.н., профессор

Ж.К. Адашев
Ученый секретарь Научного совета по присуждению ученых степеней, д.ф.-м.н., старший научный сотрудник

У.У. Жамилов
Председатель Научного семинара при Научном совете по присуждению ученых степеней, д.ф.-м.н., старший научный сотрудник

ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Целью исследования является описание предельно p-адических трансляционно-инвариантных, периодических и непериодических мер Гиббса для моделей Изинга и Поттса с конечным множеством значений спина на дереве Кэли и анализ ограниченности этих мер.

Объектом исследования являются p-адическая модель Изинга и Поттса на дереве Кэли.

Научная новизна исследования состоит в следующем:

найдены достаточные условия существования трансляционно-инвариантных p-адических обобщенных мер Гиббса для модели Изинга на дереве Кэли третьего порядка;

найдены условия существования периодических p-адических квази-гиббсовских мер для модели Поттса с тремя состояниями на дереве Кэли второго порядка и доказано существование фазовых переходов;

с использованием трансляционно-инвариантных решений функциональных уравнений для модели Поттса с тремя состояниями на дереве Кэли малого порядка, дана конструкция построения p-адических квази- гиббсовских мер на дереве Кэли высшего порядка и найдены условия ограниченности этих мер;

с использованием трансляционно-инвариантных и периодических решений функциональных уравнений модели Изинга на дереве Кэли малого порядка, были построены непериодические, различные p-адические обобщенные мер Гиббса на дереве Кэли высшего порядка.

Внедрение результатов исследования. На основе полученных результатов по p-адическим периодическим мерам Гиббса для некоторых классических моделей статистической механики:

Полученные результаты для p-адической модели Изинга на дереве Кэли использованы в зарубежном проекте G0003247 «Хаотические и смешанные p-адические динамические системы, связанные с перенормированными группами решеточных моделей» для исследования p-адических обобщенных мер Гиббса для модели Изинга на деревьях Кэли (Справка Университета Объединенных Арабских Эмиратов от 4 сентября 2023 г., ОАЭ). Применение научных результатов позволило открыть новое понимание теории p-адических динамических систем и практических исследований по изучению термодинамических свойств физических систем.

Методы доказательства существования трансляционно-инвариантных и периодических p-адических обобщенных мер Гиббса для моделей Поттса и Изинга использованы в фундаментальном проекте YOT-FTEX-2018-154 «Спектры гамильтонианов и мер Гиббса на решетках Z^d и деревьях Кэли Γ^k» для исследования существования трансляционно-инвариантных и периодических p-адических обобщенных мер Гиббса для некоторых классических моделей статистической механики с континуум значением спина (Справка Национального университета Узбекистана имени Мирзо Улугбека № 04/11-5431 от 9 сентября 2023 г.). Применение данных научных результатов позволило расширить множества трансляционно-инвариантных и периодических мер Гиббса для некоторых моделей статистической механики с континуум значениями спина.

Структура и объем диссертации. Диссертация состоит из введения, трех глав, разбитых на десять параграфов, заключения и списка использованной литературы. Объем диссертации составляет 111 страниц.

E'LON QILINGAN ILMIY ISHLAR RO'YXATI

LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo'lim (part I; часть I)

1. Rahmatullaev M.M., Khakimov O.N., Tukhtabaev A.M. A p-adic generalized Gibbs measure for the Ising model on a Cayley tree. // Theoretical and Mathematical Physics. – 2019. № 201(1).– pp. 1521-1530. (Q3. Scopus. IF=0.88).
2. Rahmatullaev M.M., Tukhtabaev A.M. Non periodic p-adic generalized Gibbs measure for Ising model. // p-Adic Numbers, Ultrametric Analysis and Applications. – 2019. Vol. 11. № 4.– pp. 319-327. (Q3. Scopus. IF=0.67).
3. Tukhtabaev A.M. On G_2-periodic quasi Gibbs measures of p-adic Potts Model on a Cayley tree. // p-Adic Numbers, Ultrametric Analysis and Applications. – 2021. Vol. 13. № 4.– pp. 291-307. (Q3. Scopus. IF=0.76).
4. Rahmatullaev M.M., Tukhtabaev A.M. On periodic *p*-adic generalized Gibbs measures for Ising model on a Cayley tree. // Letters in Mathematical Physics. – 2022. Vol. 112.issue № 6.– 112. (Q2. Scopus. IF=1.12).
5. Rahmatullaev M.M., Tukhtabaev A.M. Some non-periodic p-adic generalized Gibbs measures for the Ising model on a Cayley tree of order k. // Mathematical Physics, Analysis and Geometry. – 2023. Vol. 26. № 22. (Q2. Scopus. IF=1).
6. Rahmatullaev M.M., Tukhtabaev A.M. Some non-periodic p-adic generalized Gibbs measures for the Ising model on a Cayley tree.// Доклады Академии Наук Республики Узбекистан. – 2023. № 1. – С. 3-9. (01.00.00; № 7).

II bo'lim (part II; часть II)

7. Rahmatullaev M.M., Tukhtabaev A.M. On periodic *p*-adic generalized Gibbs measures for Ising model on a Cayley tree. On $G_k^{(2)}$-periodic Gibbs measures of p-adic Potts model on a Cayley tree // «Modern problems of differential equations and related branches of mathematics». International scientific conference, Fergana, March 12-13, 2020, – pp. 163-167.
8. Tukhtabaev A.M. On G_2-periodic Gibbs measueres of p-adic Potts model on a Cayley tree. // «Modern problems of applied mathematics and information technologies Al-Khwarizmi 2021». International scientific conference, Fergana, November 15-17, 2021, – pp. 242.
9. Rahmatullaev M.M., Tukhtabaev A.M. Two periodic *p*-adic Gibbs measures for Ising models on a Cayley tree. // «Actual problems of stochastic analysis». Republic conference, Tashkent, February 20-21, 2021, – pp. 412-416.

10. Rahmatullaev M.M., Tukhtabaev A.M. Boundedness of *p*-adic ART quasi Gibbs measures. // « Sarymsakov readings ». Republic scientific conference, Tashkent, September 16-18, 2021, – pp. 282-283.
11. Rahmatullaev M.M., Tukhtabaev A.M. On $G_k^{(2)}$-periodic p-adic generalized Gibbs measure for Ising model on the Cayley tree. // «Modern problems of applied mathematics and information technologies». International scientific and practical conference, Bukhara, May 11-12, 2022, – pp. 38-39.
12. Rahmatullaev M.M., Tukhtabaev A.M. On $G_k^{(2)}$-periodic *p*-adic generalized Gibbs measure for Ising model on a Cayley tree // «Operator algebras, non-associative structures and related problems». Republic scientific conference, Tashkent, September 14-15, 2022, – pp. 298-299.
13. Rahmatullaev M.M., Tukhtabaev A.M. p-adic analogue of the Bleher-Ganikhodjaev construction. // «Mathematics, mechanics and intellectual technologies., Tashkent-2023». Republic scientific and practical conference, Tashkent, March 28-29, 2023, – pp. 77-78.

Avtoreferat "O'zbekiston matematika jurnali" tahririyatida
2023-yil 16 - oktabrda tahrirdan o'tkazildi.

Bosmaxona litsenziyasi:

9338

Bichimi: 84x60 $^{1}/_{16}$. «Times New Roman» garniturasi.
Raqamli bosma usulda bosildi.
Shartli bosma tobog'i: 2,75. Adadi 100 dona. Buyurtma № 53/22.

Guvohnoma № 851684.
«Tipograff» MChJ bosmaxonasida chop etilgan.
Bosmaxona manzili: 100011, Toshkent sh., Beruniy ko'chasi, 83-uy.